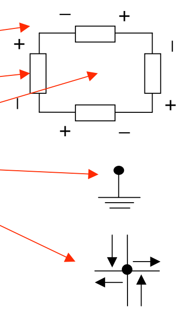
CIRCUIT ELEMENTS AND CIRCUIT ANALYSIS

Lecture 2 review:

- Terminology: Nodes and branches
- Introduce the implicit reference (ground) node – defines node voltages
- Introduce fundamental circuit laws: Kirchhoff's Current and Voltage Laws

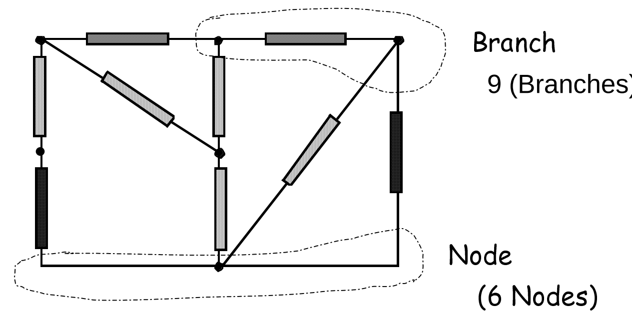


Today:

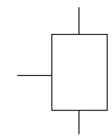
- Basic circuit elements: Ideal voltage sources and current sources; linear resistors
- Resistor formulas
- Resistors in series and the parallel
- Formal nodal analysis

ELECTRIC CIRCUITS

Circuit: Interconnection of electrical elements; modeled with two (or more) terminals   Terminology: *Nodes* and *Branches*



There are also 3-terminal elements:



CIRCUIT TERMINOLOGY

Labels:

Nodes: connection points of two or more circuit elements (together with "wires")

Branches: one two-terminal element and the nodes at either end

Idealizations:

Wires are "perfect conductors" – voltage is the same at any point on the wire (add capacitor circuit element to model this phenomenon)

Is this reasonable? Yes, in many cases.

What are the limits?

"long" wires" $\Rightarrow$ voltage propagates at nearly c = 3×10^8 m/s

Circuit Elements of DC Circuits

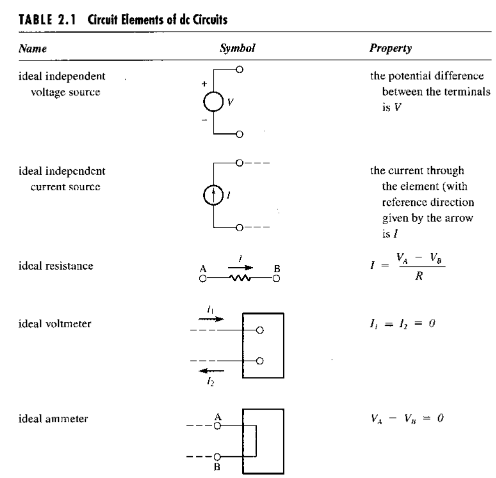


TABLE 2.1 Circuit Elements of dc Circuits

Name	*Symbol*	*Property*
ideal independent voltage source		the potential difference between the terminals is V
ideal independent current source		the current through the element (with reference direction given by the arrow) is I
ideal resistance		$I = \frac{V_A - V_B}{R}$
ideal voltmeter		$I_1 = I_2 = 0$
ideal ammeter		$V_A - V_B = 0$

