UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences
Last modified on September 24, 2003 by Eric Chung (e_chung@uclink.berkeley.edu)

Prof. Neureuther | EECS 42
Fall 2003

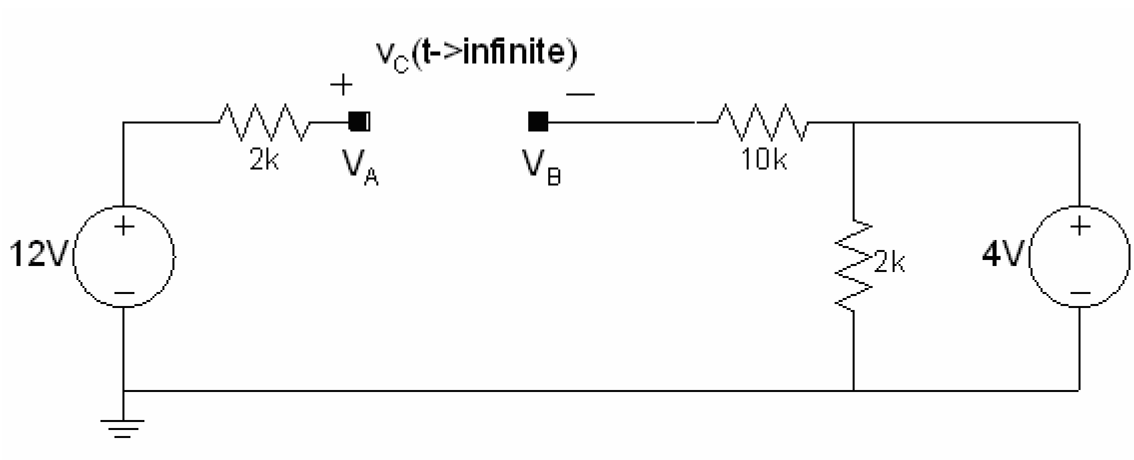
Homework 4 Solutions

Problem 4.1. Transient Analysis

a) Remember the steps for solving RC circuits:

1) Identify the initial voltage on the capacitor.
Since the capacitor is initially uncharged, $v_C(t=0)=0V$.

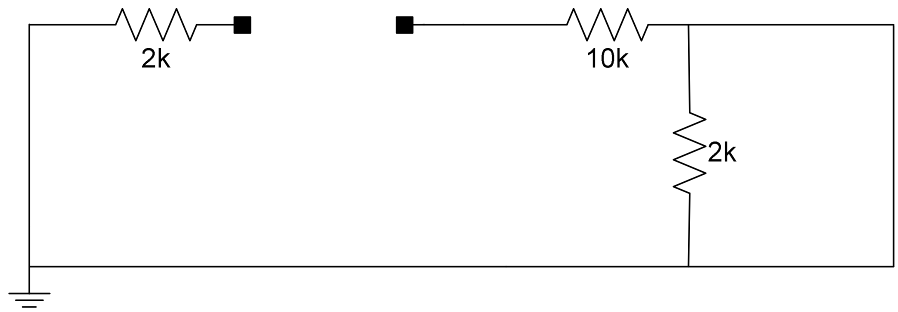
2) Identify the final voltage on the capacitor (as $t \to \infty$) by connecting the switch and open-circuiting the capacitor. Remember, when we open-circuit a capacitor, we are modeling steady-state, when no more current flows through a capacitor.



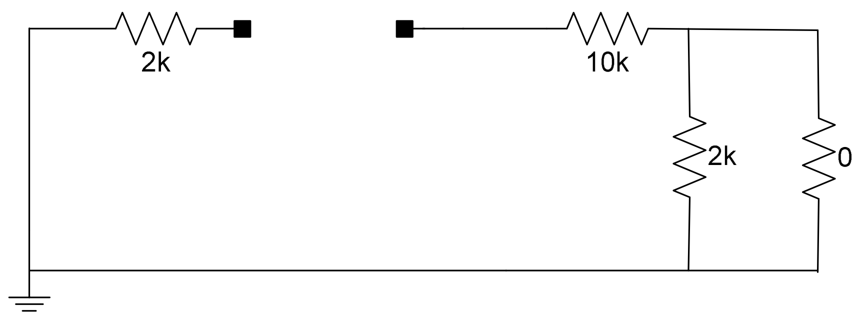
Note that $v_C(t \to \infty)=V_A - V_B$. Since no current flows across the 2k resistor on the left, then the voltage difference across the 2k resistor must be zero. Hence, $V_A = 12V$. Similarly, since no current flows across the 10k resistor, the voltage difference across it must be zero. Hence, $V_B = 4V$.

