EECS 42 Introduction to Digital Electronics
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Lecture #5

- **Recap I vs. V**
- **Thevenin and Norton Eq. Circuits**
- **Load Line Graphical Solution**
- **Nonlinear elements and power**

Schwarz and Oldham 3.1-3.3

http://inst.EECS.Berkeley.EDU/~ee42/

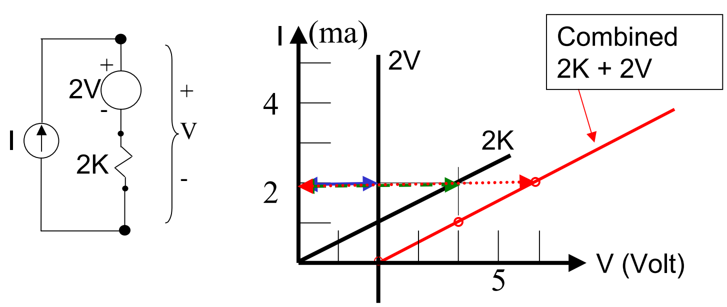
EXAMPLE of I vs V GRAPH

Simple Circuit, e.g. voltage source + resistor.

If two circuit elements are in series the current is the same; clearly the total voltage will be the sum of the voltages i.e. V_S + IR.

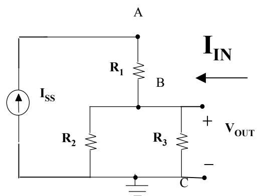
We can graph this on the I-V plane. We find the I-V graph of the combination by adding the voltages V_S and I R at each current I.

Lets do an example for =2V, R=2K

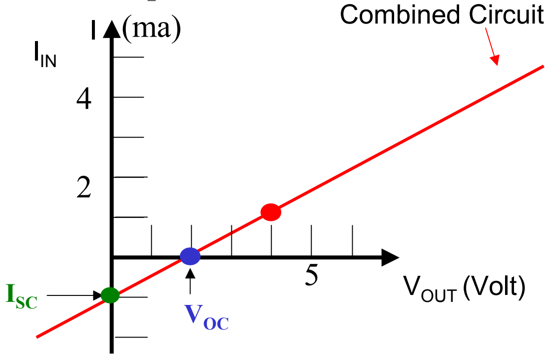


At 2 mA
2V + 4V = 6V

GRAPHICAL EQUIVALENT CIRCUIT



I_{SS} = 1 mA
R_1 = 1kΩ
R2 = 6 k Ω
R3 = 3 kΩ



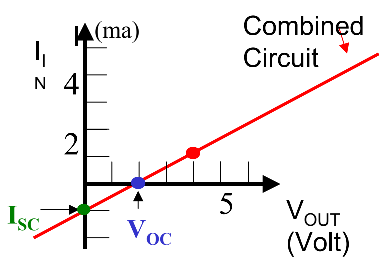
Short Circuit : $V_{OUT} = 0$
$I_{IN} = -I_{SS} = -1$ mA

Open Circuit: $I_{IN} = 0$
Note R2||R3 = 2 kΩ
$V_{OUT} = I_{SS} \times$ R2||R3
= 1 mA × 2kΩ = 2V

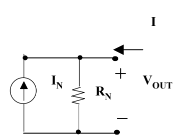
Third Point
I_{IN} = 1 mA
KCL => $I_{2||3} = I_{SS} + I_{IN}$
$V_{OUT} = I_{2||3} \times R_2||R_3$
= 2 mA × 2kΩ = 4V

Key: It will always be the case that for linear circuit elements the I vs. V is a straight line.

SIMPLEST EQUIVALENT CIRCUITS

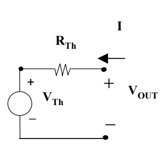


An adequately equivalent circuit is one that has an I vs. V graph that is identical to that of the original circuit.



Norton

$I_N = -I_{SC} = -(-1\text{ mA}) = 1\text{mA}$



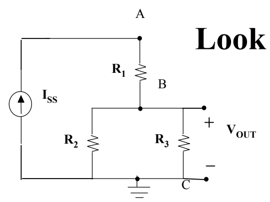
Thevenin

$V_{TH} = V_{OC} = 2V$

R_{TH} is the inverse of the slope

$R_{TH} = R_N = V_{OC} / (-I_{SC}) = 2V / (-(-1\text{ mA})) = 2$ kΩ

$R_{TH} = R_N$ SHORTCUT METHODS



Look at algebraic relation for the example circuit.

