EECS 42 Introduction to Electronics for Computer Science

Andrew R. Neureuther

Lecture #3 KCL, KVL, Circuit Elements

- **Kirchhoff Current Law (and Bag case)**
- **Kirchhoff Voltage Law**
- **Circuit elements symbols and I vs. V graphs**

Oldham and Schwarz: 2.1-2.2

http://inst.EECS.Berkeley.EDU/~ee42/

WHAT IF THE NET CURRENT WERE NOT ZERO?

Suppose imbalance in currents is 1μA = 1 μC/s (net current entering node)

Assuming that q = 0 at t = 0, the charge increase is 10^{-6} C each second

or $10^{-6}/1.6\times10^{-19}=6\times10^{12}$ charge carriers each second

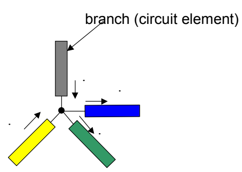
But by definition, the capacitance of a node to ground is ZERO because we show any capacitance as an explicit circuit element (branch). Thus, the voltage would be infinite (Q = CV).

Something has to give! In the limit of zero capacitance the accumulation of charge would result in infinite electric fields … there would be a spark as the air around the node broke down.

Charge is transported around the circuit branches (even stored in some branches), but it doesn't pile up at the nodes!

KIRCHHOFF'S CURRENT LAW

Circuit with several branches connected at a node:



(Sum of currents <u>entering</u> node) – (Sum of currents <u>leaving</u> node) = 0

Alternative statements of KCL

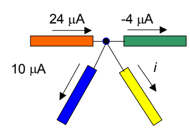
1 "Algebraic sum" of currents ***entering*** node = 0

where "algebraic sum" means currents leaving are included with a minus sign

2 "Algebraic sum" of currents leaving node = 0

where currents entering are included with a minus sign

KIRCHHOFF'S CURRENT LAW EXAMPLE



Currents entering the node: 24 μA

Currents leaving the node: –4 μA + 10 μA + i

24 = 10 + (–4) + i

i = 18 μA

Three statements of KCL

$$\sum_{IN} i_{in} = \sum_{OUT} i_{out} \qquad 24 = -4 + 10 + i \quad \Rightarrow \quad i = 18\ \mu A$$

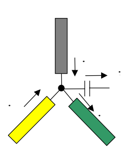
$$\sum_{ALL} i_{in} = 0 \qquad 24 - (-4) - 10 - i = 0 \quad \Rightarrow \quad i = 18\ \mu A$$

$$\sum i_{out} = 0 \qquad -24 - 4 + 10 + i = 0 \quad \Rightarrow \quad i = 18\ \mu A$$

EQUIVALENT

KIRCHHOFF"S CURRENT LAW WITH A CAPACITOR AT A NODE

Circuit with several branches, including a capacitor

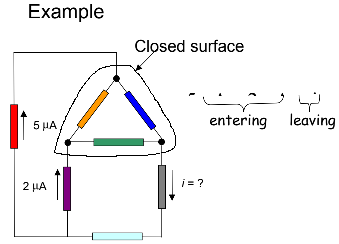


q = charge stored at node is zero. If charge *is* stored, for example in the capacitor shown as branch 3, the charge is accounted for as the time-integral of i_3. Thus the charge is not over at the node; it is on the capacitor.

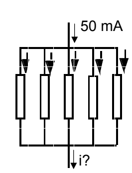
(Sum of currents <u>entering</u> node) – (Sum of currents <u>leaving</u> node) = 0

KIRCHHOFF'S CURRENT LAW USING SURFACES

Example



Another example

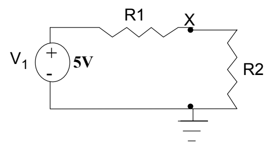


Example of the use of KCL

At node X:

Current into X from the left:
$(V_1 - v_X)/R1$

Current out of X to the right:
$v_X/R2$



R1 = 1 kΩ R2 = 2kΩ

KCL: $(V_1 - v_X)/R1 = v_X/R2$

Given V_1, This equation can be solved for v_X

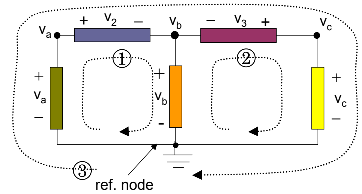
$v_X = V_1\, R2/(R1 + R2)$

Of course we just get the same result as we obtained from our series resistor formulation. (Find the current and multiply by R2)

KVL EXAMPLE

Examples of Three closed paths:

①, ②, ③



Note that:
$v_2 = v_a - v_b$
$v_3 = v_c - v_b$

Path 1:
$-v_a + v_2 + v_b = 0$
↑
$v_a - v_b$
YEP!