Therefore $v_C(t \to \infty)=V_A - V_B = 12V - 4V = 8V$.

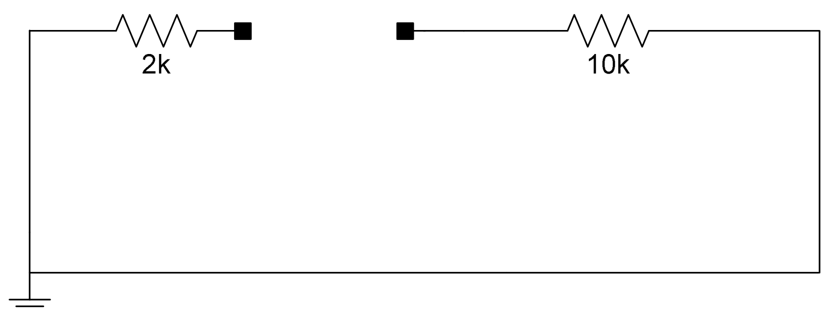
3) Identify the equivalent resistance R_{eq} seen by the capacitor. To identify this equivalent resistance, we open-circuit the capacitor and kill all sources (by open-circuiting any current sources and shorting any voltage sources):



The equivalent resistance R_{eq} seen by the two terminals of the capacitor should be $2k + 10k = 12k$. If this is not obvious to you, consider modeling the short-circuit wire on the right of the figure above as a resistor with R = 0 (ideal wires have no resistance).



Note now that the two resistors on the right are in parallel. $2k \,||\, 0k = \dfrac{(2k)(0k)}{2k+0k} = 0k$



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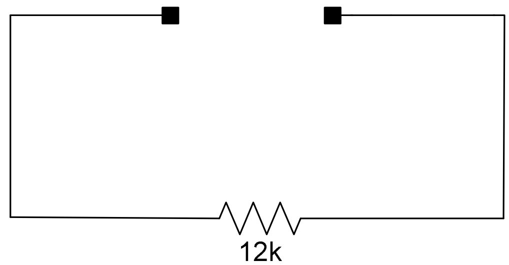


Thus $R_{eq} = 12k$.

4) Using the formula for transient voltage on a capacitor:

$v_C(t) = V_{FINAL} + (V_{INITIAL} - V_{FINAL})e^{-t/RC}$

Note: $RC = (R_{EQ})(C) = (12k)(5pF) = 60nS$

$v_C(t) = 8V + (0V - 8V)e^{-t/60nS}$

$\boxed{v_C(t) = 8V - 8e^{-t/60nS}V}$

Problem 4.2. Transient Analysis Continued

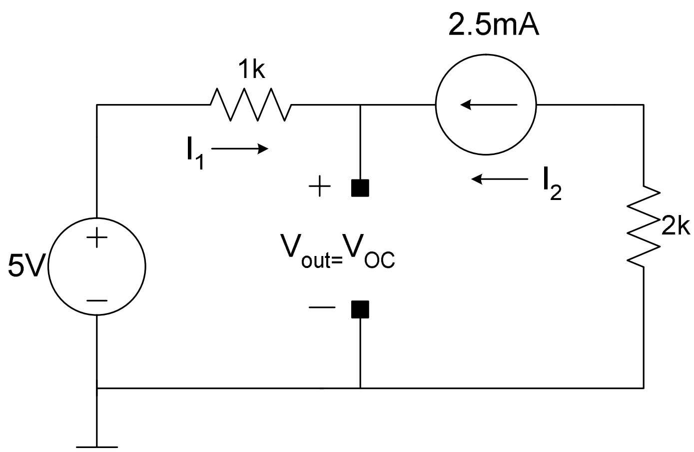
a) To compute the Thevenin equivalent circuit, it is necessary to calculate three parameters:

$V_{TH} = V_{OC}$

$I_{SC} = I_{TH}$

$R_{TH} = \dfrac{V_{TH}}{-I_{SC}}$

To calculate V_{OC} , we open-circuit the voltage of interest at V_{OUT} :

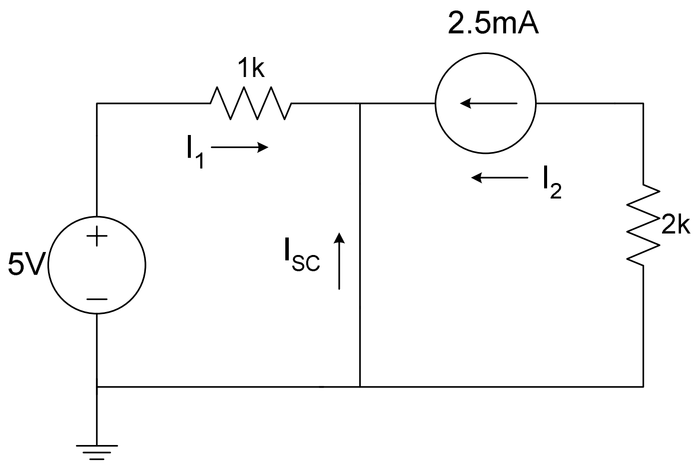


If we apply KCL at V_{OC} , then $I_1 + I_2 = 0$

$I_1 = \dfrac{5V - V_{OC}}{1k}$ and $I_2 = 2.5mA$

$$I_1 + I_2 = \frac{5V - V_{OC}}{1k} + 2.5mA = 0$$
$$V_{OC} = V_{TH} = 7.5V$$

Now we have to calculate I_{SC} (defined going into the top terminal). Short-circuiting the nodes at V_{OUT}, we obtain the new circuit:

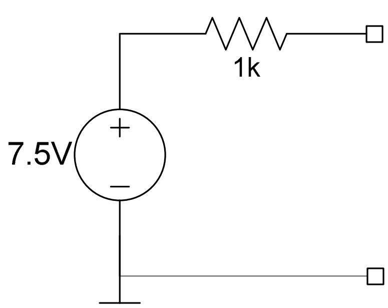


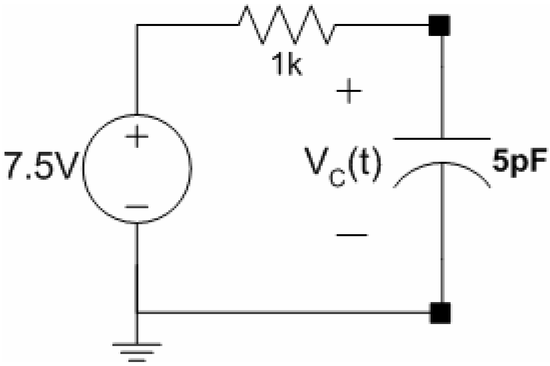
If we apply KCL at the center node, then $I_1 + I_2 + I_{SC} = 0$

$$I_1 = \frac{5V - 0V}{1k} \text{ and } I_2 = 2.5mA$$
$$I_{SC} = -\frac{5V}{1k} - 2.5mA = -7.5mA$$

Since we know I_{SC}, we can compute $R_{TH} = \frac{V_{TH}}{-I_{SC}} = \frac{7.5V}{-7.5mA} = 1k$

Knowing V_{TH} and R_{TH}, we can derive the equivalent Thevenin circuit:





Applying the steps outlined earlier, we know that $V_{INITIAL} = 0V$ (since the capacitor is uncharged) and $V_{FINAL} = 0V$. The equivalent resistance $R_{EQ} = 1k$. Therefore $RC = (1k)(2pF) = 2nS$.

