CIRCUIT ANALYSIS CONTINUED

Lecture 6 review:

• Formal nodal analysis (including floating Voltage sources)
• Voltage divider example (generalized)

Today:

• Examples of circuits and nodal analysis
• Parallel resistors
• Current dividers
• Simplifying circuits
• Real voltmeters
• Real ammeters
• Series and parallel capacitors

NODAL ANALYSIS EXAMPLE

Find \( V_a \), \( V_b \) if \( R_1 = R_2 = R_3 = R_4 = 1 \text{M} \Omega \), and \( V_1 = V_4 = 1.5 \text{V} \) with \( V_{\text{LL}} = 1 \text{V} \)

Solution: At supernode enclosing nodes \( a \) and \( b \):

\[
\frac{(V_1 - V_a)}{R_1} - \frac{V_a}{R_2} = \frac{V_{\text{LL}}}{R_3}
\]

and

\[ V_b = \text{_______} \]

Thus:

\[ V_a = \text{_______} \]

Be sure to check

\[ V_b = \text{_______} \]

answer with KCL!
CHECK ANSWER WITH KCL

Is \( V_a = 0.25 \) and \( V_b = 1.25 \) if \( R_1 = R_2 = R_3 = R_4 = 1 \text{M}\Omega \), and \( V_1 = V_4 = 1.5 \text{V} \) with \( V_{LL} = 1 \text{V} \)?

KCL at the Supernode: \( 0.25 - 1.25 + 1.25 - 0.25 = 0 \)

Clearly the current into the supernode is zero and we have verified that the solution is correct.

RESISTORS IN PARALLEL

1 Select Reference Node
2 Define unknown node voltages

Note: \( I_{ss} = I_1 + I_2 \), i.e.,

\[
I_{ss} = \frac{V_X}{R_1} + \frac{V_X}{R_2} \Rightarrow V_X = I_{ss} \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = I_{ss} \cdot \frac{R_1 R_2}{R_1 + R_2}
\]

RESULT 1 EQUIVALENT RESISTANCE: \( R_e \parallel R_1 \parallel R_2 = \text{?} \)

RESULT 2 CURRENT DIVIDER:

\[
I_1 = \frac{V_X}{R_1} = I_{ss} \times \text{?} \\
I_2 = \frac{V_X}{R_2} = I_{ss} \times \text{?}
\]
IDENTIFYING SERIES AND PARALLEL COMBINATIONS
Use series/parallel equivalents to simplify a circuit before starting KVL/KCL

\[ R_1 = R_2 = 10 \, \text{K} \]
\[ R_3 = 20 \, \text{K} \]
\[ R_4 = 5 = 5 \, \text{K} \]
\[ R_6 = 10 \, \text{K} \]

\[ R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \]

Please note the order of math operators here!

IDENTIFYING SERIES AND PARALLEL COMBINATIONS (cont.)
Some circuits must be analyzed (not amenable to simple inspection)

\[ R_1 \text{ and } R_2 \text{ are not in } || \]
\[ R_1 \text{ and } R_6 \text{ are not in series} \]

Special cases:
\[ R_3 = 0 \text{ OR } R_3 = \infty \]

Example: \[ R_3 = 0 \Rightarrow R_1 \parallel R_2; R_4 \parallel R_5 \text{ in series}; R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}} \]

OR IF \[ R_3 = \infty \Rightarrow \]
REAL VOLTMETERS

Concept of “Loading” as Application of Parallel Resistors

How is voltage measured? Modern answer: Digital multimeter (DMM)

Problem: Connecting leads from voltmeter across two nodes changes the circuit. The voltmeter is characterized by how much current it draws at a given voltage → “voltmeter input resistance,” $R_{in}$. Typical value: 10 MΩ. Let’s do an example; measure $V$ in voltage divider:

$$V_2 = V_{SS} \left( \frac{R_2}{R_1 + R_2} \right)$$

Example: $V_{SS} = 10V$, $R_2 = 100K$, $R_1 = 900K \Rightarrow V_2 = 1V$

But if $R_{in} = 10M$, $V_2' = 0.991V$, a 1% error
MEASURING CURRENT

Insert DMM (in current measurement mode) into circuit. But ammeters disturb the circuit. (Note: Ammeters are characterized by their “ammeter input resistance,” $R_{in}$. Ideally this should be very low. Typical value (in mA range) $1\,\Omega$.)

Potential measurement error due to non-zero input resistance:

$$I = \frac{V_1}{R_1 + R_2}$$

Example $V = 1\,\text{V}$; $R_1 + R_2 = 1\,\text{K}\Omega$, $R_{in} = 1\,\Omega$

$$I = 1\,\text{mA}, \quad I_{\text{meas}} = \frac{1}{1\,\text{K} + 1\,\Omega} = 0.999\,\text{mA} \quad (0.1\% \text{ error})$$

CAPACITORS IN SERIES

Equivalent capacitance defined by

$$V_{eq} = V_1 + V_2 \quad \text{and} \quad i = C_{eq} \frac{dV_{eq}}{dt} = C_{eq} \frac{d(V_1 + V_2)}{dt}$$

Clearly, $C_{eq} = \frac{i}{C_{eq}}$
CAPACITORS IN PARALLEL

Equivalent capacitance defined by
\[ i = C_{eq} \frac{dV}{dt} \]

Clearly, \[ C_{eq} = \quad \text{CAPACITORS IN PARALLEL} \]