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## Homework 6 Solutions

## Problem 6.1. Logic and Truth Table from Circuit Symbols

a) Write algebraic expressions for the logic functions X and Y .
$X=A+\bar{B}$
$Y=\overline{B C}$
b) Combine these logic functions to find F .
$F=\overline{\overline{X Y}}$
$F=\overline{(A+\bar{B}) \bullet(\overline{B C})}$
c) Truth Table

| A | B | C | X | Y | F |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |

Problem 6.2. Synthesis via Sum of Products.

| A | B | C | X | Y | F |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | 0 | 1 | $\mathbf{1}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 0 | $\mathbf{1}$ |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | 1 | 0 | $\mathbf{1}$ |

To apply the sum of products method for the variable F , we identify rows that have 1 's in the F column. We continue by creating three product terms ABC and placing a NOT over each of the variables that have a corresponding zero in the column of the variable.
$F=\bar{A} B \bar{C}+\bar{A} B C+A B C$
Note: ABC is a single product term. A NOT is placed over any of the variables that have a corresponding zero in the truth table.

Problem 6.3. Synthesis of your own logic function.
Arbitrary logic function: $F=A B+C D+E$
a) Draw a circuit that realizes your sum of products form in NAND gates.

To generate a circuit using only NAND gates, it helps to apply DeMorgan's law to function F.
$F=\overline{\overline{A B+C D+E}}=\overline{\overline{A B} \bullet \overline{C D} \bullet \bar{E}}$
Note that this expression expresses all the terms using NAND functions
The gate equivalent is shown below:

b) Apply a DeMorgan's law to convert the sum to a product

$$
F=A B+C D+E=\overline{\overline{A B+C D+E}}=\overline{\overline{A B} \bullet \overline{C D} \bullet \bar{E}}
$$

c) Apply the other DeMorgan's law to convert the initial products that still appear in b) to sums.

$$
F=A B+C D+E=\overline{\overline{A B+C D+E}}=\overline{\overline{A B} \bullet \overline{C D} \bullet \bar{E}}=\overline{(\bar{A}+\bar{B}) \bullet(\bar{C}+\bar{D}) \bullet E}
$$

d) The NOR gate representation:


Any NOR or NAND gate with a signal tied to both of its inputs will produce the inverted input signal as its output.
6.4. Timing Diagram


