EECS 42 Introduction to Electronics for Computer Science

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Lecture #4 Capacitors; I vs. V

- Capacitors and Energy Stored
- Current versus Voltage for Circuits
- Equivalent Straight Line Circuits
  - Thevenin; Norton

Schwarz and Oldham: 5.1 – 5.2; 3.1

http://inst.EECS.Berkeley.EDU/~ee42/

BASIC CIRCUIT ELEMENTS

- Voltage Source (always supplies some constant given voltage - like ideal battery)
- Current Source (always supplies some constant given current)
- Resistor (Ohm’s law)
- Wire (“short” – no voltage drop)

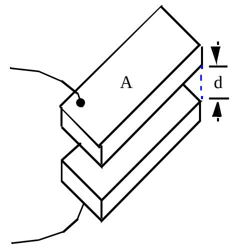
Lecture #4

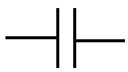
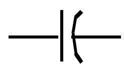
- Capacitor (capacitor law – based on energy storage in electric field of a dielectric S&O 5.1-5.2)
- Inductor (inductor law – based on energy storage in magnetic field in space S&O 5.1-5.2)

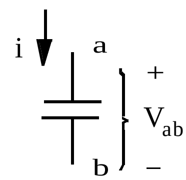
CAPACITOR CIRCUIT ELEMENT

Any two conductors a and b separated by an insulator with a difference in voltage V_{ab} will have an equal and opposite charge on their surfaces whose value is given by $Q = CV_{ab}$, where C is the **capacitance** of the structure, and the + charge is on the more positive electrode.

A simple *parallel-plate capacitor* is shown. If the area of the plate is A, the separation d, and the *dielectric constant* of the insulator is ε, the capacitance equals C = A ε/d.



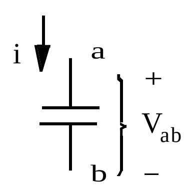
Symbol  or 



Constitutive relationship: $Q = C (V_a - V_b)$.

(Q is positive on plate a if $V_a > V_b$)

CAPACITOR CURRENT/VOLTAGE



Constitutive relationship: $Q = C (V_a - V_b)$.

(Q is positive on plate a if $V_a > V_b$)

Charge and Current are related: Charge is the time integral of Current

So $i = \frac{dQ_a}{dt}$ so $i = C\frac{dv}{dt}$ equivalent to $Q = C v$

where we use the *associated reference directions*.

Example: A 1 mA current into a 1 pF capacitance produces a voltage change rate of 1mA/1pF = 1 V/ns

ENERGY STORED IN A CAPACITOR

You might think the energy (in Joules) is QV, which has the dimension of joules. But during charging the average voltage was only half the final value of V.

Thus, energy is $\frac{1}{2}QV = \boxed{\frac{1}{2}CV^2}$.

Example: A 1 pF capacitance charged to 5 volts has
½(5V)^2 (1pF) = 12.5 pJ

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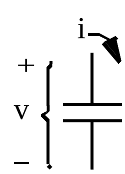
Version Date 09/01/03

ENERGY STORED IN A CAPACITOR (cont.)

More rigorous derivation: During charging, the power flow is v · i into the capacitor, where i is into + terminal. We integrate the power from t = 0 (v = 0) to t = end (v = V). The integrated power is the energy

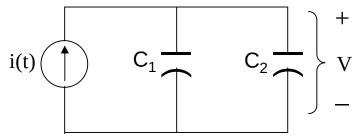


$$E = \int_{t = t_{Initial}}^{t = t_{Final}} v \cdot i \, dt = \int_{v = V_{Initial}}^{v = V_{Final}} v \frac{dq}{dt} dt = \int_{v = V_{Initial}}^{v = V_{Final}} v \, dq$$

but dq = C dv. (We are using small q instead of Q to remind us that it is time varying . Most texts use Q.)

$$E = \int_{v = V_{Initial}}^{v = V_{Final}} Cv \, dv = \frac{1}{2} C V_{Final}{}^2 - \frac{1}{2} C V_{Initial}{}^2$$

CAPACITORS IN PARALLEL

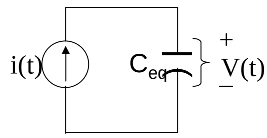


Add Currents

$$i(t) = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt}$$

Equivalent capacitance defined by

$$i = C_{eq} \frac{dV}{dt}$$



Clearly, $C_{eq} = C_1 + C_2$ CAPACITORS IN PARALLEL

Example: 1 pF and 2pF in parallel = 3 pF

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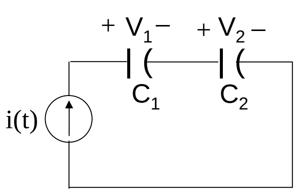
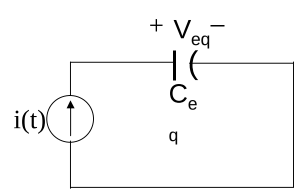
Version Date 09/01/03

