KIRCHHOFF’S CURRENT LAW

Circuit with several branches connected at a node:

(Sum of currents entering node) – (Sum of currents leaving node) = 0

Alternative statements of KCL
1. “Algebraic sum” of currents entering node = 0
   where “algebraic sum” means currents leaving are included with a minus sign
2. “Algebraic sum” of currents leaving node = 0
   where currents entering are included with a minus sign

KIRCHHOFF’S CURRENT LAW USING SURFACES

Example

Another example

50 mA

KIRCHHOFF’S CURRENT LAW

q = charge stored at node is zero. If charge is stored, for example in the capacitor shown as branch 3, the charge is accounted for as the time-integral of i. Thus the charge is not over at the node; it is on the capacitor.

(Sum of currents entering node) – (Sum of currents leaving node) = 0

WHAT IF THE NET CURRENT WERE NOT ZERO?

Suppose imbalance in currents is 1 µA = 1 µC/s (net current entering node)
Assuming that q = 0 at t = 0, the charge increase is 10⁻⁴ C each second or 10⁻⁴ / 1.6 x 10⁻¹⁸ = 6 x 10¹³ charge carriers each second

But by definition, the capacitance of a node to ground is ZERO because we show any capacitance as an explicit circuit element (branch). Thus, the voltage would be infinite (Q = CV).

Something has to give! In the limit of zero capacitance the accumulation of charge would result in infinite electric fields ... there would be a spark as the air around the node broke down.

Charge is transported around the circuit branches (even stored in some branches), but it doesn’t pile up at the nodes!
Example of the use of KCL

At node X:

Current into X from the left:
\[(V_1 - v_X)/R_1\]

Current out of X to the right:
\[v_X/R_2\]

KCL:
\[(V_1 - v_X)/R_1 = v_X/R_2\]

Given \(V_1\), this equation can be solved for \(v_X\)

Of course we just get the same result as we obtained from our series resistor formulation.

\(v_X = V_1 \times \frac{R_2}{R_1 + R_2}\)

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BRANCH AND NODE VOLTAGES

The voltage across a circuit element is defined as the difference between the node voltages at its terminals.

Specifying node voltages: Use one node as the implicit reference (the “common” node … attach special symbol to label it)

Now single subscripts can label voltages:

\[\text{e.g., } v_a \text{ means } v_a - v_b, \text{ etc.}\]

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KIRCHHOFF'S VOLTAGE LAW (KVL)

The algebraic sum of the “voltage drops” around any “closed loop” is zero.

Why? We must return to the same potential (conservation of energy).

Voltage drop \(\Delta\) defined as the branch voltage if the + sign is encountered first; it is (\(\times\)) the branch voltage if the – sign is encountered first … important bookkeeping

Closed loop: Path beginning and ending on the same node

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IDEAL VOLTAGE SOURCE

Note: The current and voltage are unassociated here.

Special cases:

\[v = v(t) = 160 \cos 377t\]

Current through voltage source can take on any value (positive or negative) but not infinite
IDEAL CURRENT SOURCE

"Complement" or "dual" of the voltage source: Current through branch is fixed and independent of the voltage across the branch.

Actual current source examples – hard to find except in electronics (transistors, etc.), as we will see.

upper-case $i$ $\rightarrow$ DC (constant) value
lower-case implies current could be time-varying $i(t)$

RESISTOR AND IT'S I vs. V Graph

We use associated current and voltage (i.e., $i$ is defined as into + terminal), then $v = iR$ (Ohm’s law).

VOLTAGE SOURCE I vs V Graph

Describe a two-terminal circuit element by plotting current vs. voltage

CURRENT SOURCE I vs. V Graph

When both $i$ and $v$ are negative is the current source absorbing or releasing power?

If $i$ is positive then we are confined to quadrants 4 and 1:
Remember the voltage across the current source can be any finite value (not just zero)
And do not forget $i$ can be positive or negative. Thus we can be in any quadrant.