$V_{OC} = I_{SS} \times R_2||R_3$

$I_{SC} = -I_{SS}$

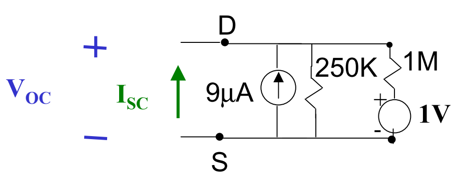
$R_{TH} = R_N = V_{OC} / (-I_{SC})$

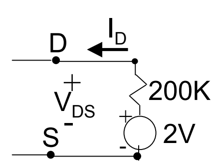
$R_{TH} = R_N = (I_{SS} \times R_2||R_3)/(-(-I_{SS})) = R_2||R_3$

In General turn all of the independent sources to zero and find the remaining equivalent resistance seen looking into the terminals.

Currents sources are turned to zero current (with any voltage) and voltage sources are turned to zero voltage (with any current).

Thevenin Equivalent Circuit Example





$V_{OC} = 2V$
from Node Analysis Method Week 5

$I_{SC} = -9\mu A - 1V/1M\Omega = -10\ \mu A$

$R_{TH} = R_N = V_{OC} / (-I_{SC}) = 2V/ 10\ \mu A = 200k\Omega$

I vs. V and Equivalent Circuits

- I vs. V for ideal voltage source is a vertical line at $V = V_{SV}$
- I vs. V for ideal current source is a horizontal line at $I = I_{SC}$
- I vs. V for a circuit made up of ideal independent sources and resistors is a straight line.
- The simplest circuit for a straight line is an ideal voltage source and a resistor (Thevenin) or a current source and a parallel resistor (Norton)
- The easiest way to find the I vs. V line is to find the intercepts where I = 0 (open circuit voltage V_T) and where V = 0 (Short circuit current I_N)
- The short-cut for finding the $(\text{slope})^{-1} = R_T = R_N$ is to turn off all of the independent sources to zero and find the remaining equivalent resistance between the terminals of the elements.

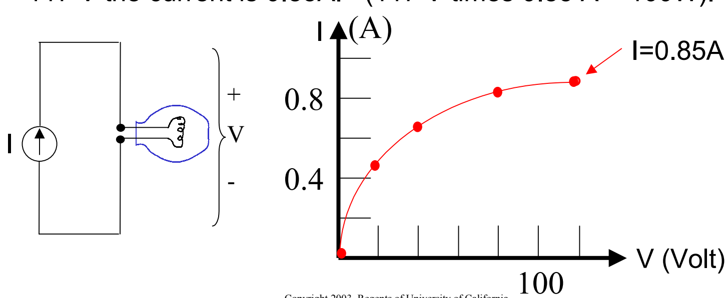
Example of I-V Graphs

Nonlinear element, e.g. lightbulb

If we think of the light bulb as a sort of resistor, then the resistance changes with current because the filament heats up. The current is reduced (sort of like the resistance increasing). “sort of” because a resistor has, by definition a linear I-V graph and R is always the same.

But for a light bulb the graph kind of “rolls over”, becoming almost flat.

Consider a 100 Watt bulb, which means at at the nominal line voltage of 117 V the current is 0.85A. (117 V times 0.85 A = 100W).

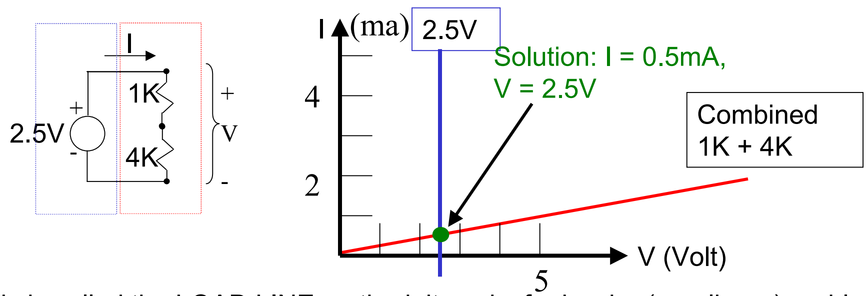


I-V Graphs as a method to solve circuits

We can find the currents and voltages in a simple circuit graphically. For example if we apply a voltage of 2.5V to the two resistors of our earlier example:

We draw the I-V of the voltage and the I-V graph of the two resistors on the same axes. Can you guess where the solution is?

At the point where the voltages of the two graphs AND the currents are equal. (Because, after all, the currents are equal, as are the voltages.)



