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Homework 5 Solutions

### 5.1 Super-node

a) Supernode consists of nodes $a, b$ and VLL. Write a KCL equation at the supernode, we obtain: (V1-Va)/R1 - Va/R2 - Vb/R3 -(Vb-V4)/R4=0 ---(eqn1). Also Vb = Va + VLL ---(eqn 2)
Plug eqn 2 into eqn 1, and solve for Va , we have $\mathrm{Va}=0.25 \mathrm{~V}$ and plug the value of Va into eqn 2, we get $\mathrm{Vb}=1.25 \mathrm{~V}$.
$\mathrm{V}_{\mathrm{a}}=0.25 \mathrm{~V}$
$\mathrm{V}_{\mathrm{b}}=1.25 \mathrm{~V}$
b) Plug in the answers from part a and see if the current flowing into the supernode sums up to zero, if it does, then the solution to part a is correct.

### 5.2 Dependent Sources

a) Do $K C L$ at node $V_{E}$, we find $V_{E}=\beta i b^{*} R_{E}+i b^{*} R_{E}$
$\mathrm{V}_{\mathrm{E}}=(\beta+1) \mathrm{ibRE}$
b) Vtest is the sum of two voltages, Ve and the voltage drop across Rin, Thus:
$V_{T E S T}=\mathrm{ibRIN}_{\mathrm{I}}+\mathrm{VE}_{\mathrm{E}} \quad$ Plug the answer from part a into this equation, we have:
$\mathrm{V}_{\text {TEST }}=\mathrm{ibRIN}+(\beta+1) \mathrm{ibRE}$
c) The current flowing between the terminals AA' is ib, and the voltage across AA' is $V_{\text {test }}$, thus the resistance seen looking into $A A^{\prime}$ is Vtestib and it is equal to:
$V_{\text {TESTIT }}=\operatorname{RIN}+(\beta+1) \operatorname{RE}^{\text {I }}$
5.3 Thevenin and Norton Equivalents Use the circuit to the right
a) This circuit is consisted of two Norton circuits in parallel, one is the 1 mA with $2 \mathrm{~K} \Omega$ and the other is 2 mA with $4 \mathrm{~K} \Omega$. Combining the two current sources, we have Itotal $=1 \mathrm{~mA}+2 \mathrm{~mA}=3 \mathrm{~mA}$. This is also the Norton equivalent current. Combining the resistors in parallel, we have
 $\underline{R N}=(2 K * 4 K) /(2 K+4 K)=(8 / 6) K \Omega=(4 / 3) K \Omega=1.33 K \Omega$
b) $\mathrm{Vth}=\mathrm{RN}^{*}$ Itotal $=4 \mathrm{~V}$, then $\underline{\mathrm{Rth}}=\mathrm{RN}=(4 / 3) \mathrm{K} \Omega=1.33 \mathrm{~K} \Omega$
c) The Thevenin equivalent circuit is:

d) The Norton equivalent circuit is:

5.4 Review of Transients Use the circuit to the right

The switch in the circuit closes at $\mathrm{t}=0$. Just before switching, the capacitor is charged to 2 V
a) Given Vinitial $=\mathrm{Vc}(\mathrm{t}=0)=2 \mathrm{~V}$

Find the final voltage on the capacitor as time go to infinity by closing the switch and open circuit the capacitor. Note, we open circuit the capacitor during steady state, therefore, no current flows into a capacitor. Vfinal is found by assuming that there is no current flowing through the


Capacitor, so
$\operatorname{Voc}(\mathrm{t}=$ infinity $)=-[\mathrm{R} 2 /(\mathrm{R} 1+\mathrm{R} 2) \mathrm{Vaa}=-[2 \mathrm{k} . /(1 \mathrm{k} .+$
2 k .)] $5 \mathrm{~V}=-3.33 \mathrm{~V}$
To identify the equivalent resistance seen by the capacitor, we open-circuit the capacitor and turn all sources to zero, that is open-circuit all current sources and short-circuit all voltage sources. As a result, the Capacitor sees Rthevenin $=\mathrm{R} 1 \| \mathrm{R} 2+\mathrm{R} 3=$ 3.67 k . and thus the time constant is:
$\tau=\mathrm{RC}=1 \mathrm{pF} 3.67 \mathrm{k} .=3.67 \mathrm{~ns}$
Recall the formula for transient voltage across a capacitor is
$\mathrm{Vc}(\mathrm{t})=$ Vfinal $+($ Vinitial-Vfinal $) \mathrm{e}^{-\mathrm{t} / \tau}$
Vfinal $=-3.33 \mathrm{~V}$
$($ Vinitial-Vfinal $)=2 \mathrm{~V}-(-3.33 \mathrm{~V})=5.33 \mathrm{~V}$, plug these values into the equation, we get $\underline{\mathrm{Vc}}(\mathrm{t})=-3.33+5.33 \mathrm{e}_{\mathrm{j}} \mathrm{Vt} / 3.67 \mathrm{~ns}$
b) To find $\mathrm{dVc}(\mathrm{t}) / \mathrm{dt}$ just before the switch closes at $\mathrm{t}=0$, we can use the formula, $\mathrm{i}(\mathrm{t})=\mathrm{c}$ $\mathrm{dV} / \mathrm{dt} \Rightarrow \mathrm{dV} / \mathrm{dt}=\mathrm{i}(\mathrm{t}) / \mathrm{C}$ where current is flowing into the + terminal. When $\mathrm{Vc}(\mathrm{t}<0)=2 \mathrm{~V}$ and the switch is open a current flows out of the positive terminal of C through R2 and R3 back to the negative terminal. Thus R2 and R 3 are in series connection. This $i(t<0)=-\mathrm{Vc} /(\mathrm{R} 2+\mathrm{R} 3)=-$ $2 \mathrm{~V} /(2 \mathrm{k} .+3 \mathrm{k})=.-0.4 \mathrm{~mA}$. Plug this value into the equation: $\mathrm{dV} / \mathrm{dt}=\mathrm{i}(\mathrm{t}) / \mathrm{C}$, we get: $\underline{\mathrm{dV} / \mathrm{dt}=-0.4 \mathrm{~mA} / 1 \mathrm{pF}=-0.4 \mathrm{~V} / \mathrm{ns}}$