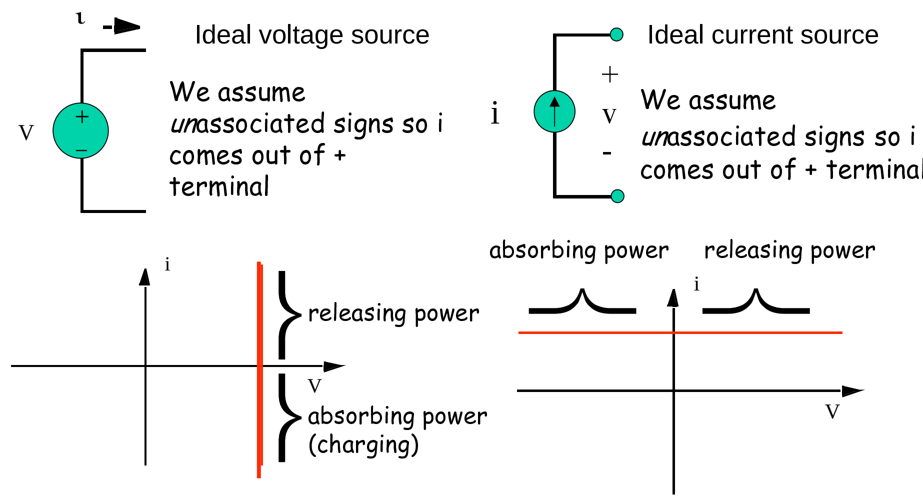
Ideal Voltmeter and Ideal Ammeter

- Ideal voltmeter is a device which measures the voltage across its terminals while drawing no current. The current thru an ideal voltmeter is zero.
- Power = Voltage X Current = ?

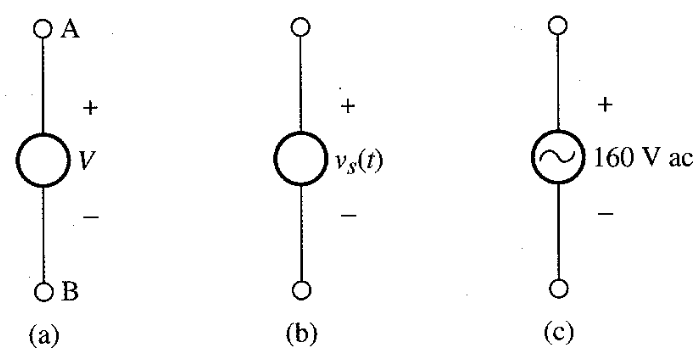
- Ideal ammeter is a device which measures the current going thru it while maintains the voltage across its terminal to be zero.
- Power = Voltage X Current = ?

CURRENT-VOLTAGE CHARACTERISTICS OF VOLTAGE & CURRENT SOURCES

Describe a two-terminal circuit element by plotting current vs. voltage

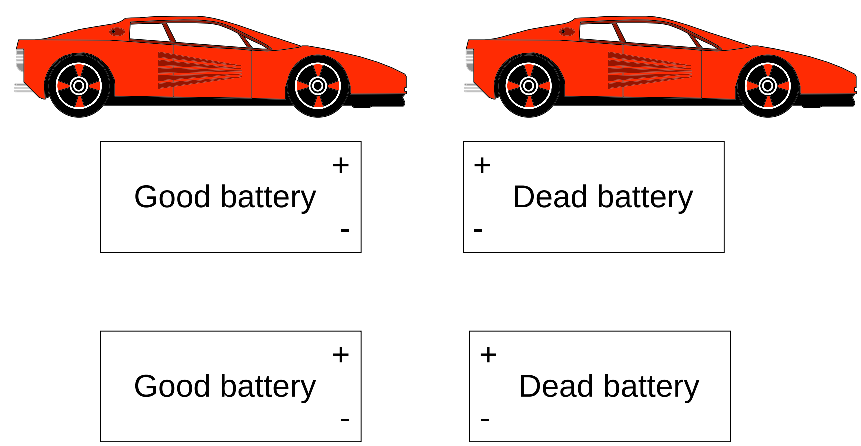


Ideal/Independent Voltage Sources



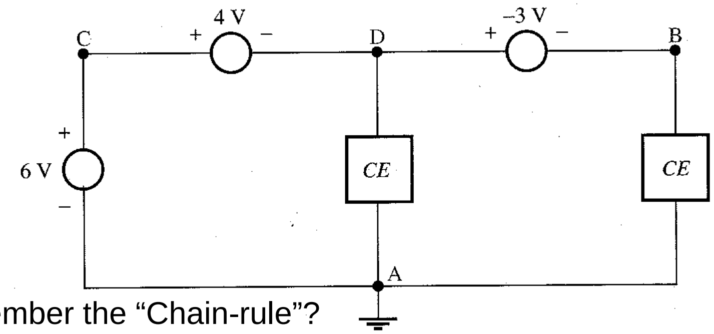
(a) + terminal is higher than - terminal by V volt.
(b) v_s(t) is time varying
(c) The voltage vary sinosoidally (sine wave) with amplitude of 160v.

