

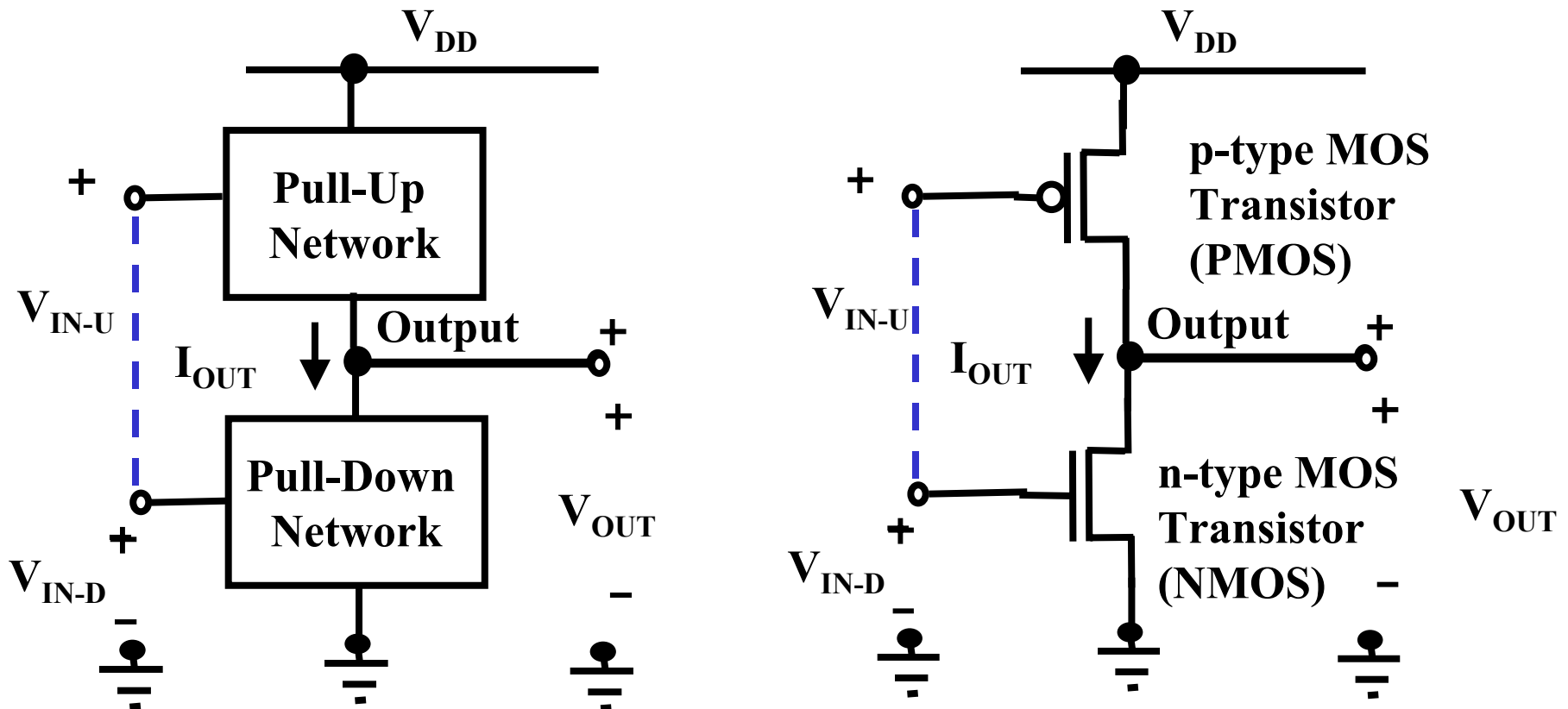
# Static and Transient Analysis of Gates

- A) Detailed Recap of Static Gate Analysis  
Transistor CMOS Inverter Example  
Terminology,  $I_{OUT-SAT-D}$ , VTC,  $V_M$
- B) **Break to Discuss Quiz and Midterm**
- C) Transient Gate Analysis
- D) Switch Resistor ( $R_{EQ}$ ) Approx. Model
- E) Logic Block and  $0.69R_{EQ}C$  Worst Case Inputs

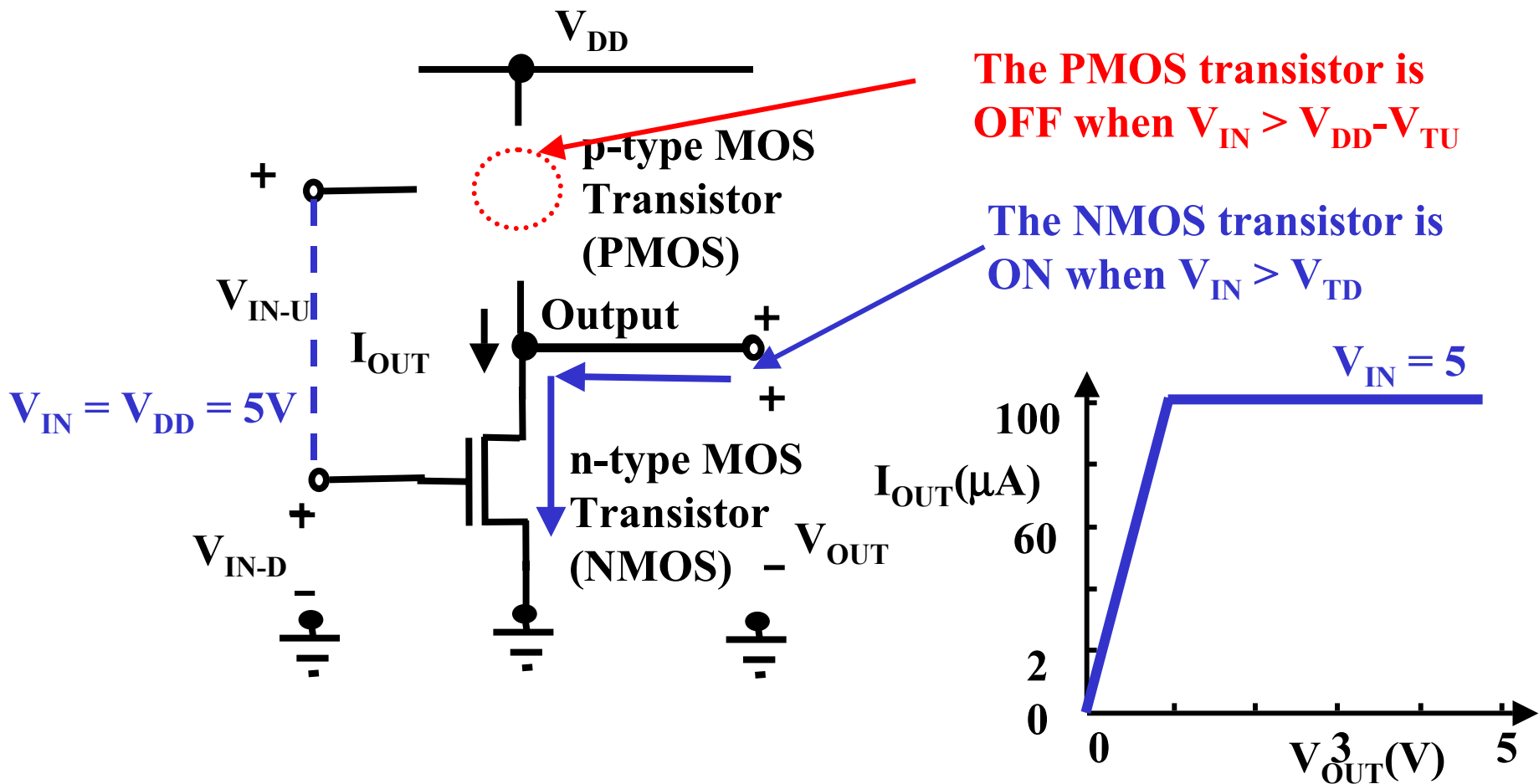
**Reading: 523-525, 604-611, (only the loadline methods) and lecture handouts**

