EECS 42 Introduction to Electronics for Computer Science
Andrew R. Neureuther

Lecture #4
- Capacitors and Inductors
- Energy Stored in C and L
- Equivalent Circuits
  - Thevenin
  - Norton
  
http://inst.EECS.Berkeley.EDU/~ee42/

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**Game Plan 02/03/03**

- Capacitors and Inductors; Equivalent Sources
  - Schwarz and Oldham: 5.1-5.2, 3.1
- N-L Elements; Graphical Solutions; Power
  - Schwarz and Oldham: 3.2-3.4

Next (4th) Week
- RC Transient
  - Schwarz and Oldham: 8.1 plus Handouts

Problem Set #2 – Out 1/27/03 - Due 2/5/03 2:30 in box near 275 Cory
  - 2.1 Flow; 2.2 KCL; 2.3 KVL; 2.4 resistor circuit; 2.5 Power

Problem Set #3 – Out 2/2/03 - Due 2/12/03 2:30 in box near 275 Cory
  - 3.1 and 3.2 charging capacitors; 3.3–3.5; Equivalent Circuits;

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**What Goes in the Circuit Element Boxes?**

![Circuit Element Boxes](image)

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**BASIC CIRCUIT ELEMENTS**

- **Voltage Source**
  - (always supplies some constant given voltage - like ideal battery)
- **Current Source**
  - (always supplies some constant given current)
- **Resistor**
  - (Ohm’s law)
- **Wire**
  - ("short" – no voltage drop)
- **Capacitor**
  - (capacitor law – based on energy storage in electric field of a dielectric S&O 5.1)
- **Inductor**
  - (inductor law – based on energy storage in magnetic field in space S&O 5.1)

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**CAPACITOR**

- Any two conductors a and b separated by an insulator with a difference in voltage \( V_{ab} \) will have an equal and opposite charge on their surfaces whose value is given by \( Q = CV_{ab} \), where \( C \) is the capacitance of the structure, and the charge is on the more positive electrode.

- A simple parallel-plate capacitor is shown. If the area of the plate is \( A \), the separation \( d \), and the dielectric constant of the insulator is \( \varepsilon \), the capacitance equals \( C = \frac{A \varepsilon}{d} \).

  - **Symbol**
  - **Constitutive relationship**: \( Q = C (V_a - V_b) \).

  - \( Q \) is positive on plate a if \( V_a > V_b \).

  - But \( \frac{Q}{V} \) is minus, so equivalent to \( \frac{Q}{V} = C \), where we use the associated reference directions.

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**ENERGY STORED IN A CAPACITOR**

- You might think the energy (in Joules) is \( QV \), which has the dimension of joules. But during charging the average voltage was only half the final value of \( V \).

  - Thus, energy is \( \frac{1}{2} CV^2 \)
ENERGY STORED IN A CAPACITOR (cont.)
More rigorous derivation: During charging, the power flow is \( i \) into the capacitor, where \( i \) is into + terminal. We integrate the power from \( t = 0 \) (v = 0) to \( t = \text{end} \) (v = V). The integrated power is the energy

\[
E = \int_{t_\text{Initial}}^{t_\text{Final}} v \cdot i \, dt = \frac{1}{2} CV_\text{Final}^2 - \frac{1}{2} CV_\text{Initial}^2
\]

but \( dq = C \, dv \). (We are using small \( q \) instead of \( Q \) to remind us that it is time varying. Most texts use \( Q \).

\[
E = \frac{1}{2} CV_\text{Final}^2 - \frac{1}{2} CV_\text{Initial}^2
\]

Clearly,

\[
E = \int_0^t v \, i \, dt = \int_0^t \frac{1}{C} \, dq = \int_0^t \frac{1}{C} \, dv
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CAPACITORS IN PARALLEL
Add Currents

\[
\begin{align*}
&\text{Add Currents} \\
&i(t) = \frac{C_1}{C_1 + C_2} \, v(t) \\
&i(t) = \frac{C_2}{C_1 + C_2} \, v(t)
\end{align*}
\]

Equivalent capacitance defined by

\[
i = C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}
\]

Clearly,

\[
C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}
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CAPACITORS IN SERIES
Add Voltages

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&\text{Add Voltages} \\
&i(t) = \frac{C_1}{C_1 + C_2} \, v(t) \\
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CAPACITANCE AND INDUCTANCE

- Capacitors: two-plate example; Store energy in the electric field \( Q = CV \), \( I = C \, dV/dt \) and \( V = \frac{1}{C} \) integral of voltage
- Computer example 1 mAh current charging 1 PF
  \( V(t) = \frac{1}{C} \frac{q}{dt} = \frac{1}{C} \frac{0.1 \, \mu A}{1 \, \mu F} = 10^9 \text{V/s} \)
- At D.C. time derivatives are zero \( \Rightarrow C \) is open circuit
- \( C \) in parallel add; series \( 1/C = \text{sum} (1/C_i); \) short together (infinite current but conserve charge)
- Inductors: coil example; Store energy in the magnetic field; Flux = LI, \( V = L \, dI/dt \) and \( I = \frac{1}{L} \) integral of voltage
- At D.C. time derivatives are zero \( \Rightarrow L \) is short circuit
- \( L \) in parallel \( 1/L = \text{sum} (1/L_i) \); series add; connect in series when have different currents \( \Rightarrow L_1 I_1 + L_2 I_2 = (L_1 + L_2) \text{NEW} \)

EXAMPLES OF I-V GRAPHS
Resistors in Series
If two resistors are in series the current is the same; clearly the total voltage will be the sum of the two IR values i.e. \( IR_1 + IR_2 \).
Thus the equivalent resistance is \( R_1 + R_2 \) and the I-V graph of the series pair is the same as that of the equivalent resistance.
Of course we can also find the I-V graph of the combination by adding the voltages directly on the I-V axes. Lets do an example for 1K + 4K resistors

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&\text{Example of I-V Graphs} \\
&\text{Resistors in Series} \\
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Simplest Equivalent Circuits

An adequately equivalent circuit is one that has an \( I \) vs. \( V \) graph that is identical to that of the original circuit.

**Combined Circuit**

\[
\begin{align*}
V_{IN} & \quad I_1 \quad R_1 \\
I_2 & \quad R_2 \\
V_{OUT} & \quad R_3
\end{align*}
\]

\[
R_{TH} = RN = (-ISC/VOC)
\]

\[
V_{TH} = VOC
\]

\[
IN = -ISC
\]

**I vs. V and Equivalent Circuits**

- \( I \) vs. \( V \) for ideal voltage source is a vertical line at \( V = V_{SV} \)
- \( I \) vs. \( V \) for ideal current source is a horizontal line at \( I = I_{SC} \)
- \( I \) vs. \( V \) for a circuit made up of ideal independent sources and resistors is a straight line.
- The simplest circuit for a straight line is an ideal voltage source and a resistor (Thevenin) or a current source and a parallel resistor (Norton)
- The easiest way to find the \( I \) vs. \( V \) line is to find the intercepts where \( I = 0 \) (open circuit voltage \( V_{OC} \)) and where \( V = 0 \) (short circuit current \( I_{SC} \))
- The shortcut for finding the \( \text{slope}^{-1} = R_T = R_N \) is to turn off all of the dependent sources to zero and find the remaining equivalent resistance between the terminals of the elements.