EECS 42 Introduction to Digital Electronics

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Lecture #9 Prof. King: Node Equations - Advanced

- **Supernode for voltage supplies**
- **Checking Solutions**

Schwarz and Oldham 53-58, 2.5 and 2.6

Quiz 9/25 20 min:

Basic Circuit Analysis and Basic Transient

Midterm 10/2: Lectures # 1-9: ForTopics – See slide 2

Length/Credit Review TBA

http://inst.EECS.Berkeley.EDU/~ee42/

First Midterm Exam: Topics

- Basic Circuit Analysis (KVL, KCL)
- Equivalent Circuits and Graphical Solutions for Nonlinear Loads
- Transients in Single Capacitor Circuits
- Node Analysis Technique and Checking Solutions

Exam is in class 9:40-10:45 AM, Closed book, Closed notes, Bring a calculator, Paper provided

ReCap

FORMAL CIRCUIT ANALYSIS USING KCL: NODAL ANALYSIS

(Memorize these steps and apply them rigorously!)

1 Choose a Reference Node ⏚

2 Define unknown node voltages (those not fixed by voltage sources)

3 Write KCL at each unknown node, expressing current in terms of the node voltages (using the constitutive relationships of branch elements*)

4 Solve the set of equations (N equations for N unknown node voltages)

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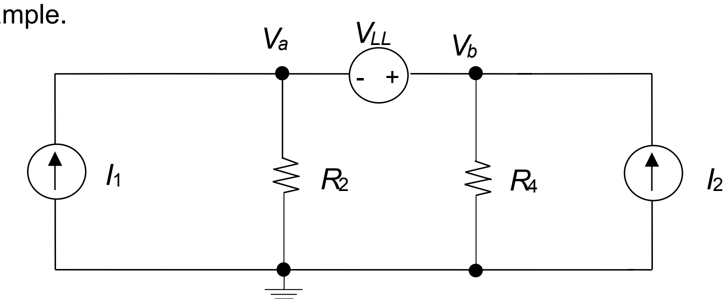
* With inductors or floating voltages we will use a modified Step 3: The Supernode Method – see slide 10

NODAL ANALYSIS WITH "FLOATING" VOLTAGE SOURCES

A "floating" voltage source is a voltage source for which neither side is connected to the reference node. V_{LL} in the circuit below is an example.

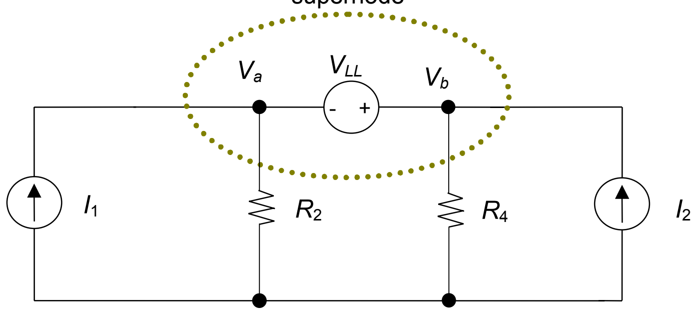


What is the problem? → We cannot write KCL at node a or b because there is no way to express the current through the voltage source in terms of .

Solution: Define a "supernode" – that chunk of the circuit containing nodes a and b. Express KCL at this supernode.

FLOATING VOLTAGE SOURCES (cont.)

Use a Gaussian surface to enclose the floating voltage source; write KCL for that surface



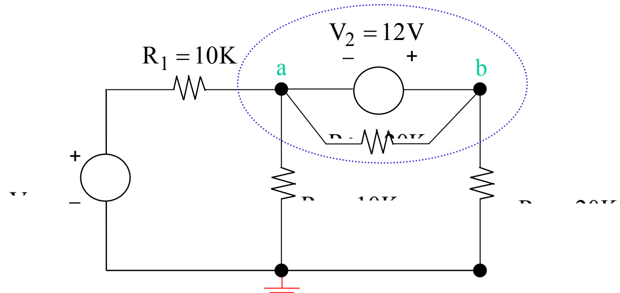
We have two unknowns: V_a and V_b.

We obtain one equation from KCL at supernode: $I_1 - \frac{V_a}{R_2} - \frac{V_b}{R_4} + I_2 = 0$

We obtain a second "auxiliary" equation from the property of the voltage source: $V_{LL} = V_b - V_a$ (often called the "constraint")

$\Rightarrow$ 2 Equations & 2 Unknowns

ANOTHER EXAMPLE



1 Choose reference node (can it be chosen to avoid floating voltage source?)

2 Label unknowns V_a and V_b

3 Equation at supernode: $\frac{V_1 - V_a}{} = \frac{V_b}{} + \frac{V_a}{}$ $\rightarrow$ $V_a\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{V_b}{R_4} = \frac{V_1}{R_1}$