# Transistor Inverter Example

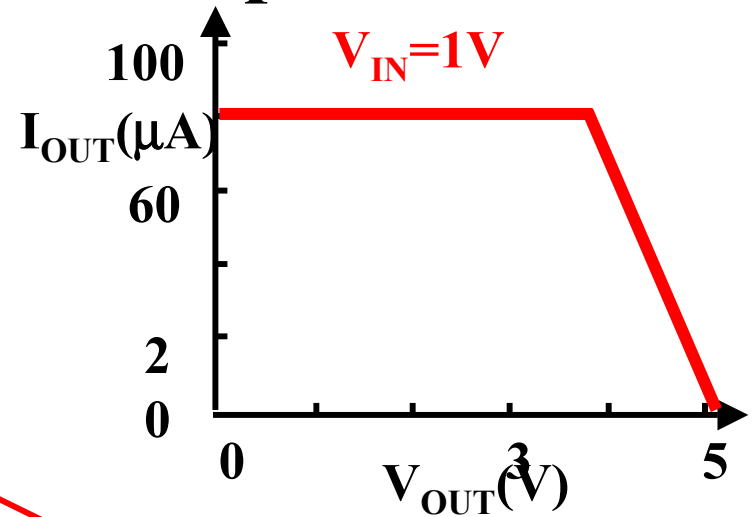
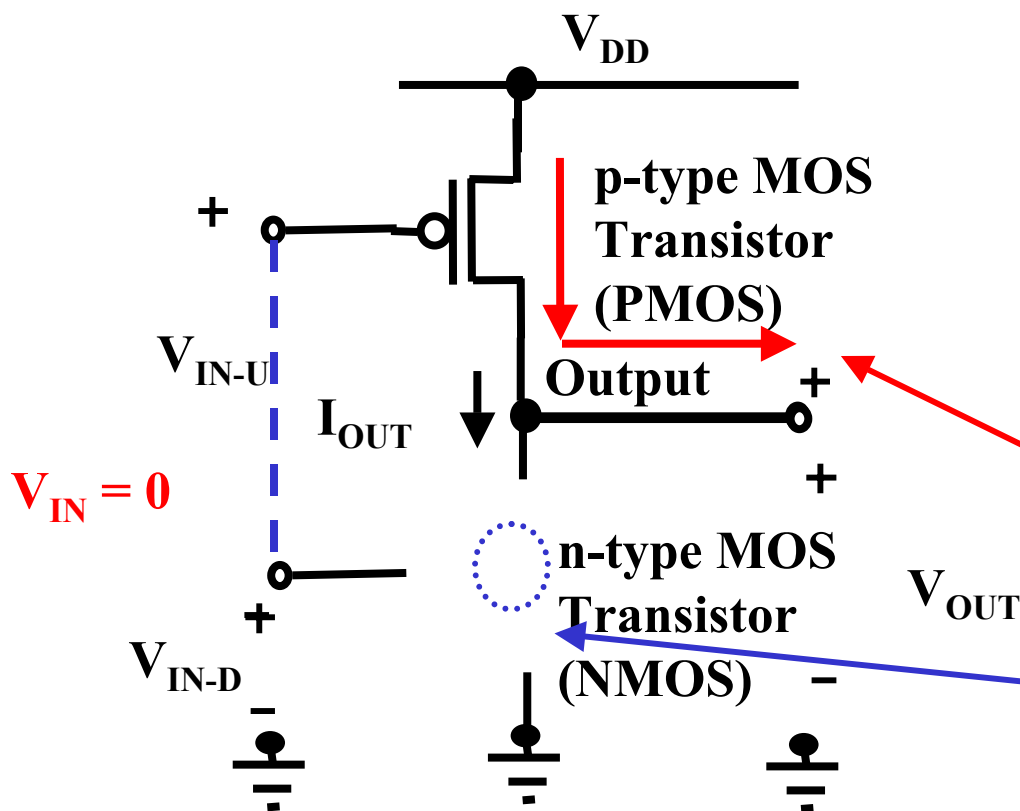
It may be simpler to just **think of PMOS and NMOS transistors** instead of a general 3 terminal pull-up or pull-down devices or networks.



# Case #1: $V_{IN} = V_{DD} = 5V$ The Output is Pulled-Down



## Case #2: $V_{IN} = 0$ The Output is Pulled-Up



The PMOS transistor is ON when  $V_{IN} < V_{DD} - V_{TU}$

The NMOS transistor is OFF when  $V_{IN} < V_{TD}$

# Terminology

$V_{DD}$  = Power supply voltage (D is from Drain)

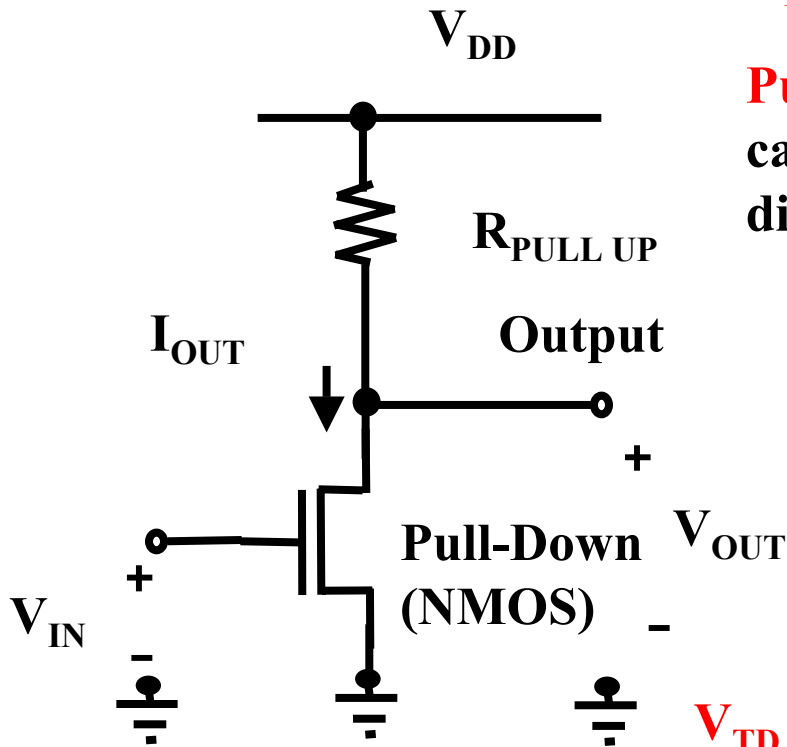
**Pull-Down Network** = Set of devices used to carry current from the output node to ground to discharge the output node to ground.

