CIRCUIT ELEMENTS AND CIRCUIT ANALYSIS

Lecture 2 review:
- Terminology: Nodes and branches
- Introduce the implicit reference (ground) node – defines node voltages
- Introduce fundamental circuit laws: Kirchhoff’s Current and Voltage Laws

Today:
- Basic circuit elements: Ideal voltage sources and current sources; linear resistors
- Resistor formulas
- Resistors in series and the parallel
- Formal nodal analysis

ELECTRIC CIRCUITS

Circuit: Interconnection of electrical elements; modeled with two (or more) terminals

Terminology: Nodes and Branches

Node (6 Nodes)

There are also 3-terminal elements:

CIRCUIT TERMINOLOGY

Labels:
- Nodes: connection points of two or more circuit elements (together with “wires”)
- Branches: one two-terminal element and the nodes at either end

Idealizations:
- Wires are “perfect conductors” – voltage is the same at any point on the wire (add capacitor circuit element to model this phenomenon)
- Is this reasonable? Yes, in many cases.
- What are the limits?
  - “long” wires ⇒ voltage propagates at nearly $c = 3 \times 10^8$ m/s

Circuit Elements of DC Circuits

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal independent voltage source</td>
<td>$V$</td>
<td>the potential difference between the terminals is $V$</td>
</tr>
<tr>
<td>Ideal independent current source</td>
<td>$I$</td>
<td>the current through the element with reference direction given by the arrow is $I$</td>
</tr>
<tr>
<td>Ideal resistance</td>
<td>$R$</td>
<td>$V = IR$</td>
</tr>
<tr>
<td>Ideal voltmeter</td>
<td>$V$</td>
<td>$V = V_i$</td>
</tr>
<tr>
<td>Ideal ammeter</td>
<td>$V$</td>
<td>$V = 0$</td>
</tr>
</tbody>
</table>
Ideal Voltmeter and Ideal Ammeter

- Ideal voltmeter is a device which measures the voltage across its terminals while drawing no current. The current thru an ideal voltmeter is zero.
- Power = Voltage $\times$ Current = ?

- Ideal ammeter is a device which measures the current going thru it while maintains the voltage across its terminal to be zero.
- Power = Voltage $\times$ Current = ?

Ideal/Independent Voltage Sources

(a) + terminal is higher than - terminal by V volt.
(b) $v_s(t)$ is time varying
(c) The voltage vary sinusoidally (sine wave) with amplitude of 160v.

How (not) to jump start a car?

Good battery +
- Dead battery

Good battery +
- Dead battery
### Example

- Find the potential difference $V_{AB}$.

\[ V_{AB} = V_{AC} + V_{CD} + V_{DB} = (V_A - V_C) + (V_C - V_D) + (V_D - V_B) \]

or

\[ V_{AB} = V_{AD} + V_{DB} \]

Remember the “Chain-rule”?

27 milliamperes is going in the direction as shown.

If CE is any circuit element, will it affect the result?

### Ideal Current Sources

- Ideal current source (similar to voltage source)

A sinusoidal current source with a magnitude of 16 milliamperes is going in the direction as shown.

### Current Source Example

- Find the current $I$.

By itself, the 10V voltage source does not affect the current.

Apply KCL at node A:

\[ I + 3A = 0 \]

\[ I = -3A \]

### Exercise

- Find the current $I$.

By itself, the 10V voltage source does not affect the current.

At node A, apply KCL:

\[ -3mA -5mA + I = 0 \]

\[ I = 8mA \]
Ideal Resistance

- Ohm’s law stated that:
  \[ I_{A \rightarrow B} = \frac{(V_A - V_B)}{R} \]

Examples

- Terminal A is at potential -3v with respect to ground, and terminal B is at -2v. The resistance is 10Ω. What is the current?
  - Apply Ohm’s law:
    \[ I_{A \rightarrow B} = \frac{(V_A - V_B)}{R} = \frac{(-3 - (-2))}{10} = -0.1\text{A}. \]

Examples (cont.)