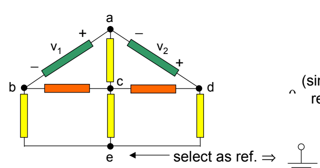
Path 2:
$-v_b - v_3 + v_c = 0$

Path 3:
$-v_a + v_2 - v_3 + v_c = 0$

$\mathbf{V_a = 5V \quad V_b = 3V \quad V_3 = 1V}$

BRANCH AND NODE VOLTAGES

The voltage across a circuit element is defined as the difference between the node voltages at its terminals



^ (since it's the reference)

Specifying node voltages: Use one node as the implicit reference (the "common" node … attach special symbol to label it)

Now single subscripts can label voltages:

e.g., v_b means $v_b - v_e$, v_a means $v_a - v_e$, etc.

BASIC CIRCUIT ELEMENTS

- Voltage Source (always supplies some constant given voltage - like ideal battery)
- Current Source (always supplies some constant given current)
- Resistor (Ohm's law)
- Wire ("short" – no voltage drop)

Lecture #4

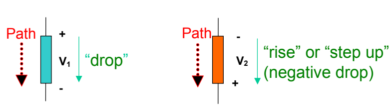
- Capacitor (capacitor law – based on energy storage in electric field of a dielectric S&O 5.1)
- Inductor (inductor law – based on energy storage in magnetic field in space S&O 5.1)

KIRCHHOFF'S VOLTAGE LAW (KVL)

The algebraic sum of the "voltage drops" around any "closed loop" is zero.

Why? We must return to the same potential (conservation of energy).

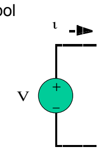
Voltage drop → defined as the branch voltage if the + sign is encountered first; it is (-) the branch voltage if the – sign is encountered first … important bookkeeping



Closed loop: Path beginning and ending on the same node

IDEAL VOLTAGE SOURCE

Symbol



Note: The current and voltage are unassociated here.

Examples:

1) V = 3V

2) v = v(t) = 160 cos 377t

☞Special cases:

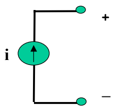
upper case V → constant voltage … called "DC"

lower case v → general voltage, may vary with time

Current through voltage source can take on *any* value (positive or negative) *but not infinite*

IDEAL CURRENT SOURCE

"Complement" or "<u>dual</u>" of the voltage source: Current though branch is fixed and independent of the voltage across the branch



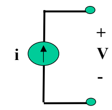
note **un**associated direction

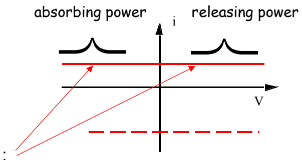
Actual current source examples – hard to find except in electronics (transistors, etc.), as we will see

upper-case I → DC (constant) value

lower-case implies current could be time-varying i(t)

CURRENT SOURCE I vs. V Graph





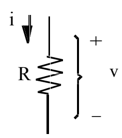
If i is positive then we are confined to quadrants 4 and 1:

Remember the voltage across the current source can be *any* finite value (not just zero)

When both I and V are negative is the current source absorbing or releasing power?

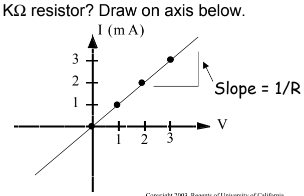
And do not forget i can be positive or negative. Thus we can be in any quadrant.

RESISTOR AND IT'S I vs. V Graph



We use associated current and voltage (i.e., i is defined as into + terminal), then v = iR (Ohm's law).

Question: What is the current versus voltage (I vs. V) characteristic for a 1KΩ resistor? Draw on axis below.



☞Answer: V = 0 ⇒ I = 0

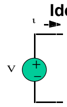
V = 1V ⇒ I = 1 mA
V = 2V ⇒ I = 2 mA

etc

VOLTAGE SOURCE I vs V Graph

Describe a two-terminal circuit element by plotting current vs. voltage

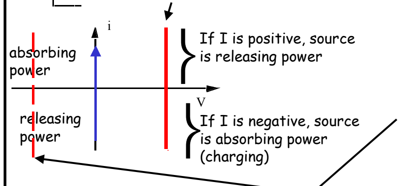
Ideal voltage source



Assume **un**associated signs

What is the I vs. V graph for an ideal wire?

If V is positive



But if V is negative