**Pull-Up Network** = Set of devices used to carry current from the power supply to the output node to charge the output node to the power supply voltage.

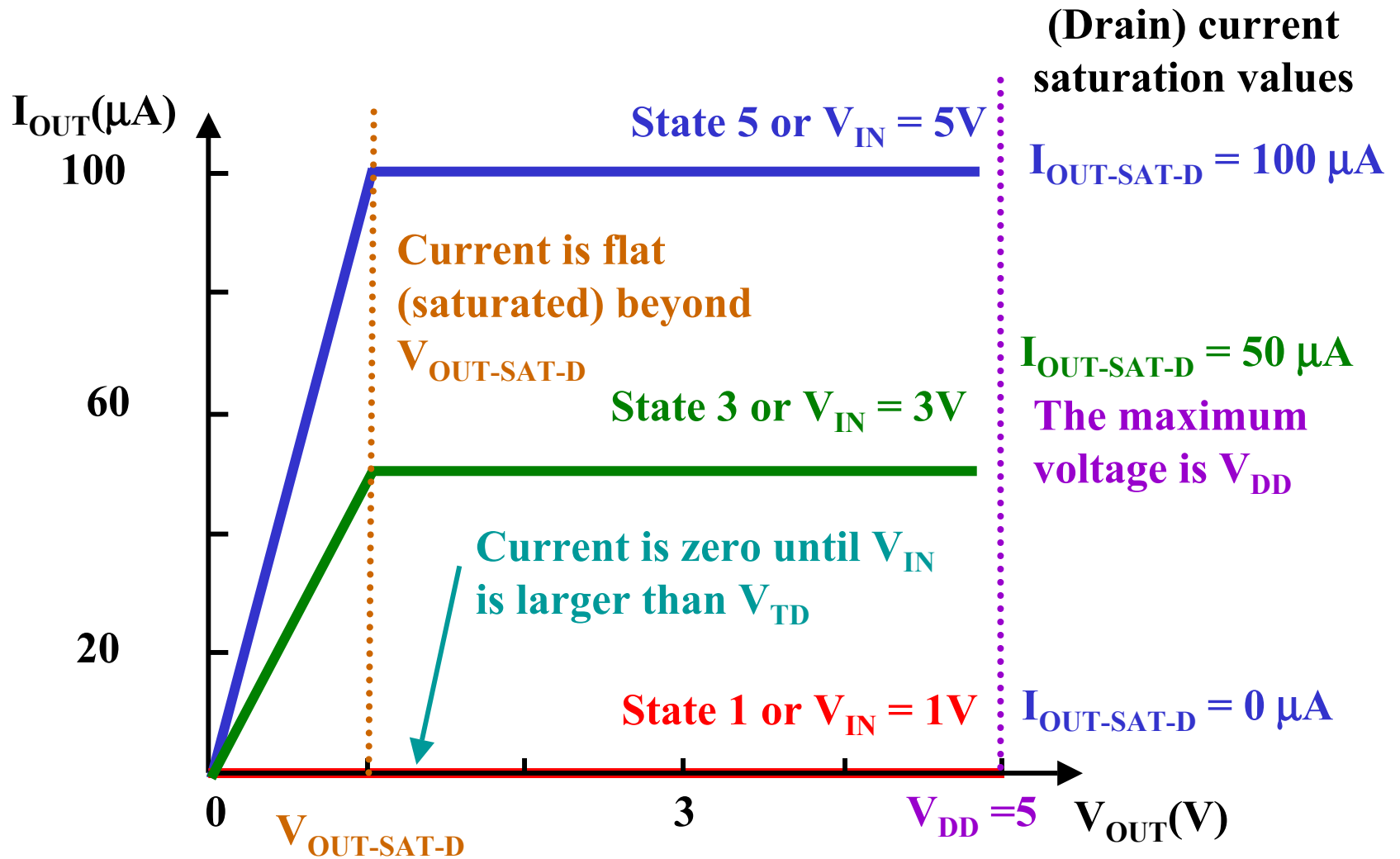
$I_{OUT}$  = Current for the device under study.

$V_{TD}$  = Threshold Voltage value of  $V_{IN}$  at which the Pull-Down (**NMOS** transistor) begins to conduct.

$V_{OUT-SAT-D}$  = Value of  $V_{OUT}$  beyond which the current  $I_{OUT-D}$  saturates at the (drain) current saturation value  $I_{OUT-SAT-D}$ .



# States are Voltage Levels of $V_{IN}$



## Saturation Current NMOS Model

Current  $I_{OUT}$  only flows when  $V_{IN}$  is larger than the threshold value  $V_{TD}$  and the current is proportional to  $V_{OUT}$  up to  $V_{OUT-SAT-D}$  where it reaches the saturation current

$$I_{OUT-SAT-D} = k_D (V_{IN} - V_{TD}) V_{OUT-SAT-D}$$

Note that we have added an extra parameter to distinguish between threshold ( $V_{TD}$ ) and saturation ( $V_{OUT-SAT-D}$ ).

Example:

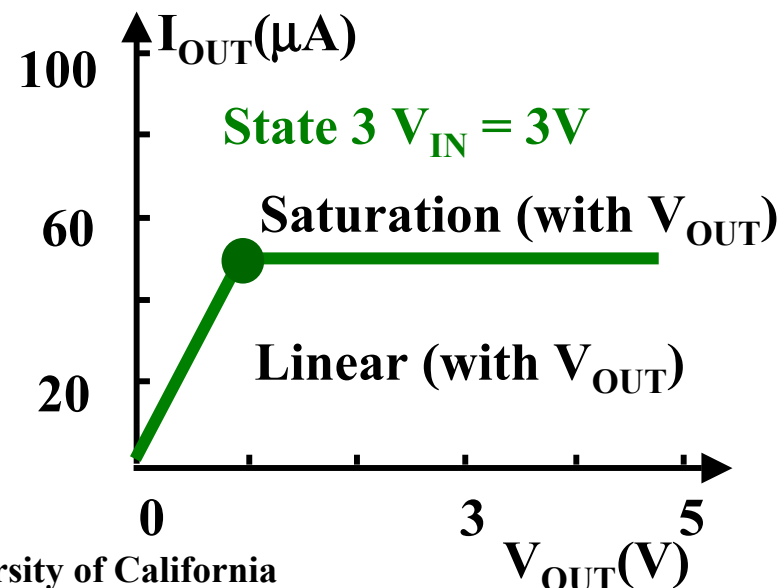
$$k_D = 25 \mu\text{A}/\text{V}^2$$

$$V_{TD} = 1\text{V}$$

$$V_{OUT-SAT-D} = 1\text{V}$$

Use these  
values in the  
homework.