- Suppose the current \( I_{A \rightarrow B} \) is known to be positive, does the voltage drop from A to B positive or negative?
  - If \( I_{A \rightarrow B} > 0 \), then \( V_A - V_B > 0 \) \( \Rightarrow \) \( V_A > V_B \)

Examples (cont.)

- Suppose the “actual direction” of current is from B to A with a magnitude of 10A, the resistant is 3Ω. What is \( V_{AB} \)?
  - Remember the direction of potential drop is also the direction of the current flow.
  - By Ohm’s law: \( I_{B \rightarrow A} = \frac{(V_B - V_A)}{R} \)
    \[ \Rightarrow (V_B - V_A) = V_{BA} = R \cdot I_{B \rightarrow A} \]
    \[ \Rightarrow V_{AB} = -V_{BA} = -R \cdot I_{B \rightarrow A} = -3 \cdot (10) = -30\text{v}. \]

Example

- Reference for the voltage \( V_1 \) and current \( I_1 \) have been chosen as indicated. Measurements show that the value of \( I_1 \) is -200A. If \( R_1 = 3\Omega \), what is \( V_1 \)?
  - \[ I_1 = \frac{(V_A - V_B)}{R_1} \]
    - Notice the direction of \( I_1 \) and \( V_{AB}(=V_1) \) is consistent.
    - \[ V_1 = I_1 R_1 = (-200)(3) = -600\text{v} \]
Example

- \( R_1 = 3000 \Omega \), \( R_2 = 4700 \Omega \), and \( R_3 = 3300 \Omega \). From measurement \( V_A = -2v \), \( V_B = 3v \), \( V_C = -3v \), \( V_D = 12.735v \) (w.r.t.). Find \( I_1 \), \( I_2 \), and \( I_3 \).

\[
I_1 = \frac{(V_B - V_A)}{R_1} = \frac{(3 - (-2))}{3000} = 1.666\text{mA}
\]

\[
I_2 = \frac{(V_C - V_B)}{R_2} = \frac{(-3 - 3)}{4700} = -1.28\text{mA}
\]

\[
I_3 = \frac{(V_B - V_D)}{R_3} = \frac{(3 - 12.735)}{3300} = -2.95\text{mA}.
\]

I-V Graph for Ideal Resistance

- For an ideal resistor, the slope \((1/R)\) is constant.

Resistors in Parallel and Series

In series:

\[
v = i_{R_1} + i_{R_2} = i \left( R_1 + R_2 \right) = i \left( R \right), \text{ where } R = R_1 + R_2
\]
continue

**In Parallel**

Apply KCL at node D

\[ i = i_1 + i_2 = (v/R_1 - v/R_2) = v \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = v \frac{(R_2 + R_1)}{(R_1 R_2)} = v \frac{1}{R} \]

\[ R = \left( \frac{R_1 R_2}{R_2 + R_1} \right) \text{, where } R_1 \text{ is in parallel with } R_2 \]

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**Resistors example**

- The resistors shown has 2 terminals A and B. It is desirable to replace it with a single resistor connected between terminals A and B. What should the value of this resistor be so that the resistance between the terminals is unchanged?

By inspection, \( R_2 \) and \( R_3 \) are in series. And this series combination is in parallel with \( R_1 \).

Resistant between A and B:

\[ R = \left( \frac{R_2 + R_3}{R_1} \right) = \frac{R_1 (R_2 + R_3)}{(R_1 + (R_2 + R_3))} \]

\[ R = \frac{(10,000)(100,000+47,000)}{(10,000+100,000+47,000)} = 9360\Omega \]

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**Power Dissipation in Ideal Resistor**

Resistor can convert electrical energy into thermal energy (heat).

In Chapter 1, we learned that in any circuit element:

Power = Voltage across the circuit element x Current flow thru the circuit element

\[ \text{Power} = V \times I \]

But in the case of resistor, Ohm’s law hold

\[ V = I \times R \] (voltage difference between resistor terminals equals current flow thru resistor x resistance)

\[ \text{Power} = (I \times R) = P \times R \]

Or Power = \( V (V/R) = V^2/R \)

In practice, manufacturers state the max. power dissipation of a resistor in watts.