CAPACITORS IN SERIES



Equivalent to →



Add Voltages

Equivalent capacitance defined by

$$V_{eq} = V_1 + V_2 \text{ and } i = C_{eq} \frac{dV_{eq}}{dt} = C_{eq} \frac{d(V_1 + V_2)}{dt}$$

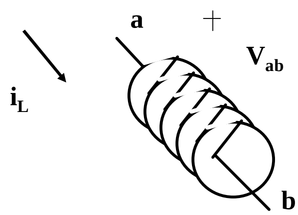
$$i = C_1 \frac{dV_1}{dt} = C_2 \frac{dV_2}{dt}$$

So $\frac{dV_1}{dt} = \frac{i}{C_1}$, $\frac{dV_2}{dt} = \frac{i}{C_2}$, so $\frac{dV_{eq}}{dt} = i\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \equiv \frac{i}{C_{eq}}$

Clearly, $C_{eq} = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}} = \dfrac{C_1 C_2}{C_1 + C_2}$ CAPACITORS IN SERIES

Example: 1 pF and 2pF in series = 0.67 pF

INDUCTOR CIRCUIT ELEMENT



Constitutive relationship Magnetic Flux = Li_L

Time-rate of change of flux = Voltage

$V_{ab} = L\, di_L/dt$

Energy = $\frac{1}{2}(L(i_L)^2)$

In EE42 we do not analyze circuits with inductance due to the time limitations

Capacitance and Inductance

- Capacitors: two plate example; Store energy in the electric field Q = CV, I = C dV/dt and V = (1/C) integral of voltage
  1 mA current charging 1 pF V(t) = (I/C)t = (10-3 A/10^{-12} F) t = 10^9 V/s t
- Energy = $\frac{1}{2}(CV^2)$ (because average voltage is V/2)
- At D.C. time derivatives are zero => C is open circuit
- C in parallel add; series 1/C = sum ($1/C_i$); short together (infinite current but conserve charge)

Not used in EE 42

- Inductors: coil example; Store energy in the magnetic field; Flux = LI, V = L dI/dt and I = (1/L) (integral of voltage)
- At D.C. time derivatives are zero => L is short circuit
- L in parallel 1/L = sum ($1/L_i$); series add; connect in series when have different currents => $L_1I_1+L_2I_2 = (L_1+L_2)I_{NEW}$

EXAMPLE of I vs V GRAPH

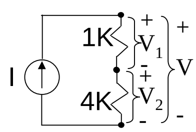
Version Date 09/01/03

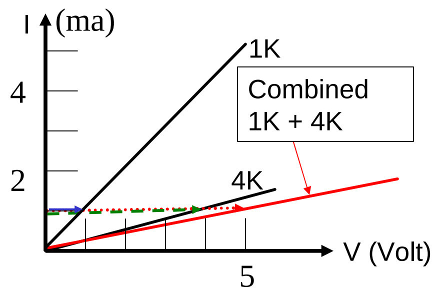
Resistors in Series

If two resistors are in series the current is the same; clearly the total voltage will be the sum of the two IR values i.e. $I(R_1+R_2)$.

Thus the equivalent resistance is R_1+R_2 and the I-V graph of the series pair is the same as that of the equivalent resistance.

Of course we can also find the I-V graph of the combination by adding the voltages directly on the I-V axes. Lets do an example for 1K + 4K resistors





At 1 mA
1V + **4V** = **5V**

Resistors in series add voltages

Resistors in parallel add currents

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Version Date 09/01/03

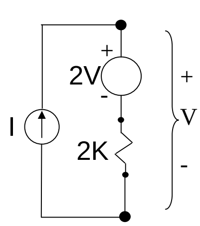
EXAMPLE of I vs V GRAPH

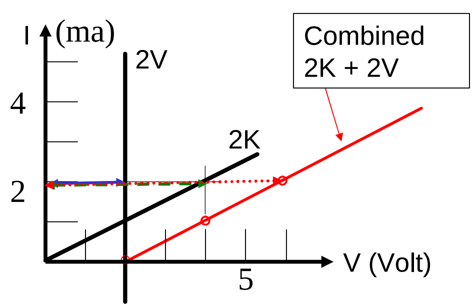
Simple Circuit, e.g. voltage source + resistor.