$$I_{OUT-SAT-PD} = 25 \frac{\mu\text{A}}{\text{V}^2} (3\text{V} - 1\text{V}) 1\text{V} = 50 \mu\text{A}$$



## Saturation Current PMOS Model

Current  $I_{OUT}$  only flows when  $V_{IN}$  is smaller than  $V_{DD}$  minus the threshold value  $V_{TU}$  and the current is proportional to  $(V_{DD}-V_{OUT})$  up to  $(V_{DD}-V_{OUT-SAT-U})$  where it reaches the saturation current

$$I_{OUT-SAT-U} = k_U (V_{DD} - V_{IN} - V_{TU}) V_{OUT-SAT-U}$$

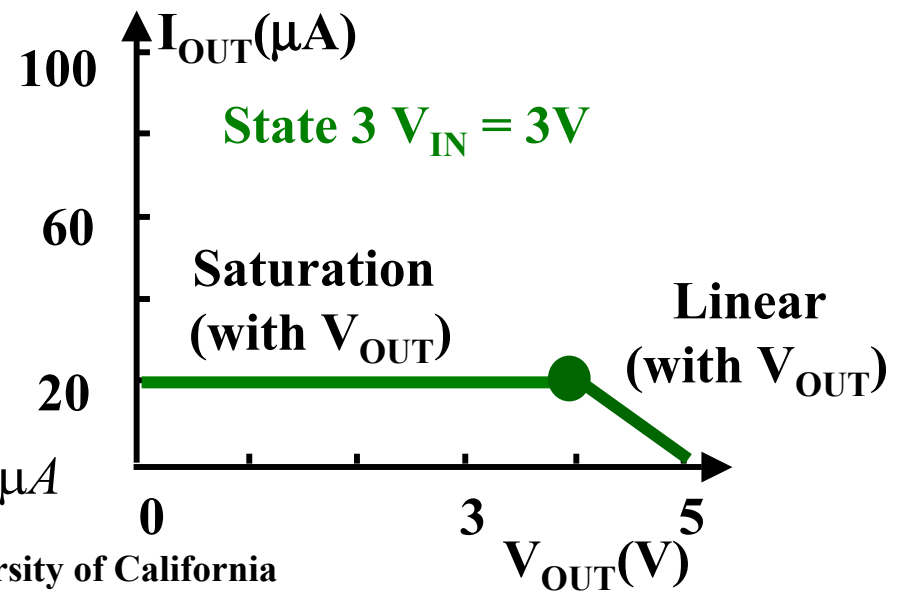
**Example:**

$$k_U = 20 \mu A/V^2$$

$$V_{TU} = 1V$$

$$V_{OUT-SAT-U} = 1V$$

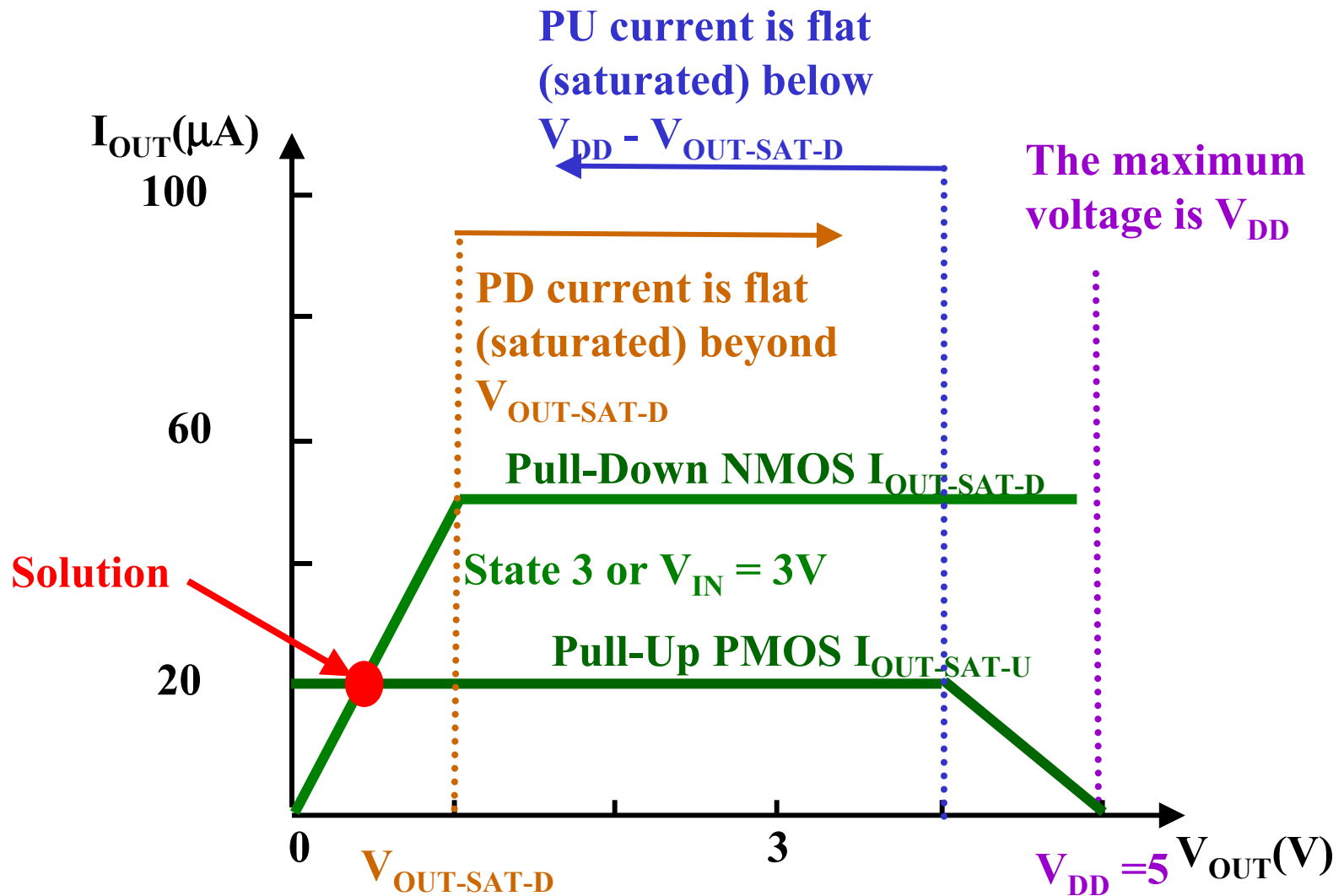
**Use these values in the homework.**



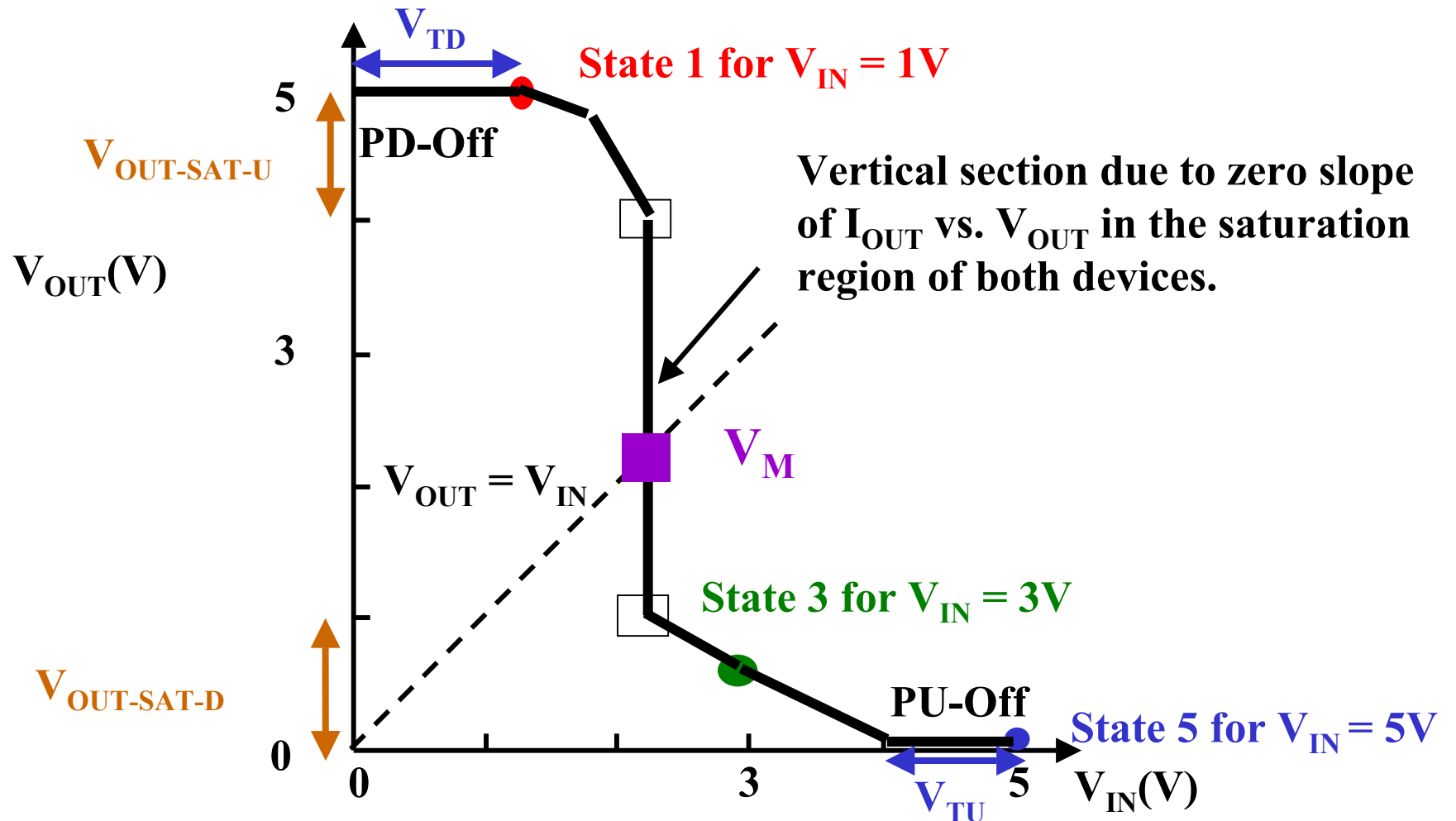
$$I_{OUT-SAT-U} = 20 \frac{\mu A}{V^2} (5V - 3V - 1V) 1V = 20 \mu A$$



# Composite $I_{OUT}$ vs. $V_{OUT}$ for CMOS



# Voltage Transfer Function for the Complementary Logic Circuit



## Method for Finding $V_M$

At  $V_M$ ,

1)  $V_{OUT} = V_{IN} = V_M$

2) Both devices are in saturation

3)  $I_{OUT-SAT-D} = I_{OUT-SAT-U}$

$$I_{OUT-SAT-D} = k_D (V_{IN} - V_{TD}) V_{OUT-SAT-D}$$

$$= I_{OUT-SAT-U} = k_U (V_{DD} - V_{IN} - V_{TU}) V_{OUT-SAT-U}$$

Substitute  $V_M$

**Solve for  $V_M$**

**Example Result:** When  $k_D = k_P$ ,  $V_{OUT-SAT-D} = V_{OUT-SAT-U}$   
and  $V_{TD} = V_{TU}$ , then  $V_M = V_{DD}/2$

