Lecture #3

- Kirchhoff’s Laws
- Ideal independent sources
- Resistors

BRANCHES AND NODES

Circuit with several branches connected at a node:

\[ \begin{align*}
    i_1 & \quad i_2 \\
    i_3 & \quad i_4
\end{align*} \]

KIRCHHOFF’S CURRENT LAW “KCL”:

(See Text 1.2 and 1.3)

\[ \text{(Sum of currents entering node)} – \text{(Sum of currents leaving node)} = 0 \]

Charge stored at node is zero. If charge is stored, for example in a capacitor, then the capacitor is a branch and the charge is stored there NOT at the node.

WHAT IF THE NET CURRENT WERE NOT ZERO?

Suppose imbalance in currents is 1 μA = 1 μC/s (net current entering node)

Assuming that \( q = 0 \) at \( t = 0 \), the charge increase is \( 10^{-6} \) C each second or \( 10^{-6}/1.6 \times 10^{-19} = 6 \times 10^{-2} \) charge carriers each second

But by definition, the capacitance of a node to ground is ZERO because we show any capacitance as an explicit circuit element (branch). Thus, the voltage would be infinite (\( Q = CV \)).

Something has to give! In the limit of zero capacitance the accumulation of charge would result in infinite electric fields … there would be a spark as the air around the node broke down.

Charge is transported around the circuit branches (even stored in some branches), but it doesn’t pile up at the nodes!

SIGN CONVENTIONS FOR SUMMING CURRENTS

Kirchhoff’s Current Law (KCL)

Sum of currents entering node = sum of currents leaving node

Use reference directions to determine “entering” and “leaving” currents … no concern about actual polarities

KCL yields one equation per node

Alternative statements of KCL

1. “Algebraic sum” of currents entering node = 0
   where “algebraic sum” means currents leaving are included with a minus sign

2. “Algebraic sum” of currents leaving node = 0
   where currents entering are included with a minus sign
KIRCHHOFF’S CURRENT LAW EXAMPLE

Currents entering the node: 24 µA
Currents leaving the node: -4 µA + 10 µA + i

\[24 = 10 + (-4) + i\]
\[i = 18 \text{ µA}\]

Three statements of KCL

\[\sum I_{in} = \sum I_{out}\]

\[\sum I_{in} = 24 + 10 + i \Rightarrow i = 18 \text{ µA}\]
\[\sum I_{out} = -4 + 10 + i \Rightarrow i = 18 \text{ µA}\]

GENERALIZATION OF KCL TO SURFACES

Sum of currents entering and leaving any “black box” is zero

Could be a big chunk of circuit in here, e.g., could be a “Black Box”

In other words there can be lots of nodes and branches inside the box.

KIRCHHOFF’S CURRENT LAW USING SURFACES

Example

\[i = 7 \text{ µA}\]
\[i must be 50 \text{ mA}\]

Another example

\[V_1, V_2, V_3, \ldots\]

KCL: \((V_1 - V_Y) \cdot R_1 = V_Y \cdot R_2\)

Given \(V_Y\), this equation can be solved for \(V_X\)

Of course we just get the same result as we obtained from our series resistor formulation. (Find the current and multiply by \(R_2\)).

BRANCH AND NODE VOLTAGES

The voltage across a circuit element is defined as the difference between the node voltages at its terminals

Specifying node voltages: Use one node as the implicit reference (the “common” node ... attach special symbol to label it)

Now single subscripts can label voltages:

\[e.g., V_a \text{ means } V_b - V_a, V_b \text{ means } V_a - V_b, \text{ etc.}\]

KIRCHHOFF’S VOLTAGE LAW (KVL)

The algebraic sum of the “voltage drops” around any “closed loop” is zero.

Why? We must return to the same potential (conservation of energy).

Voltage drop \(\rightarrow\) defined as the branch voltage if the + sign is encountered first; it is (-) the branch voltage if the – sign is encountered first ... important bookkeeping

Closed loop: Path beginning and ending on the same node

Path \(\rightarrow\) “drop”

Path \(\rightarrow\) “rise” or “step up” (negative drop)
KVL EXAMPLE

Path 1:
\[ v_a - v_b = 0 \]

YEP!

Path 2:
\[ v_b - v_c = 0 \]

Path 3:
\[ v_c - v_a = 0 \]

Note that:
\[ v_2 = v_3 = v_b \]

Examples of Three closed paths:
[Diagram of circuit with KVL example]

BASIC CIRCUIT ELEMENTS

• Voltage Source
  (always supplies some constant given voltage - like ideal battery)

• Current Source
  (always supplies some constant given current)

• Resistor
  (Ohm’s law)

• Wire
  (‘‘short’’ - no voltage drop)

• Capacitor
  (capacitor law – based on energy storage in electric field of a dielectric S&O 5.1)

• Inductor
  (inductor law – based on energy storage in magnetic field in space S&O 5.1)

DEFINITION OF IDEAL VOLTAGE SOURCE

Symbol: \( V \)

Note: The current and voltage are unassociated here.

Examples:
1) \( V = 3V \)
2) \( v = v(t) = 160 \cos 377t \)

Special cases:
- upper case \( V \) → constant voltage … called “DC”
- lower case \( v \) → general voltage, may vary with time

Current through voltage source can take on any value (positive or negative) but not infinite

IDEAL CURRENT SOURCE

“Complement” or “dual” of the voltage source: Current through branch is fixed and independent of the voltage across the branch

Actual current source examples – hard to find except in electronics (transistors, etc.), as we will see

upper-case \( I \) → DC (constant) value
lower-case implies current could be time-varying (i)

RESISTOR

We use associated current and voltage (i.e., \( i \) is defined as into + terminal), then \( v = iR \) (Ohm’s law).

Question: What is the I-V characteristic for a 1K\( \Omega \) resistor? Draw on axis below.

\[ V = 0 \Rightarrow I = 0 \]
\[ V = 1\text{m}A \Rightarrow I = 1\text{mA} \]
\[ V = 2\text{m}A \Rightarrow I = 2\text{mA} \]

etc

IDEAL WIRE

Think of a resistor with zero resistance. Clearly \( V \) is identically zero, for any current.

Question: What is the I-V characteristic?

\[ V = 0 \text{ for all } I \]

Note that all real wires and circuit connections have resistance, but we will most often approximate it to be zero, that is assume an ideal wire.
CURRENT-VOLTAGE CHARACTERISTICS OF VOLTAGE & CURRENT SOURCES

Describe a two-terminal circuit element by plotting current vs. voltage

Ideal voltage source

Assume unassociated signs

If V is positive, source is releasing power

If I is positive, source is releasing power

But if V is negative, source is absorbing power (charging)

If I is negative, source is absorbing power

Remember the voltage across the current source can be any finite value (not just zero)

And do not forget i can be positive or negative. Thus we can be in any quadrant.