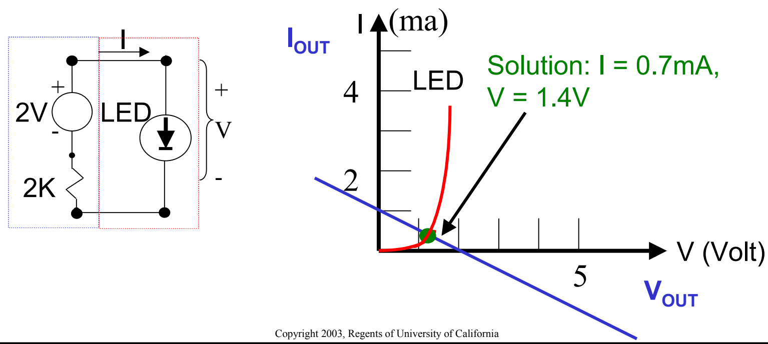
This is called the LOAD LINE method; it works for harder (non-linear) problems.

Another Example of the Load-Line Method

Lets hook a 2K resistor + 2V source circuit up to an LED (light-emitting diode), which is a very nonlinear element with the IV graph shown below.

Again we draw the I-V graph of the 2V/2K circuit on the same axes as the graph of the LED. Note that we have to get the sign of the voltage and current correct!! (Think of **I_{OUT} versus V_{OUT}** for the 2V/2K circuit)

At the point where the two graphs intersect, the voltages and the currents are equal, in other words we have the solution.

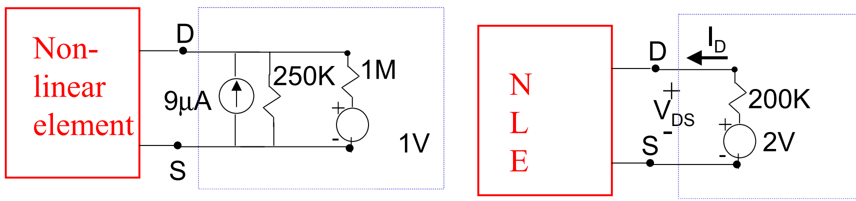


The Load-Line Method

Given a circuit containing a two-terminal non-linear element "NLE", and some linear components.

First replace the entire linear part of the circuit by its Thevenin equivalent (which is a resistor in series with a voltage source).

Then define I and V at the NLE terminals (typically associated signs)



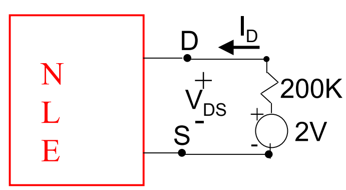
This is the circuit from slide 8

Example of Load-Line method (con't)

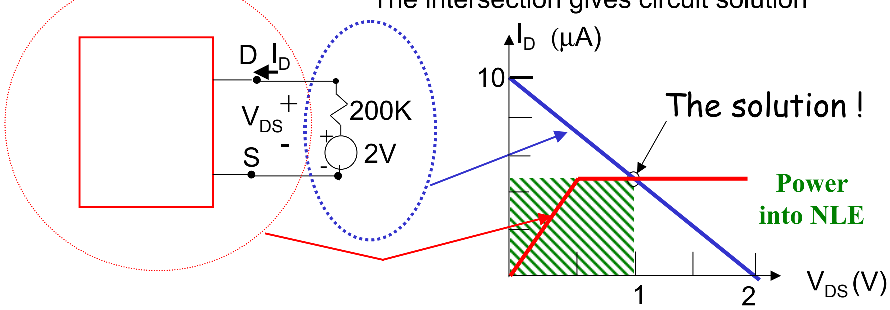
Given the graphical properties of two terminal non-linear circuit (i.e. the graph of a two terminal device)

And have this connected to a linear (Thévenin) circuit

Whose I-V can also be graphed on the same axes ("load line")



The intersection gives circuit solution



Power Calculation Review

Power is calculated the same way for linear and non-linear elements and circuits.

For any circuit or element the dc power is I X V and, if associated signs are used, represents heating for positive power or extraction of energy for negative signs.

For example in the last example the NLE has a power of +1V X 5μA or 5μW . It is absorbing power. The rest of the circuit has a power of - 1V X 5μA or - 5μW , because the signs are unassociated. It is delivering the 5μW to the NLE.

So what it the power absorbed by the 200K resistor?

Answer: I X V is + 5μA X (5μA X 200K) = 5μW. Then the voltage source must be supplying a total of 10μW. Can you show this?

Load-Line method

The method is graphical, and therefore approximate

But if we use equations instead of graphs, it could be accurate

It can also be use to find solutions to circuits with three terminal nonlinear devices (like transistors) for which the third input controls the device state.