## Lecture 17: October 29, 2001

# Reminder: Quiz and Midterm

**Quiz 20 minutes At Start of Class Wed. Oct 31**

**Covers Material 6<sup>th</sup>-9<sup>th</sup> week including HW#9**

**Midterm in Class Wed. Nov 7<sup>th</sup>**

**Covers Material 6<sup>th</sup>-10<sup>th</sup> week including HW#10**

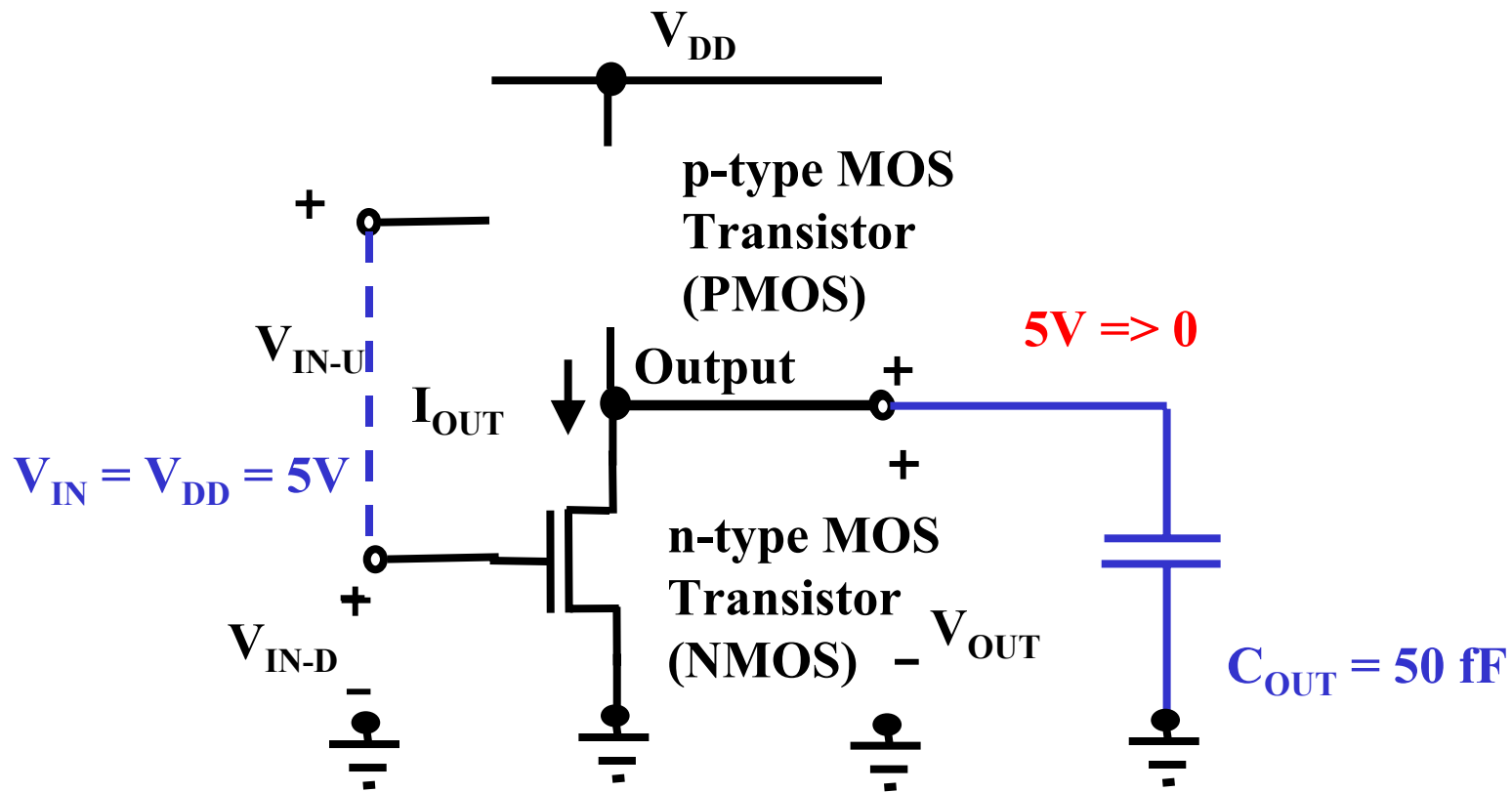
**Closed Book, Closed Notes, Bring Calculator, Paper Provided  
Last Name A-K 2040 Valley LSB; Last Name L-Z in 10 Evans**

**Topic Coverage Review in class Oct 31; Old Exams on Web**

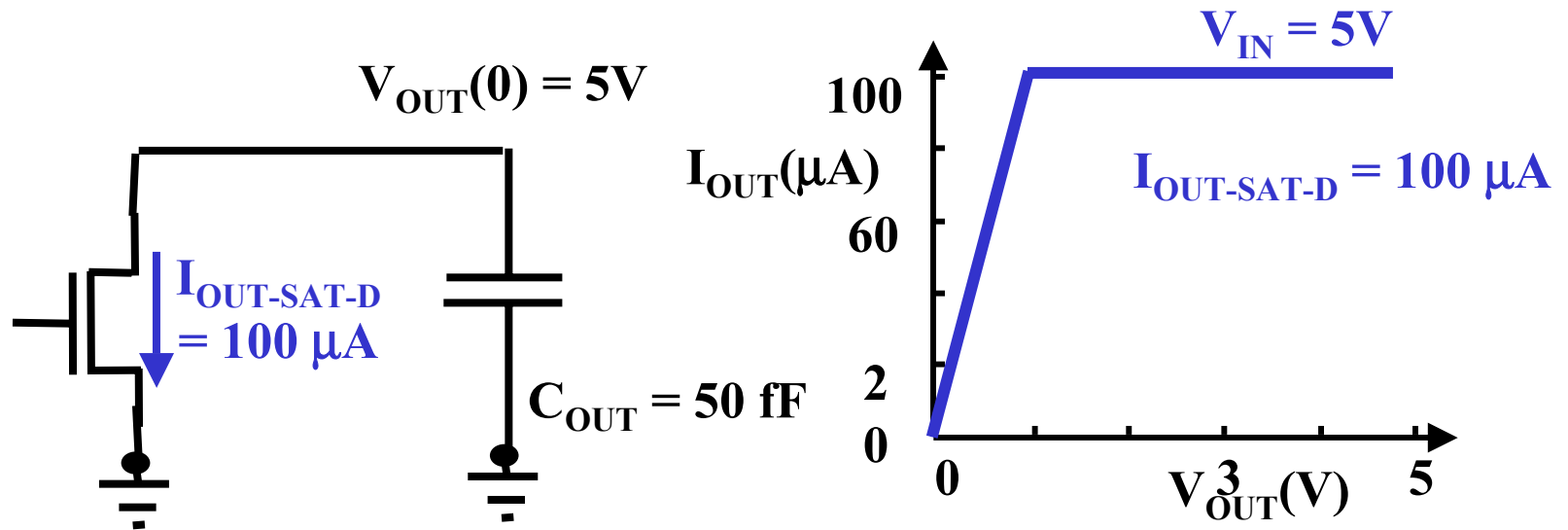
**Review Session: Sat 1-2:30 (TBA Evans); Tu 5-6:00 (? Cory)**

**EE 43 Labs Are **Not** Cancelled:**

# Transient Gate Problem: Discharging and Charging Capacitance on the Output



# Output Capacitance Voltage vs. Time



When  $V_{OUT} > V_{OUT-SAT-D}$  the available current is  $I_{OUT-SAT-D}$

Assume that the necessary voltage swing to cause the next downstream gate to begin to switch is  $V_{DD}/2$  or 2.5V. The **propagation delay** is thus

$$\Delta t = \frac{C_{OUT} \Delta V}{I_{OUT-SAT-D}} = \frac{C_{OUT} V_{DD}}{2 I_{OUT-SAT-D}} = \frac{50 \text{ fF} \cdot 2.5 \text{ V}}{100 \mu\text{A}} = 1.25 \text{ ns}$$

# Switched Equivalent Resistance Model

The above model assumes the device is an ideal constant current source.

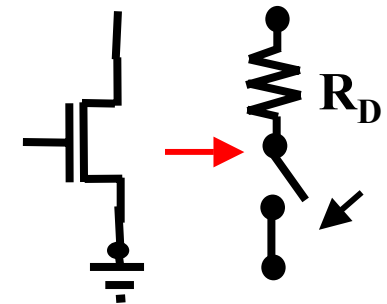
- 1) This is not true below  $V_{OUT-SAT-D}$  and leads to inaccuracies.
- 2) Combining ideal current sources in networks with series and parallel connections is problematic.

