Charging and Discharging RC Circuits
Handout for EECS 42

Developed by Professor W.G. Oldham to provide understanding of transient issues in computer logic.

Extensions by Professor A.R. Neureuther in Spring 2003 to include sequential switching of logic gates as occurs in the EECS 43 logic gate experiment.

Time delay $\tau_0$ occurs between input and output: “computation” is not instantaneous.

Value of input at $t = 0^+$ determines value of output at later time $t = \tau_0$.

Logic states and capacitance to ground:

Input (A and B tied together) vs. time.

Output (Ideal) vs. time.

Actual voltage versus time.

Simplification for time behavior of RC Circuits

Before any input change occurs we have a dc circuit problem (that is we can use dc circuit analysis to relate the output to the input).

Long after the input change occurs things settle down .... Nothing is changing .... So again we have a dc circuit problem.

We call the time period during which the output changes transient.

We can predict a lot about the transient behavior from the pre- and post-transient dc solutions.

Charging and discharging in RC Circuits
(an enlightened approach)

Before we analyze real electronic circuits - lets study RC circuits.

Relevance to digital circuits:
We communicate with pulses
We send beautiful pulses out
But we receive lousy-looking pulses and must restore them

RC charging effects are responsible .... So lets review them.

What environment do pulses face?

Every real wire in a circuit has resistance.
Every junction (node) has capacitance to ground and to other nodes.
The active circuit elements (transistors) add additional resistance in series with the wires, and additional capacitance in parallel with the node capacitance.

Thus the most basic model circuit for studying transients consists of a resistor driving a capacitor.

A pulse originating at node I will arrive delayed and distorted at node O because it takes time to charge C through R.

If we focus on the circuit which distorts the pulses produced by $V_{in}$, its simplest form consists simply of an R and C. ($V_{in}$ represents the time-varying source which produces the input pulse.)
After the transient is over (nothing changing anymore) it means $\frac{dV}{dt}$ → 0.

- $V_{out}$ approaches its final value asymptotically (it never actually gets exactly to $V_1$, but it gets arbitrarily close). Why?
- After the transient is over (nothing changing anymore) it means $\frac{dV}{dt}$ = 0; that is all currents must be zero. From Ohm's law, the voltage across R must be zero, i.e. $V_{in} = V_{out}$. That is, $V_{out} \to V_1$ as $t \to \infty$.

Review of simple exponentials.

### Rising Exponential from Zero

$$V_{out} = V_1(1-e^{-t/\tau})$$

- at $t = 0$, $V_{out} = 0$, and $V_{in} \to V_1$ also.
- at $t \to \infty$, $V_{out} = 0.63V_1$.

### Falling Exponential to Zero

$$V_{out} = V_1e^{-t/\tau}$$

- at $t = 0$, $V_{out} = V_1$, and $V_{in} \to 0$, also.
- at $t \to \infty$, $V_{out} = 0.37V_1$.

Exact form of $V_{out}$?

- $V_{out} = V_1e^{-t/\tau}$

Equation for $V_{out}$: Do you remember general form?

- $V_{out} = V_1(1-e^{-t/\tau})$
Further Review of simple exponentials

Rising Exponential from Zero
\[ V_{out} = V_i(1-e^{-t/RC}) \]
Falling Exponential to Zero
\[ V_{out} = V_i e^{t/RC} \]

We can add a constant (positive or negative)
\[ \tau \]
where \( \tau \) is the time constant related to \( R \) and \( C \).

\( \tau \) in RC RESPONSE

- If \( C \) is bigger, it takes longer (\( \tau \)).
- If \( R \) is bigger, it takes longer (\( \tau \)).

Thus, \( \tau \) is proportional to \( RC \).

In fact, \( \tau = RC \).

Thus, \( V_{out} = V_i(1-e^{RC}) \)

Generalization

Vin switches at \( t = 0 \), then for any time interval \( t > 0 \), in which Vin is a constant, Vout is always of the form:
\[ V_{out} = A + B e^{-t/RC} \]

We determine \( A \) and \( B \) from the initial voltage on \( C \), and the value of \( Vin \). Assume \( Vin \) "switches" at \( t = 0 \) from \( V_{co} \) to \( V_{1} \):

Thus, \( V_{out} = V_{co} + (V_{1} - V_{co}) e^{-t/RC} \)

You may choose to solve RC problems using this "A and B" formulation, but in the next lecture we show you an easier way.
Re-Cap: Charging and discharging in RC Circuits

Last Time:
We learned that simple RC circuit with a step input has a universal exponential solution of the form:
\[ V_{\text{out}} = A + Be^{-t/RC} \]
Example 0: \( R = 1K, C = 1\mu F \), \( V_{\text{in}} \) steps from zero to 10V at \( t=0 \):
1) Initial value of \( V_{\text{out}} \) is 0
2) Final value of \( V_{\text{out}} \) is 10V
3) Time constant is \( RC = 10^{-9} \) sec
4) \( V_{\text{out}} \) reaches 0.63 X 10% of 10V in 10 seconds
5) Sketch waveform (starts at \( V_{\text{co}} \), ends asymptotically at \( V_{\text{1}} \), initial value of \( V_{\text{out}} \) is 0)

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Charging and discharging in RC Circuits

(Example 1 of the EE42 Easy Method)

Find \( V_{\text{c}}(t) \) for the following circuit: (input switches from 2V to -1V at \( t = 0 \))

1) Simplify the circuit:
2) The time constant of the transient is \( \tau = RC \)
3) Solve the dc problem for the capacitor voltage before the transient.
   This is the starting value (initial value) for the transient voltage.
4) Solve the dc problem for the capacitor voltage after the transient is over.
   This is the asymptotic value.
5) Sketch the Transient. It is 63% complete after one time constant.
6) Write the equation by inspection.

Charging and discharging in RC Circuits

(Example 1 of the EE42 Easy Method)

Find \( V_{\text{c}}(t) \) for the following circuit: (input switches from 2V to -1V at \( t = 0 \))

1) Simplify the circuit:
2) The time constant of the transient is \( \tau = RC = 20\mu \text{sec} \)
3) Before the transient \( V_{\text{in}} = 2V \) so \( V_{\text{c}} = 2V \)
4) After the transient is over \( V_{\text{in}} = -1V \) so \( V_{\text{c}} = -1V \). This is the asymptotic value.

2 MORE EXAMPLES ---OUR METHOD AVOIDS ALL MATH!

a. Sketch waveform (starts at \( V_{\text{co}} \), ends asymptotically at \( V_1 \), initial slope intersects at \( t = RC \) or transient is 63% complete at \( t=RC \))
   Write equation:
   \[ V_{\text{c}}(t) = V(0) \left[ 1 - e^{-t/RC} \right] \]
   vs. \[ V(t) = V(0) \left[ 1 - e^{-t/RC} \right] \]
   \( V(t) \) vs. \( t \)
   \( 2 \) More Examples ---Our Method Avoilds All Math!

b. pre-exponent \( B = \) initial value - constant term
EXAMPLE of CHARGING to 95%

Your photo flash charges a 1000 μF capacitor from a 50 V source through a 2K resistor. If the capacitor is initially uncharged, how long must you wait for it to reach 95% charged (47.5 V)?

Solution: RC = 2K × 10⁻³ = 2 sec

By inspection: \( V_c = 50 - 50e^{-t/2} \), so

\[
47.5 = 50(1 - e^{-t/2}) \\
\Rightarrow \frac{t}{2} = (1 - \frac{47.5}{50}) \\
\Rightarrow t = 6 \text{ sec}
\]