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**Example**

- The power dissipation of a 47000Ω resistor is stated by the manufacturer to be 1/4 Watt. What is the maximum dc voltage that may be applied? What is the largest dc current that can be made to flow through the resistor without damaging it?

From the formula:

\[ P = \frac{V^2}{R} \]

\[ P_{\text{max}} = \frac{V_{\text{max}}^2}{R} \]

\[ V_{\text{max}} = \sqrt{\left(\frac{1}{4}\right)(47000)} = 108.4V \]

From the formula:

\[ P = fR \]

\[ P_{\text{max}} = \frac{I_{\text{max}}^2}{R} \]

\[ I_{\text{max}} = \sqrt{\left(\frac{1}{4}\right)(47000)} = 2.3mA \]
CONDUCTIVITY, RESISTIVITY, AND RESISTANCE
(optional-for those of you who are interested)

If we apply a voltage $V$ across a block of conducting material of length $L$ and cross-sectional area $A$, the current, $I$, is proportional to $V$ and $A$, and inversely, proportional to $L$. $I = \frac{VA}{L}$.

The proportionality constant $\sigma$ is called the conductivity and has units:

$$(\text{Current/voltage})(\text{length}) = S/\text{cm} \quad S = \text{Siemens} = \frac{1}{\Omega}$$

$\sigma = \frac{A}{V}$

More familiar, resistivity, $\rho$

$$\rho = \frac{1}{\sigma} \quad \text{units (voltage/current) \times (length) = \Omega \text{cm}}$$

Define resistance and Ohm’s Law:

$$R = \frac{\rho L}{Wt}$$

FORMAL CIRCUIT ANALYSIS USING KCL: NODAL ANALYSIS

1 Choose a Reference Node \[ \equiv \]

2 Define unknown node voltages (those not fixed by voltage sources)

3 Write KCL at each unknown node, expressing current in terms of the node voltages (using the constitutive relationships of branch elements*)

4 Solve the set of equations (N equations for N unknown node voltages)

* with inductors or floating voltages we will use a modified Step 3

NODAL ANALYSIS USING KCL – The Voltage Divider –

1 Choose reference node

2 Define unknown node voltages

3 Write KCL at unknown nodes

$$V_{ss} - V_2 = \frac{V_2}{R_1} - 0$$

4 Solve:

$$V_2 = V_{ss} \frac{R_2}{R_1 + R_2}$$

EXAMPLE OF NODE ANALYSIS

Define the node voltages (except reference node and the one set by the voltage source); write down set of equations for node voltages $V_a$ and $V_b$

Apply KCL:

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_b}{R_2} + \frac{V_a}{R_3} = 0$$

$$\frac{V_b - V_2}{R_3} + \frac{V_b}{R_4} - I_s = 0$$

You can solve for $V_a$.

What if we used different ref node?
RESISTORS IN SERIES
(Here it's more convenient to use KVL than node analysis)

Circuit with several resistors in series – Can we find an equivalent resistance?

- KCL tells us same current flows through every resistor
- KVL tells us
  \[ I \cdot R_1 + I \cdot R_2 + I \cdot R_3 + I \cdot R_4 = V_{SS} \]
- Clearly,
  \[ I = V_{SS}/(R_1 + R_2 + R_3 + R_4) \]

Thus, equivalent resistance of resistors in series is the simple sum

GENERALIZED VOLTAGE DIVIDER

Circuit with several resistors in series

- We know
  \[ I = V_{SS}/(R_1 + R_2 + R_3 + R_4) \]
- Thus,
  \[ V_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS} \]
  and
  \[ V_3 = \frac{R_3}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS} \]

etc.. etc..

WHEN IS VOLTAGE DIVIDER FORMULA CORRECT?

- Correct if nothing else connected to nodes

What is \( V_2 \)?
Answer: \[ \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS} \]