Instead define an equivalent resistance for the device by setting  $0.69R_D C$  equal to the  $\Delta t$  found above

$$\Delta t = \frac{C_{OUT} V_{DD}}{2I_{OUT-SAT-D}} = 0.69R_D C_{OUT}$$

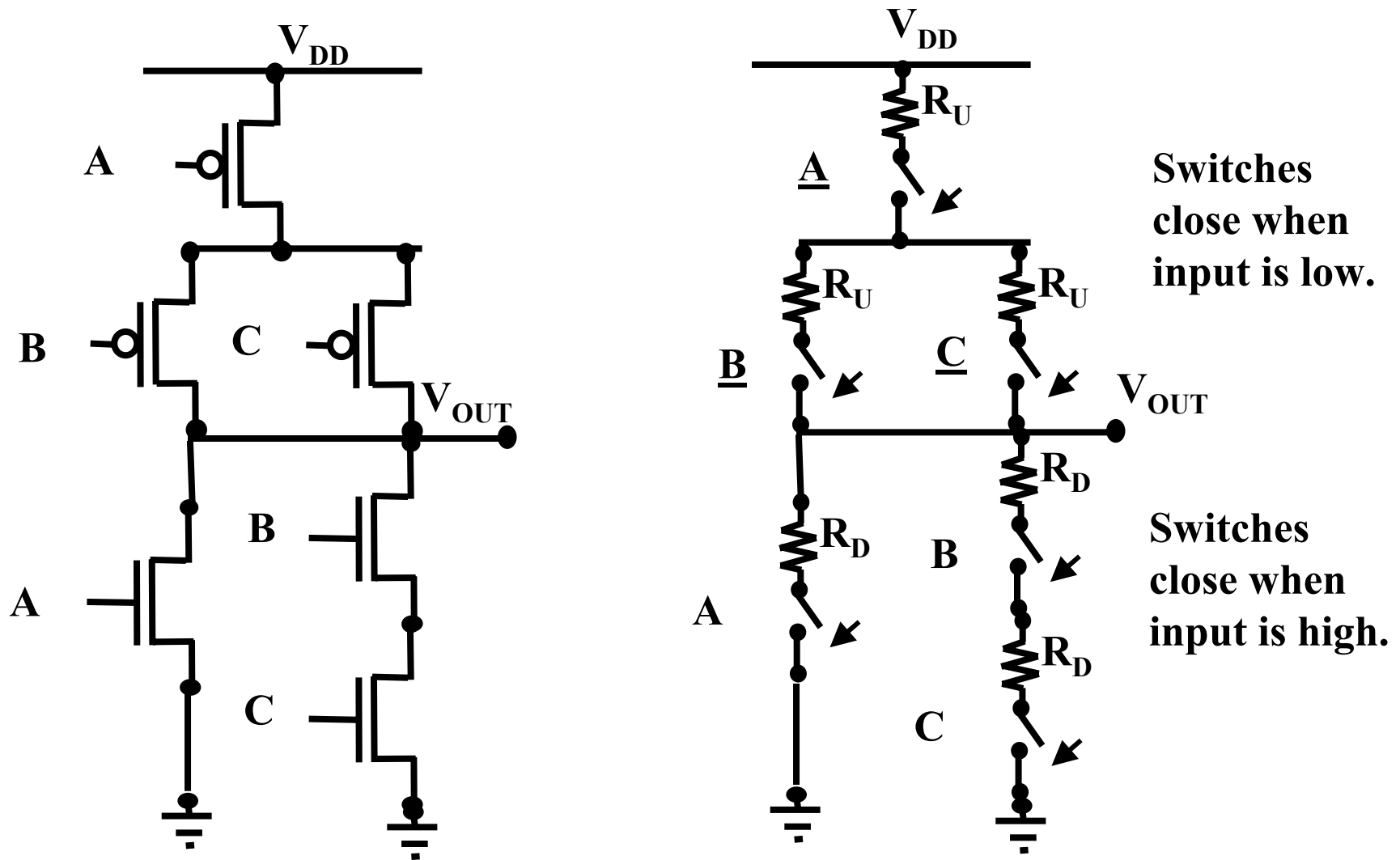
This gives

$$R_D = \frac{V_{DD}}{2 \cdot (0.69) I_{OUT-SAT-D}} \approx \frac{3}{4} \frac{V_{DD}}{I_{OUT-SAT-D}} = \frac{3}{4} \frac{5V}{100\mu A} = 37.5k\Omega$$



Each device can now be replaced by this equivalent resistor.

# Switched Equivalent Resistance Network





# Switched Equivalent Resistance Values

The resistor values depend on the properties of silicon, geometrical layout, design style and technology node.

n-type silicon has a carrier mobility that is 2 to 3 times higher than p-type.

The resistance is inversely proportion to the gate width/length in the geometrical layout.

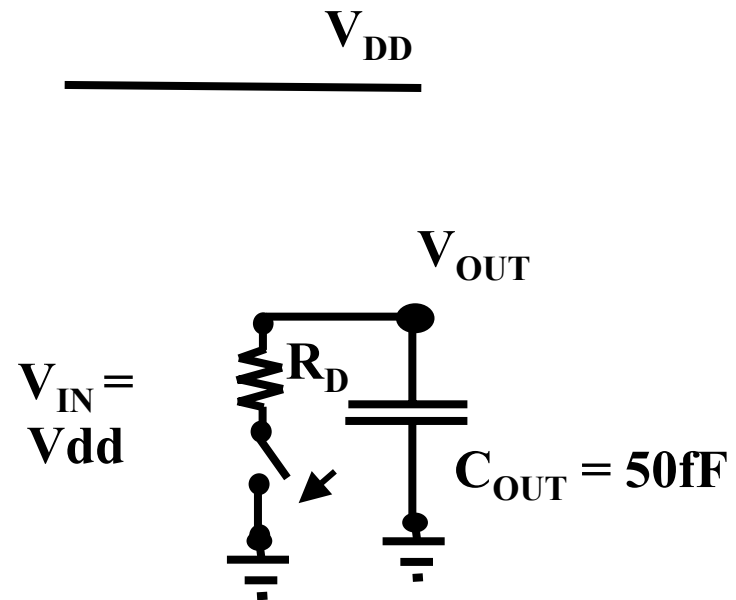
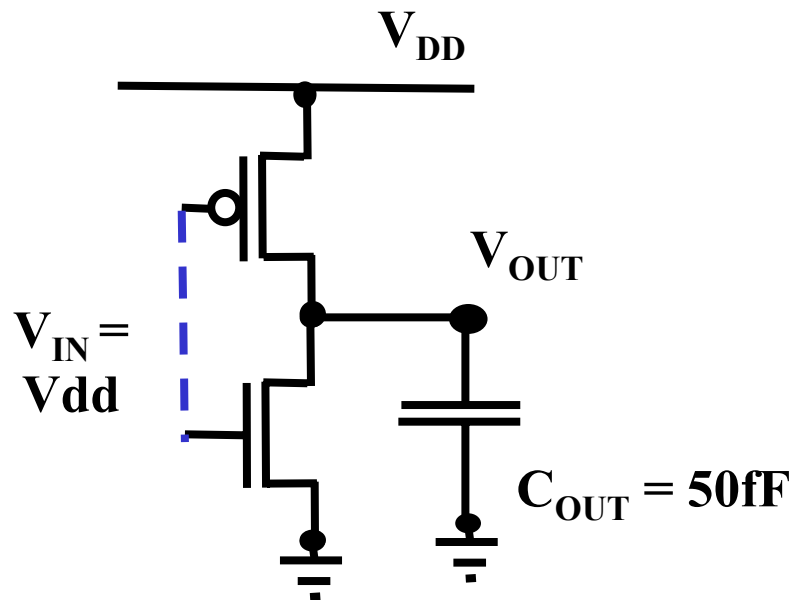
Design styles may restrict all NMOS and PMOS to be of a predetermined fixed size.