4 Auxiliary equation: $\rightarrow$ $V_a \qquad - V_b = -V_2$

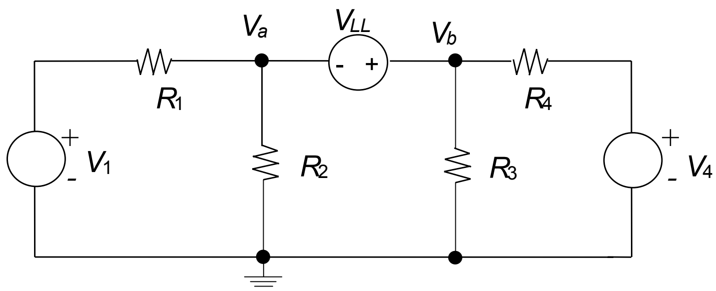
Solve: $V_a\left(\frac{R_4}{R_1} + \frac{R_4}{R_2} + 1\right) = V_1\frac{R_4}{R_1} - V_2$

$V_b = V_a + V_2$

SOLUTION : $V_a = 0$

NODAL ANALYSIS COMPLETE EXAMPLE

Find V_a, V_b if $R_1 = R_2 = R_3 = R_4 = 1M\Omega$, and $V_1 = V_4 = 1.5V$ with $V_{LL} = 1V$



Solution: At supernode enclosing nodes a and b :

$(V_1 - V_a)/R_1 - V_a/R_2 = V_b/R_3 + (V_b - V_4)/R_4$ *and*

$V_b = V_a + V_{LL}$ *Thus:* $V_a = 0.25$, $V_b = 1.25$

Be sure to check answer with KCL!

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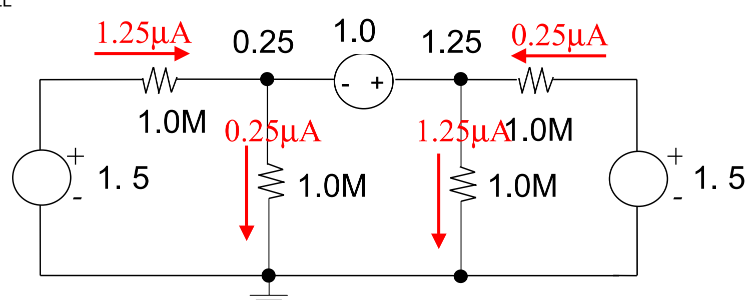
ANSWER CHECKING TECHNIQUE: USE KCL

Is $V_a = 1.25$ and $V_b = 0.25$ if $R_1 = R_2 = R_3 = R_4 = 1M\Omega$, and $V_1 = V_4 = 1.5V$ with $V_{LL} = 1V$????



KCL at the Supernode:

Clearly the current into the supernode is zero and we have verified that the solution is correct. :

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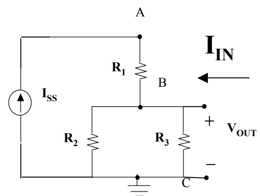
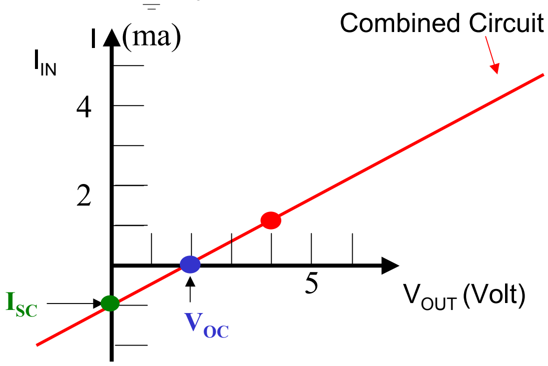
GRAPHICAL EQUIVALENT CIRCUIT



$I_{SS} = 1$ mA
$R_1 = 1$kΩ
R2 = 6 k Ω
R3 = 3 kΩ

Short Circuit : $V_{OUT} = 0$
$I_{IN} = -I_{SS} = -1$ mA

Open Circuit: $I_{IN} = 0$
Note R2||R3 = 2 kΩ
$V_{OUT} = I_{SS} \times$ R2||R3
= 1 mA × 2kΩ = 2V



Third Point
$I_{IN} = 1$ mA
KCL => $I_{2||3} = I_{SS} + I_{IN}$
$V_{OUT} = I_{2||3} \times R_2||R_3$
= 2 mA × 2kΩ = 4V

Key: It will always be the case that for linear circuit elements the I vs. V is a straight line.

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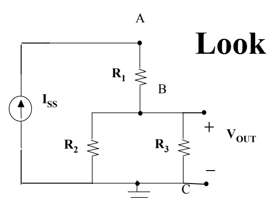
$R_{TH} = R_N$ SHORTCUT METHODS



Look at algebraic relation for the example circuit.

$V_{OC} = I_{SS} \times R_2||R_3$

$I_{SC} = -I_{SS}$

$R_{TH} = R_N = V_{OC} / (-I_{SC})$

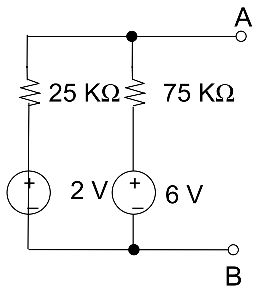
$R_{TH} = R_N = (I_{SS} \times R_2||R_3)/(-(-I_{SS})) = R_2||R_3$

In General turn all of the independent sources to zero and find the remaining equivalent resistance seen looking into the terminals.