How (not) to jump start a car?



Example

- Find the potential difference V_{AB}.



Remember the "Chain-rule"?

$V_{AB} = V_{AC}+V_{CD}+V_{DB} = (V_A-V_C) + (V_C-V_D) + (V_D-V_B)$

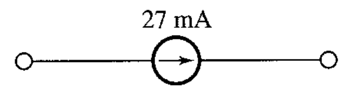
or $\quad = -6v \quad + 4v \quad + (-3v) = -5v.$

$V_{AB} = V_{AD}+V_{DB}$

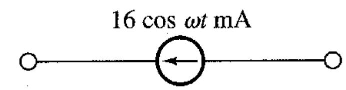
If CE is any circuit element, will it affect the result?

Ideal Current Sources

- Ideal current source (similar to voltage source)



27 milliampere is going in the direction as shown



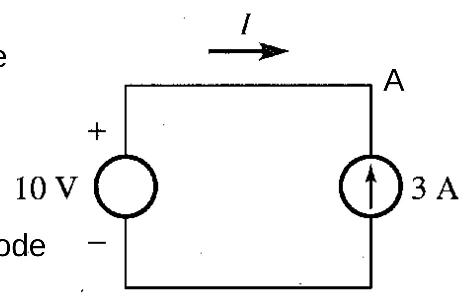
A sinusoidal current source with a magnitude of 16 milliampere is going in the direction as shown

Current Source Example

- Find the current I.

By itself, the 10V voltage source does not affect the current.

Apply KCL at node A:
Sum of all current into node A equal to zero.

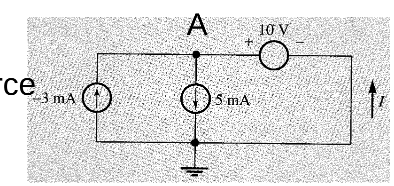


$I + 3A = 0$

$I \quad = -3A$

Exercise

- Find the current I.



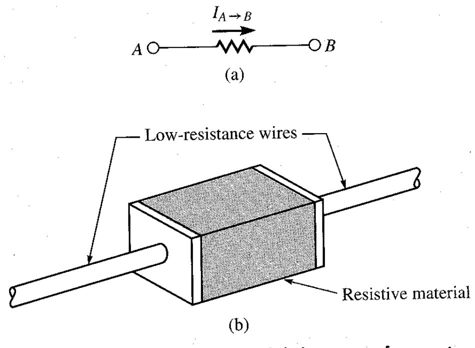
By itself, the 10V voltage source does not affect the current.

At node A, apply KCL:
Sum of all the current going into node A:
$-3mA -5mA + I = 0$
$I = 8mA.$

Ideal Resistance

- Ohm's law stated that:

$I_{A\to B} = (V_A - V_B)/R$



Examples

- Terminal A is at potential -3v with respect to ground, and terminal B is at -2v. The resistance is 10Ω. What is the current?
- Apply Ohm's law:

$$I_{A\to B} = (V_A - V_B)/R = (-3 - (-2))/(10) = -0.1A.$$

- Suppose the current $I_{A\to B}$ is known to be positive, does the voltage drop from A to B positive or negative?

If $I_{A\to B} > 0$, then $V_A - V_B > 0 \Rightarrow V_A > V_B$

Examples (cont.)

- Suppose the "actual direction" of current is from B to A with a magnitude of 10A, the resistant is 3Ω. What is V_{AB}?

Remember the direction of potential drop is also the direction of the current flow.

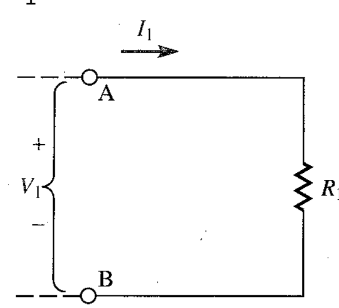
By Ohm's law: $I_{B\to A} = (V_B - V_A)/R$

$\Rightarrow (V_B - V_A) = V_{BA} = R\, I_{B\to A}$

$\Rightarrow V_{AB} = -V_{BA} = -R\, I_{B\to A} = -3\,(10) = -30v.$

Example

- Reference for the voltage V_1 and current I_1 have been chosen as indicated. Measurements show that the value of I_1 is -200A. If R_1=3Ω, what is V_1?



