EECS 42 Introduction Digital Electronics
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Lecture #8 Node Equations

- **Systematic Node Equations**
- **Example: Voltage and Current Dividers**
- **Example 5 Element Circuit**

Schwarz and Oldham 53-58, 2.5, & 2.6

Quiz 9/25 20 min:

Basic Circuit Analysis and Basic Transient

Midterm 10/2: Lectures # 1-9: 4 Topics – See slide 2

Length/Credit Review TBA

http://inst.EECS.Berkeley.EDU/~ee42/

FORMAL CIRCUIT ANALYSIS

Systematic approaches to writing down KCL and KVL: Section 2.3 of Text - In particular use of KCL gives NODAL ANALYSIS

Mathematical foundation is rigorous: EE 104 Circuit Theory

Nodal Analysis: Node voltages are the unknowns
Mesh Analysis: Branch currents are the unknowns
} Use one **or** the other for circuit analysis

We will do only nodal analysis – (because voltages make more convenient variables than currents) Thus omit Text Section 2.4 ; it is redundant.

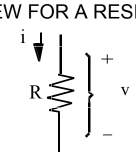
First Midterm Exam: Topics

- Basic Circuit Analysis (KVL, KCL)
- Equivalent Circuits and Graphical Solutions for Nonlinear Loads
- Transients in Single Capacitor Circuits
- Node Analysis Technique and Checking Solutions

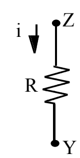
Exam is in class 9:40-10:45 AM, Closed book, Closed notes, Bring a calculator, Paper provided

THE IV CHARACTERISTICS OF AN ELEMENT ALLOW EITHER NODAL OR MESH (LOOP) ANALYSIS

REVIEW FOR A RESISTOR: OHM'S Law



If we use associated current and voltage (i.e., i is defined as into + terminal), then $v = iR$ (Ohm's law)



Another version of the same statement, and the one most important to us:

$i = (V_Z - V_Y)/R$ (Ohm's law)

NOTE ORDER OF NODES: $V_Z - V_Y$!

Class Input on Midterm/Review

- Midterm Options
  - Option 1: 65 min and 23% of grade in course (Final is 49%)
  - Option 2: 80 min and 28% of grade in course (Final is 39%)
- Review Options
  - Date Tu 9/30 or Wed 10/1
  - Time 5-6:30 PM or Other

FORMAL CIRCUIT ANALYSIS USING KCL: NODAL ANALYSIS

(Memorize these steps and apply them rigorously!)

1 Choose a Reference Node ⏚

2 Define unknown node voltages (those not fixed by voltage sources)

3 Write KCL at each unknown node, expressing current in terms of the node voltages (using the constitutive relationships of branch elements*)

4 Solve the set of equations (N equations for N unknown node voltages)

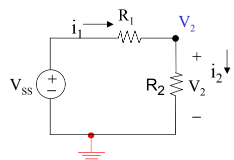
* With inductors or floating voltages we will use a modified Step 3: The Supernode Method – see Lecture #8

NODAL ANALYSIS USING KCL
–Example: The Voltage Divider –

1 Choose reference node

2 Define unknown node voltages

3 Write KCL at unknown nodes

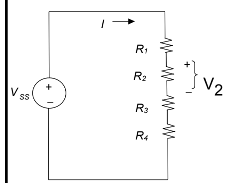


$$\frac{V_{SS}-V_2}{R_1} = \frac{V_2 - 0}{R_2}$$

4 Solve: $V_2 = V_{SS}\cdot\frac{R_2}{R_1+R_2}$

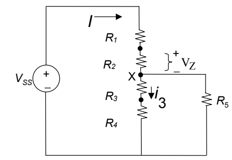
This is of course the voltage divider formula and is by itself very useful.

WHEN IS VOLTAGE DIVIDER FORMULA CORRECT?



$$V_2 = \frac{R_2}{R_1+R_2+R_3+R_4}\cdot V_{SS}$$

Correct if nothing else connected to nodes



$$V_Z \neq \frac{R_2}{R_1+R_2+R_3+R_4}\cdot V_{SS}$$

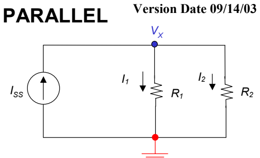
because R_5 removes condition of resistors in series – i.e. $i_3 \neq I$

What is V_Z?

RESISTORS IN PARALLEL

1 Select Reference Node

2 Define unknown node voltages



Note: $I_{ss} = I_1 + I_2$, i.e.,

$$I_{SS} = \frac{V_X}{R_1} + \frac{V_X}{R_2} \Rightarrow V_X = I_{SS}\cdot\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = I_{SS}\cdot\frac{R_1R_2}{R_1+R_2}$$

RESULT 1 EQUIVALENT RESISTANCE: $R_\parallel \equiv R_1 \parallel R_2 = \frac{R_1R_2}{R_1+R_2}$

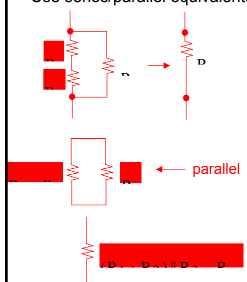
RESULT 2 CURRENT DIVIDER:

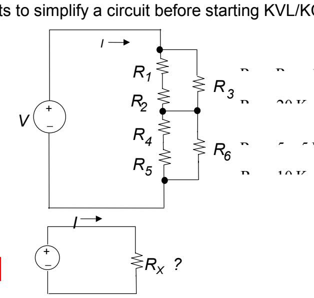
$$I_1 = \frac{V_X}{R_1} = I_{SS}\times\frac{R_2}{R_1+R_2}$$

$$I_2 = \frac{V_X}{R_2} = I_{SS}\times\frac{R_1}{R_1+R_2}$$

IDENTIFYING SERIES AND PARALLEL COMBINATIONS

Use series/parallel equivalents to simplify a circuit before starting KVL/KCL

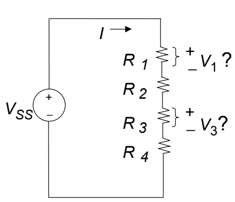




Please note the order of math operators here!

GENERALIZED VOLTAGE DIVIDER
(solved without Nodal Analysis)

Circuit with several resistors *in series*



- We know

$I = V_{SS}/(R_1 + R_2 + R_3 + R_4)$

- Thus,

$$V_1 = \frac{R_1}{R_1+R_2+R_3+R_4}\cdot V_{SS}$$

and

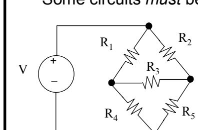
$$V_3 = \frac{R_3}{R_1+R_2+R_3+R_4}\cdot V_{SS}$$

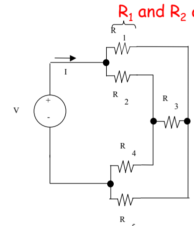
etc.. etc..

IDENTIFYING SERIES AND PARALLEL COMBINATIONS (cont.)

Some circuits *must* be analyzed (not amenable to simple inspection)

R_1 and R_2 are **not** in ||





R_1 and R_5 are **not** in series

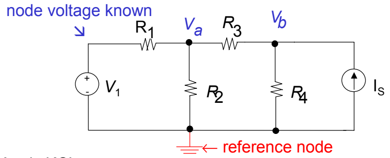
Special cases:
$R_3 = 0$ **OR** $R_3 = \infty$

Example: $R_3 = 0 \Rightarrow R_1 \parallel R_2$; $R_4 \parallel R_5$ in series; $R_{eq} = R_1 \parallel R_2 + R_4 \parallel R_5$

OR IF $R_3 = \infty \Rightarrow (R_1 + R_5) \parallel (R_2 + R_4)$

ANOTHER EXAMPLE OF NODAL ANALYSIS

Choose a reference node and define the node voltages (except reference node and the one set by the voltage source);



Apply KCL:

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_b}{R_3} + \frac{V_a}{R_2} = 0$$

$$\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} - I_S = 0$$

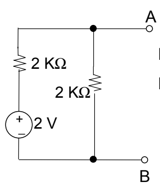
Note the systematic use of the present node minus each of the other nodes divided by the element resistance that connects them.

You can solve for V_a, V_b.

What if we used different ref node?

EXAMPLE 1: Thevenin/Norton

Find the Thévenin and Norton equivalents of:



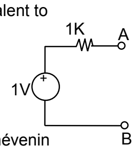
Find $V_{AB} = V_{OC}$ from voltage divider. :

If open circuit, $V_{AB} = V_{OC} = 2 \text{ X } 2/4 = 1\text{V}$

If A-B is shorted, $I_{SC} = -2\text{V}/2\text{K} = -1 \text{ mA}$ (into A)

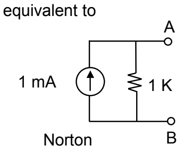
$$R_{TH} = \frac{1}{10^{-3}} = 1\text{K}$$

equivalent to



Thévenin

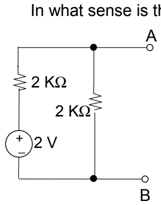
and equivalent to

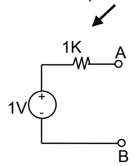


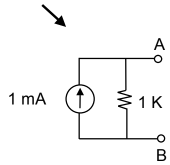
Norton

EXAMPLE 1 Thevenin/Norton Continued

In what sense is this circuit equivalent to these?







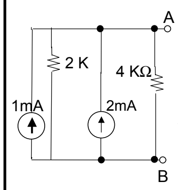
They have identical I-V characteristics and therefore have

The same open circuit voltage

The same short circuit current

EXAMPLE 2 Thevenin/Norton

Find the Thévenin and Norton equivalents of:



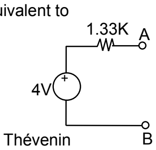
Find Norton equivalent of circuit leg on the left: (1mA and 2K) ⇒ Two Norton circuits in parallel

Thus combine current sources (2+1=3) = I_N and combine resistors in parallel: 8/6 K = R_N

So, $V_{TH} = R_N I_N = 4\text{V}$

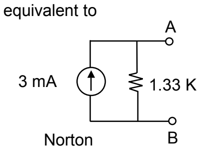
$R_{TH} = R_N = 4/3 \text{ K}$

equivalent to



Thévenin

and equivalent to



Norton