Currents sources are turned to zero current (with any voltage) => OPEN CIRCUIT. Voltage sources are turned to zero voltage (with any current)=> SHORT CIRCUIT.

EXAMPLE 3 Thevenin/Norton

Find the Thévenin and Norton equivalents of:



Find $V_{AB} = V_{OC}$ from voltage divider. Left to right:

(4 V rise across 25K + 75K) ⇒

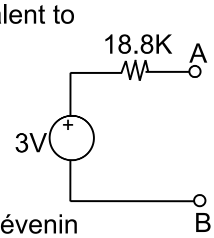
3 V across 75K, 1 V across 25k.

So, $V_{AB} = -3 + 6 = 3V = V_{OC}$

I_{SC} = - 6V/75K+2V/25K = -0.16 mA

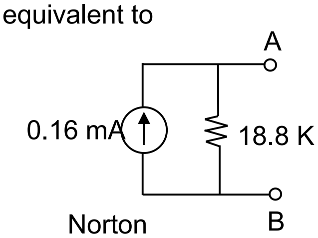
$$R_{TH} = \frac{3}{.16\times 10^{-3}} = 18.8K$$

equivalent to



Thévenin

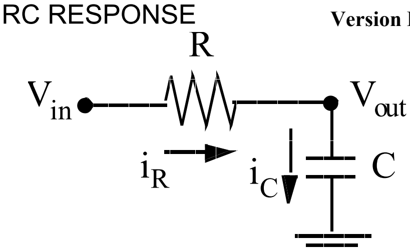
and equivalent to



Norton

RC RESPONSE

Generalization



Vin switches at t = 0; then for any time interval t > 0, in which Vin is a constant, Vout is ***always*** of the form: $V_{out} = A + Be^{-t/RC}$

We determine A and B from the initial voltage on C, and the value of Vin. Assume Vin "switches" at t=0 from Vco to V1:

First, at $t = 0 \quad V_C \equiv V_{Co}$ initial voltage

☞ Thus, $A + B = V_{Co}$

as $t \to \infty, V_C \to V_1$

☞ Thus, $A = V_1 \Rightarrow B = V_{Co} - V_1$

RC is $R_{TH}C$ where R_{TH} is the Thevenin Resistance seen C with independent sources set to zero.

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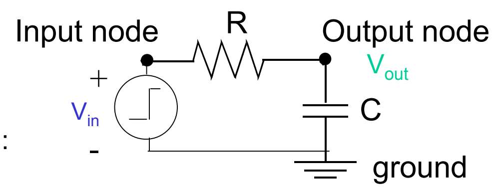
Re-Cap: Charging and discharging in RC Circuits

Last Time:

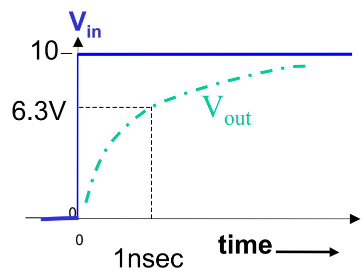
We learned that simple the simple RC circuit with a step input has a universal exponential solution of the form:

$$V_{out} = A + Be^{-t/RC}$$



Example 0: R = 1K, C = 1pF, V_{in} steps from zero to 10V at t=0:

1) Initial value of V_{out} is 0
2) Final value of V_{out} is 10V
3) Time constant is RC = 10^{-9} sec
4) V_{out} reaches 0.63 X 10 in 10^{-9} sec



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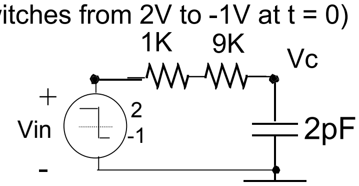
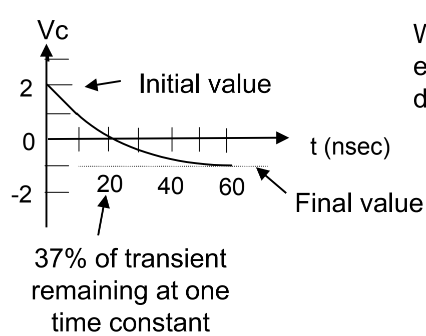
Charging and discharging in RC Circuits

Find Vc(t) for the following circuit: (input switches from 2V to -1V at t = 0)



We have : Initial value of Vc is 2V, final value is -1V and τ = 20nsec

5) Sketch Vc (t) :



What is the equation for an exponential beginning at 2V, decaying to -1V, with τ = 20nsec?

$$V_c(t) = -1 + 3e^{-t/20nsec}$$

V_{FINAL} =

$V_{INITIAL}$ =

$B = V_{INTIAL} - V_{FINAL}$ =