$I_1 = (V_A - V_B)/R_1$

Notice the direction of I_1 and $V_{AB}(=V_1)$ is consistent

$V_1 = I_1 R_1 = (-200)(3) = -600v$

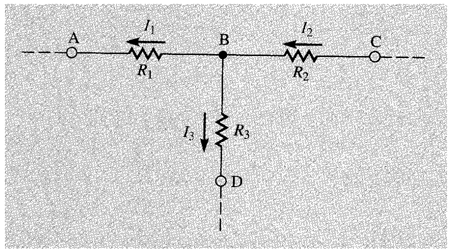
Example

- R_1=3000Ω, R_2=4700Ω, and R_3=3300Ω. From measurement V_A=-2v, V_B=3v, V_C=-3v, V_D=12.735V (w.r.t.). Find I_1, I_2,and I_3.

I_1= (V_B-V_A)/R_1=(3-(-2))/3000
= 1.666mA

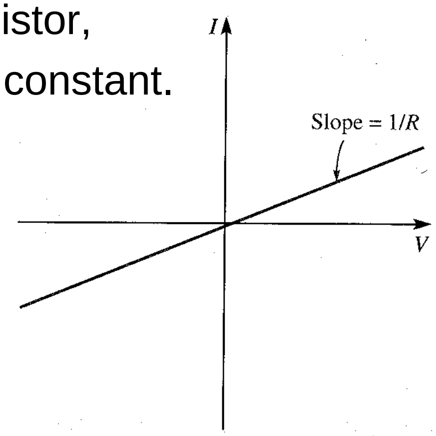
I_2= (V_C-V_B)/R_2=(-3-3)/4700
= -1.28mA

I_3= (V_B-V_D)/R_3
=(3-12.735)/3300=-2.95mA.

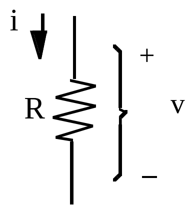


I-V Graph for Ideal Resistance

- For an ideal resistor,
the slope (1/R) is constant.

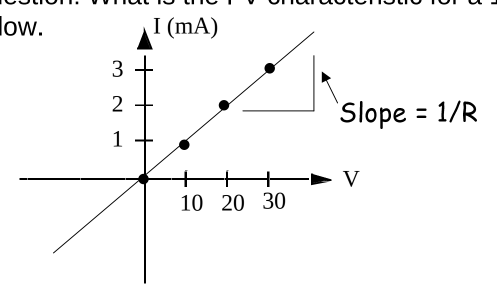


IV CHARACTERISTICS OF A RESISTOR



If we use associated current and voltage (i.e., i is defined as into + terminal), then v = iR (Ohm's law)

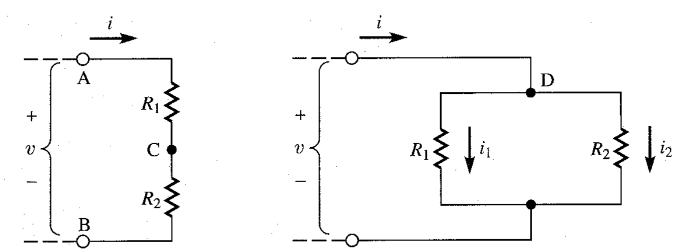
Question: What is the I-V characteristic for a 10KΩ resistor? Draw on axes below.