Applying the formula for transient voltage across a capacitor, we obtain:

$$v_C(t) = V_{FINAL} + (V_{INITIAL} - V_{FINAL})e^{-t/RC}$$
$$v_C(t) = 7.5V + (0V - 7.5)e^{-t/2nS}$$
$$\boxed{v_C(t) = 7.5V - 7.5e^{-t/2nS}}$$

c) Even if we have a time-varying voltage, Ohm's law still applies:

$$\frac{v_C(t) - 5V}{1k} = i_R(t) = \frac{(7.5V - 7.5Ve^{-t/RC}) - 5V}{1k}$$
$$\boxed{i_R(t) = 2.5mA - 7.5e^{-t/RC}mA}$$

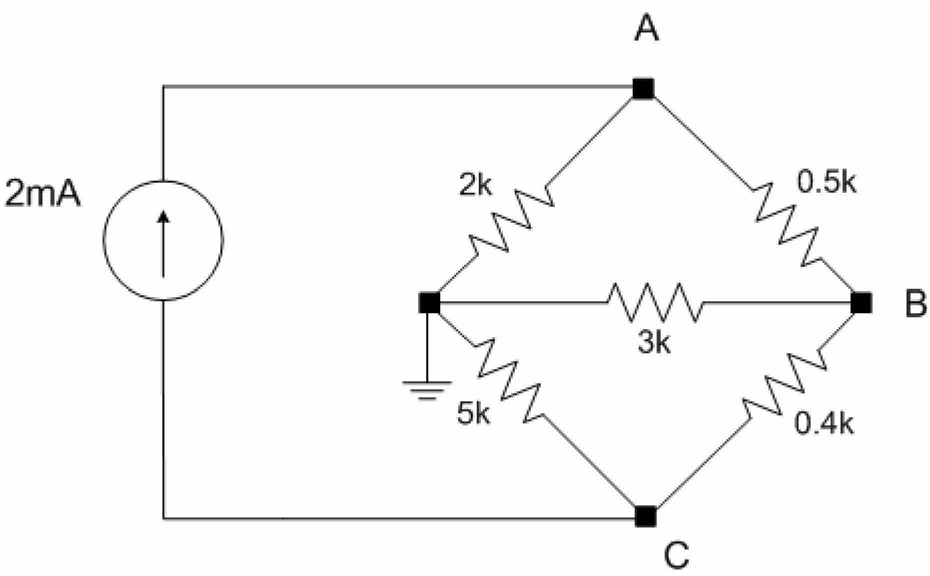
d) $E = \frac{1}{2}CV_{FINAL}^2 - \frac{1}{2}CV_{INITIAL}^2 = \frac{1}{2}(2pF)(7.5V)^2 - \frac{1}{2}(2pF)(0V)^2$
$\boxed{E = 56.25pJ}$

Problem 4.3. Nodal Analysis.
a) The two unknowns in the circuit are V_A and V_B. If we perform KCL at both nodes, we obtain:

$$\frac{V_A - 5V}{1k} + (-2.5mA) + \frac{V_A - V_B}{2k} = 0$$
$$\frac{V_B - V_A}{2k} + 2.5mA + \frac{V_B - 0V}{5k} = 0$$

b) Solving for both variables, we obtain: $V_A = 5V, V_B = 0V$.

Problem 4.4. Nodal Analysis Continued.
a) We can make this circuit easier to solve by placing the ground node where three resistors touch (gives us more zeros in our equations):



We can perform KCL at each of the three unknown nodes (A, B, C):

Node A: $\frac{V_A - 0V}{2k} - 2mA + \frac{V_A - V_B}{0.5k} = 0$

Node B: $\frac{V_A - V_B}{0.5k} + \frac{V_C - V_B}{0.4k} + \frac{0V - V_B}{3k} = 0$

Node C: $2mA + \frac{V_C - 0V}{5k} + \frac{V_C - V_B}{0.4k} = 0$

b) Solving for all the voltages, we obtain:

$\boxed{V_A = 0.58V, V_B = -0.27V, V_C = -1V}$