EECS 42 Introduction to Digital Electronics
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Lecture # 14 Circuit analysis with dependent sources (4.1-4.3)
   A) Node Equations
   B) Equivalent Sources
   C) Amplifier Parameters:
      Gain, $R_{IN}$, $R_{OUT}$
   D) Non-Ideal Op-Amp Model

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The 4 Basic Linear Dependent Sources

- Voltage-controlled voltage source ...
  \[ V = A_v V_{cd} \]

- Current-controlled voltage source ...
  \[ V = R_m I_c \]

- Current-controlled current source ...
  \[ I = A_i I_c \]

- Voltage-controlled current source ...
  \[ I = G_m V_{cd} \]
EXAMPLE OF NODAL ANALYSIS WITH DEPENDENT SOURCES

Standard technique, except an additional equation is needed if the dependent variable is an unknown current as here. Note Vb is redundant.

\[ I = \frac{V_a}{R_2} + \frac{(V_a - R_m I_2)}{R_3} \]  \[ \text{and} \]  \[ I_2 = \frac{V_a}{R_2} \]

Solving:
\[ I = \frac{V_a}{1/R_2 + 1/R_3 - R_m / R_2 R_3} \]

So \[ V_a = I R_2 R_3 / (R_2 + R_3 - R_m) \]

THEVENIN EQUIVALENT WITH DEPENDENT SOURCES

Method 1: Use \( V_{oc} \) and \( I_{sc} \) as usual to find \( V_T \) and \( R_T \) (and \( I_N \) as well)

Method 2: To find \( R_T \) by the “ohmmeter method” turn off only the independent sources; let the dependent sources just do their thing.

See examples in text (such as Example 4.3).

This method also works when computing incremental signals such as a change in the source \( V_S \) (given by \( \Delta V_S \) or \( v_S \)) produces a change in \( V_{IN} \) or \( V_{OUT} \) (given by \( \Delta V_{IN} \) or \( \Delta V_{OUT} \) also written \( v_{IN} \) and \( v_{OUT} \)), and their ratio called the small-signal gain (\( \Delta V_{OUT} / \Delta V_S \) or \( v_{OUT} / v_S \))
NODAL ANALYSIS WITH DEPENDENT SOURCES

Example: Find Thévenin equivalent of stuff in red box.

With method 2 we first find open circuit voltage \( V_T \) and then we "measure" input resistance with source \( I_{SS} \) turned off.

You verify the solution:

\[
V_{TH} = \frac{I_{SS} R_6 (R_2 + AR_3)}{R_2 + R_3 + R_6 (1 - A)} \quad R_{TH} = \frac{R_2 (R_6 + R_3)}{R_2 + R_3 + R_6 (1 - A)}
\]

EXAMPLE CIRCUIT WITH MULTIPLE SOURCES

Original circuit

Circuit with independent sources turned to zero

Note \( R_6 \) has been rotated down.
EXAMPLE CIRCUIT: GAIN = \( \frac{\Delta V_{\text{OUT}}}{\Delta V_S} = \frac{v_{\text{OUT}}}{v_S} \)

\[
R_{\text{IN}} = \frac{R_a R_b}{R_a + R_b} \quad v_{\text{IN}} = \frac{R_{\text{IN}}}{R_S + R_{\text{IN}}} v_S \\
v_{\text{OUT}} = -G_m v_{\text{IN}} \frac{R_O R_L}{R_O + R_L} = -G_m \frac{R_{\text{IN}}}{R_S + R_{\text{IN}}} \frac{R_O R_L}{R_O + R_L} v_S
\]

Input voltage divider and output current divider reduce the gain

EXAMPLE CIRCUIT: INPUT/OUTPUT RESISTANCE

\[
R_{\text{IN}} = \frac{R_a R_b}{R_a + R_b} \quad \text{Output does not feed back to input} \\
R_{\text{OUT}} = R_O \quad \text{Assume } v_S = 0 = v_{\text{IN}} = 0 = \text{no current in dependent source}
\]

Can circuit design improve \( R_{\text{IN}} \) and \( R_{\text{OUT}} \) or do we need better devices?
EXAMPLE CIRCUIT: INCREASED INPUT RESISTANCE

Add resistor $R_E$

Analysis: apply $i_{\text{TEST}}$ and evaluate $v_{\text{TEST}}$

$$v_{\text{IN}} = R_{\text{IN}}i_{\text{TEST}}$$
$$v_{\text{TEST}} = R_{\text{IN}}i_{\text{TEST}} + v_E$$

KCL
$$\frac{v_E}{R_E} + \frac{v_E}{R_0} - i_{\text{TEST}} - G_m R_{\text{IN}} i_{\text{TEST}} = 0$$

Check for special case for $R_e$ infinite

Intuitive Explanation: $R_e$ puts $R_{\text{IN}}$ on a node whose voltage increases in response to current in $R_{\text{IN}}$.

EXAMPLE CIRCUIT: INCREASED OUTPUT RESISTANCE

Add resistor $R_E$

The input has been assumed to be shorted

Analysis: apply $i_{\text{TEST}}$ and evaluate $v_{\text{TEST}}$

$$v_{\text{IN}} = -v_E$$
$$v_{\text{TEST}} = -v_E$$

Unknowns: $i_{\text{TEST}}, v_{\text{TEST}}, v_{\text{IN}}, v_E$

Need 3 equations to find the ratio of $i_{\text{TEST}}/v_{\text{TEST}}$

Intuitive Explanation: $GmV_{\text{IN}}$ burps current which has to also go through $R_0$. This raises $v_{\text{TEST}}$ and the output impedance $v_{\text{TEST}}/i_{\text{TEST}}$

Finish this in the homework
NON-IDEAL OP-AMPS

JUST ANOTHER CASE OF ANALYSIS WITH DEPENDENT SOURCES

Example:

\[
\begin{align*}
\text{Circuit (assume } R_{\text{IN}} \cong \infty) &
\end{align*}
\]

\[
\begin{align*}
\text{Analysis:} & \\
& \text{Outline your circuit analysis strategy here.}
\end{align*}
\]

Answer:

\[
\begin{align*}
V_0 &= V_{\text{IN}} \frac{A(R_1 + R_2)}{(A + 1)R_1 + R_2} \\
& \cong V_{\text{IN}} \frac{R_1 + R_2}{R_1} = 10V_{\text{IN}} \\
& \text{if } A \to \infty
\end{align*}
\]