☞Answer: V = 0 ⇒ I = 0

V = IR ⇒ I = 1 mA
when V = 10V

Resistors in Parallel and Series



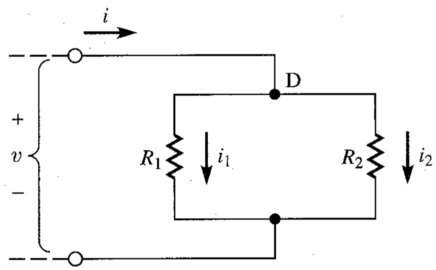
Series Parallel

In series:

$v = i R_1 + i R_2 = i (R_1+R_2) = i (R)$, where $R=R_1+R_2$

continue

- **In Parallel**



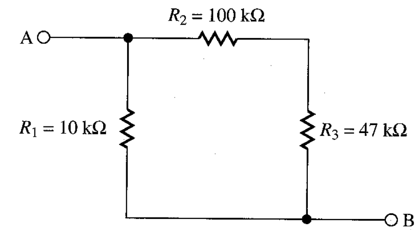
Apply KCL at node D
$i = i_1 + i_2$ = (v/R_1-v/R_2)
= v (1/R_1-1/R_2)
= v (R_2 + R_1)/($R_1 R_2$)
= v /[($R_1 R_2$)/(R_2 + R_1)]
= v /R

R = [($R_1 R_2$)/(R_2 + R_1)], where R_1 is in parallel with R_2

Resistors example

- The resistors shown has 2 terminals A and B. It is desirable to replace it with a single resistor connected between terminals A and B. What should the value of this resistor be so that the resistance between the terminals is unchanged?

By inspection, R_2 and R_3 are in series. And this series combination is in parallel with R_1.



Resistant between A and B:

R = (R_2+R_3) || R_1
= R_1(R_2+R_3)/(R_1+(R_2+R_3))
remember product over sum.
=(10,000)(100,000+47,000)/(10,000+100,000+47,000)
= 9360Ω

Power Dissipation in Ideal Resistor

Resistor can convert electrical energy into thermal energy (heat).

In Chapter 1, we learned that in any circuit element:

Power = Voltage across the circuit element X Current flow thru the circuit element

Power = V I

But in the case of resistor, Ohm's law hold

V = I R (voltage difference between resistor terminals equals current flow thru resistor X resistance)

Power = (I R) I = I^2 R

Or Power = V (V/R) = V^2/R

In practice, manufacturers state the max. power dissipation of a resistor in watts.

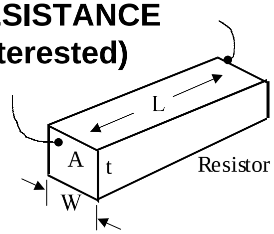
Example

- The power dissipation of a 47000Ω resistor is stated by the manufacturer to be 1/4 Watt. What is the maximum dc voltage that may be applied? What is the largest dc current that can be made to flow through the resistor without damaging it?

From the formula: P = V^2/R
P_{max} = V_{max}^2/R
get V_{max}^2 = P_{max}R
V_{max} = √[(1/4)(47000)]
= 108.4V

From the formula: P = I^2R
P_{max} = I_{max}^2 R
get I_{max}^2 = P_{max}/R
= √[(1/4)/(47000)]
= 2.3mA

CONDUCTIVITY, RESISTIVITY, AND RESISTANCE (optional-for those of you who are interested)



If we apply a voltage V across a block of conducting material of length L and cross-sectional area A, the current, I, is proportional to V and A, and inversely, proportional to L.

$$I \sim VA/L$$

The proportionality constant σ is called the conductivity and has units:

$$(\text{Current/voltage})(1/\text{length}) = \text{S/cm} \qquad \text{S} = \text{Siemens} = \frac{1}{\Omega}$$

$$I = \sigma \frac{A}{L} V$$

More familiar, resistivity, ρ

$$\rho = 1/\sigma \qquad \text{units (voltage/current)} \times (\text{length}) = \Omega\,\text{cm} = \frac{V}{A}\text{cm}$$

Define resistance and Ohm's Law: $R \equiv \frac{V}{I} = \frac{1}{\sigma}\frac{L}{A} = \rho\frac{L}{A}$ $\underline{\underline{\text{if}}}$ $A = W \times t$

$$R = \rho \frac{L}{Wt}$$

FORMAL CIRCUIT ANALYSIS USING KCL: NODAL ANALYSIS

1 Choose a Reference Node

2 Define unknown node voltages (those not fixed by voltage sources)

3 Write KCL at each unknown node, expressing current in terms of the node voltages (using the constitutive relationships of branch elements*)