If two circuit elements are in series the current is the same; clearly the total voltage will be the sum of the voltages i.e. V_S + IR.

We can graph this on the I-V plane. We find the I-V graph of the combination by adding the voltages V_S and I R at each current I.

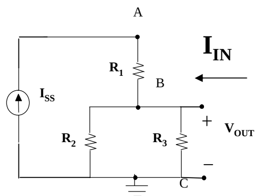
Lets do an example for =2V, R=2K





At 2 mA
2V + **4V** = **6V**

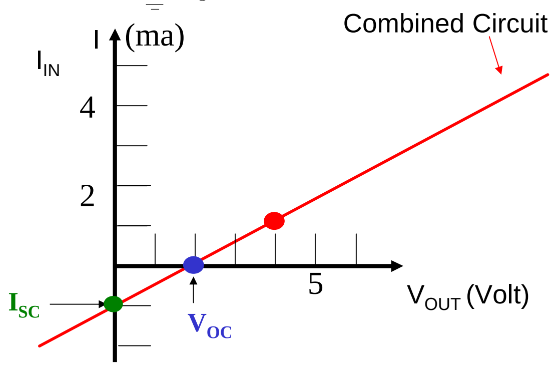
GRAPHICAL EQUIVALENT CIRCUIT



I_{SS} = 1 mA
R_1 = 1kW
R2 = 6 k W
R3 = 3 kW

Short Circuit : V_{OUT} = 0
I_{IN} = -I_{SS} = - 1 mA

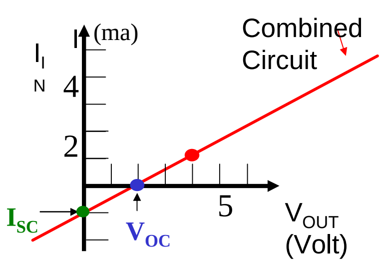
Open Circuit: I_{IN} = 0
Note R1||R2 = 2 kW
V_{OUT} = I_{SS} R1||R2
= 1 mA 2kW = 2V



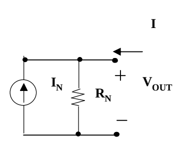
Third Point
I_{IN} = 1 mA
KCL => $I_{2||3}$ = I_{SS} + I_{IN}
V_{OUT} = $I_{2||3}$ $R_1||R_2$
= 2 mA 2kW = 4V

Key: It will always be the case that for linear circuit elements the I vs. V is a straight line.

SIMPLEST EQUIVALENT CIRCUITS

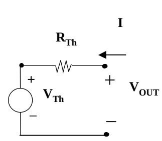


An adequately equivalent circuit is one that has an I vs. V graph that is identical to that of the original circuit.



Norton

I_N = -I_{SC} = - (-1 mA) = 1mA



Thevenin

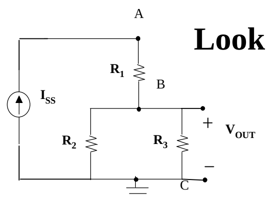
V_{TH} = V_{OC} = 2V

R_{TH} is the inverse of the slope

R_{TH} = R_N = V_{OC} / (-I_{SC}) = 2V /(- (-1 mA)) = 2 kW

R_{TH} = $R_1||R_2$

$R_{TH} = R_N$ SHORTCUT METHOS



Look at algebraic relation for the example circuit.

$V_{OC} = I_{SS} R_1||R_2$

$I_{SC} = -I_{SS}$

$R_{TH} = R_N = V_{OC} / (-I_{SC})$

$R_{TH} = R_N = (I_{SS} R_1||R_2)/(-(-I_{SS})) = R_1||R_2$

In General turn all of the independent sources to zero and find the remaining equivalent resistance seen looking into the terminals.

Currents sources are turned to zero current (with any voltage) and voltage sources are turned to zero voltage (with any current).

I vs. V and Equivalent Circuits

- I vs. V for ideal voltage source is a vertical line at $V = V_{SV}$
- I vs. V for ideal current source is a horizontal line at $I = I_{SC}$
- I vs. V for a circuit made up of ideal independent sources and resistors is a straight line.
- The simplest circuit for a straight line is an ideal voltage source and a resistor (Thevenin) or a current source and a parallel resistor (Norton)
- The easiest way to find the I vs. V line is to find the intercepts where I = 0 (open circuit voltage V_T) and where V = 0 (Short circuit current I_N)
- The short-cut for finding the $(\text{slope})^{-1} = R_T = R_N$ is to turn off all of the independent sources to zero and find the remaining equivalent resistance between the terminals of the elements.