The current per unit width of the gate increases nearly inversely with the linewidth.

**For convenience in EE 42 we assume  $R_D = R_U = 10 \text{ k}\Omega$**

# Inverter Propagation Delay

Discharge (pull-down)

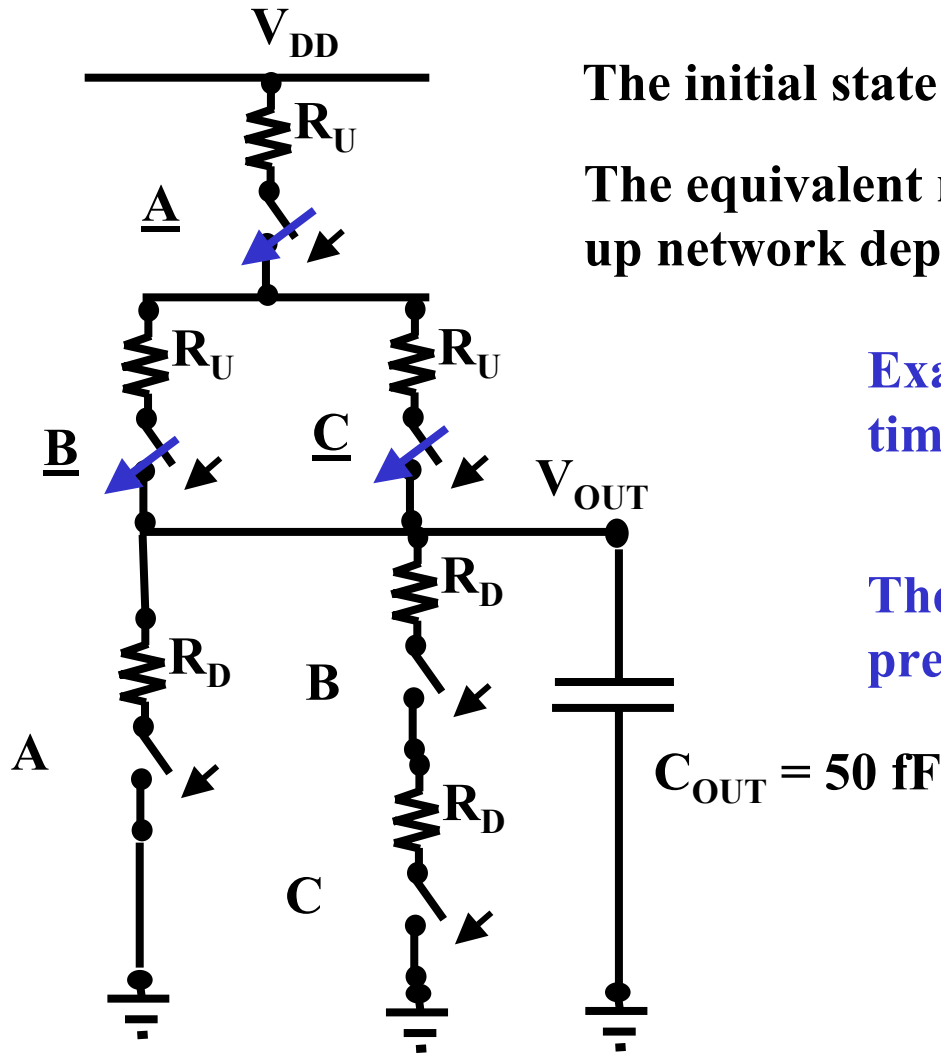


$$\Delta t = 0.69R_D C_{OUT} = 0.69(10\text{k}\Omega)(50\text{fF}) = 345 \text{ ps}$$

Discharge (pull-up)

$$\Delta t = 0.69R_U C_{OUT} = 0.69(10\text{k}\Omega)(50\text{fF}) = 345 \text{ ps}$$

# Logic Gate Propagation Delay



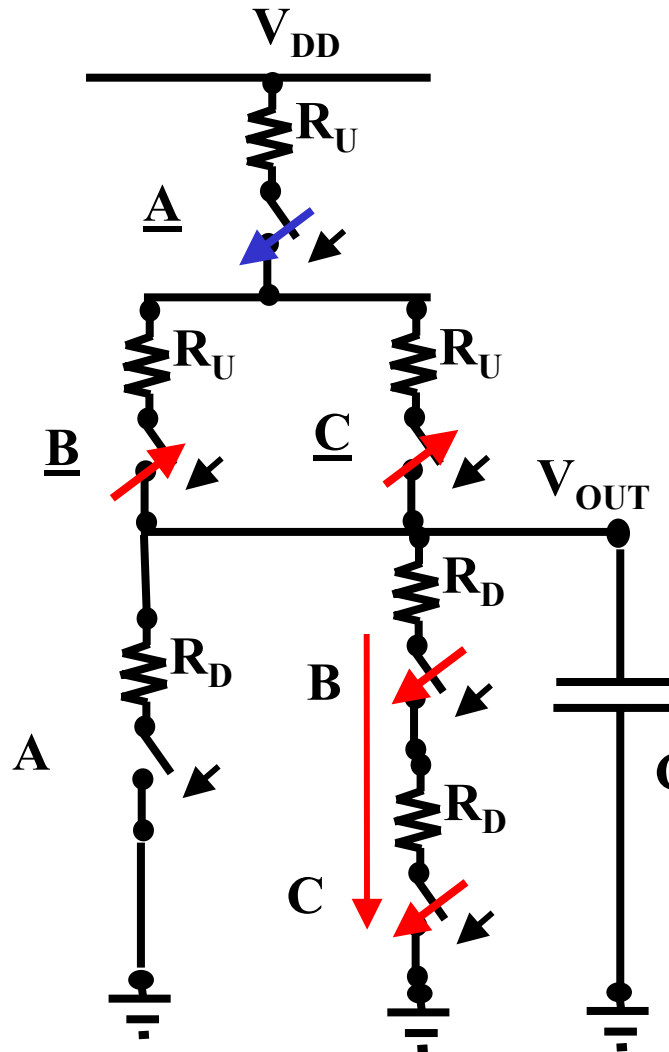
The initial state depends on the previous inputs.

The equivalent resistance of the pull-down or pull-up network depends on the current input state.

Example:  $A=0, B=0, C=0$  for a long time.

The capacitor has precharged up to  $V_{DD} = 5V$ .

# Logic Gate Propagation Delay (Cont.)



At  $t=0$ , B and C switch to high =  $V_{DD}$  and A remains low.

$C_{OUT}$  discharges through the pull-down resistance of gates B and C in series.

$$\Delta t = 0.69(R_{DB} + R_{DC})C_{OUT} = 0.69(20\text{k}\Omega)(50\text{fF}) = 690 \text{ ps}$$

The propagation delay is **two times longer** than that for the inverter!