4 Solve the set of equations (N equations for N unknown node voltages)

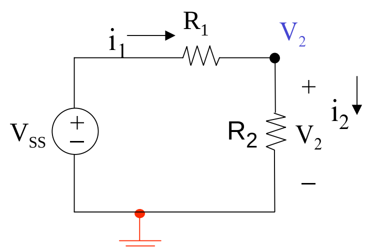
* with inductors or floating voltages we will use a modified Step 3

NODAL ANALYSIS USING KCL – The Voltage Divider –

1 Choose reference node

2 Define unknown node voltages

3 Write KCL at unknown nodes



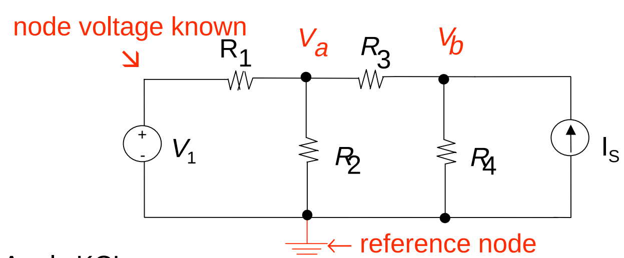
$$\frac{V_{SS} - V_2}{R_1} = \frac{V_2 - 0}{R_2}$$

4 Solve:

$$V_2 = V_{SS} \cdot \frac{R_2}{R_1 + R_2}$$

EXAMPLE OF NODE ANALYSIS

Define the node voltages (except reference node and the one set by the voltage source); write down set of equations for node voltages V_a and V_b



Apply KCL:

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_b}{R_3} + \frac{V_a}{R_2} = 0$$

$$\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} - I_S = 0$$

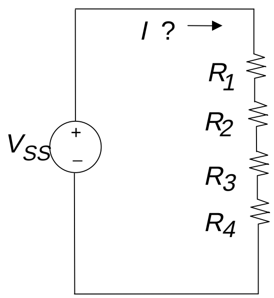
You can solve for V_a, V_b.

What if we used different ref node?

RESISTORS IN SERIES

(Here its more convenient to use KVL than node analysis)

Circuit with several resistors in series – Can we find an equivalent resistance?

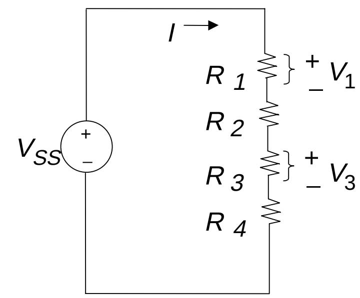


- KCL tells us same current flows through every resistor
- KVL tells us
  $I \cdot R_1 + I \cdot R_2 + I \cdot R_3 + I \cdot R_4 = V_{SS}$
- Clearly,
  $I = V_{SS}/(R_1 + R_2 + R_3 + R_4)$

☞ Thus, equivalent resistance of resistors in series is the simple sum

GENERALIZED VOLTAGE DIVIDER

Circuit with several resistors *in series*



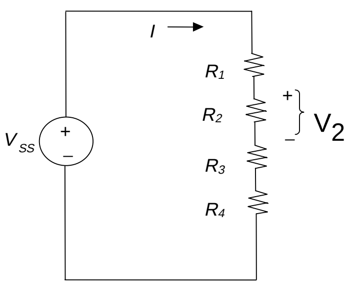
- We know
  $I = V_{SS}/(R_1 + R_2 + R_3 + R_4)$
- Thus,
  $$V_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

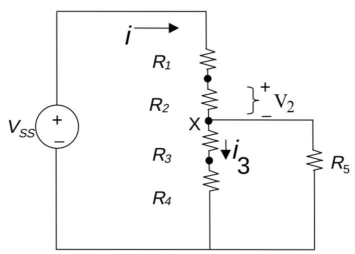
and

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS}$$

etc.. etc..

WHEN IS VOLTAGE DIVIDER FORMULA CORRECT?





$$V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \times V_{SS}$$

Correct if nothing else connected to nodes

$$V_2 \neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \times V_{SS}$$

because R_5 removes condition of resistors in series – i.e. $i_3 \neq I$

What is V_2?

Answer: $$\frac{R_2}{R_1 + R_2 + R_5 \| (R_3 + R_4)} \times V_{SS}$$