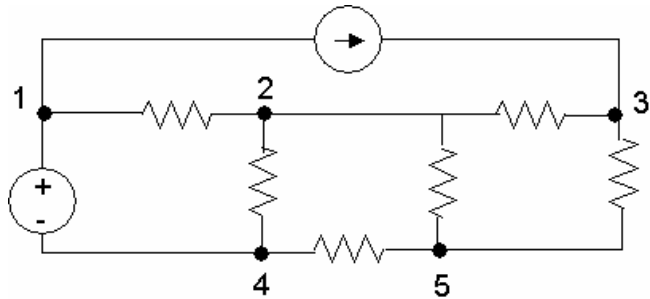## Lecture 27: Graph Theory in Circuit Analysis

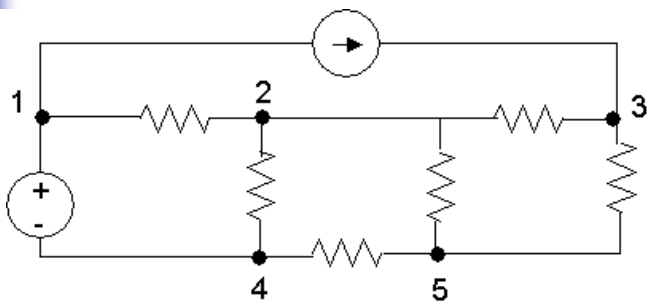Suppose we wish to find the node voltages of the circuit below.

We know how to do this by hand.

For large-scale circuits, we may wish to do this via a computer simulation (i.e. PSpice). We will need to express this circuit in a standard form for input to the program.

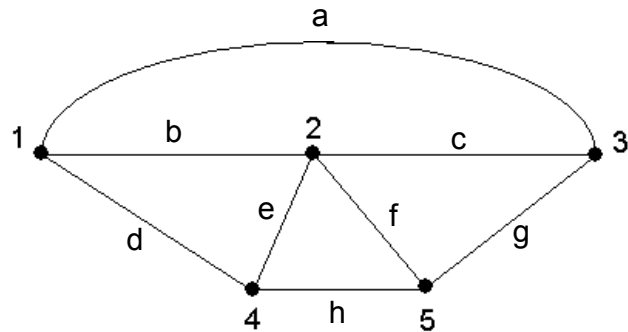## Graph Theory in Circuit Analysis

Whether the circuit is input via a GUI or as a text file, at some level the circuit will be represented as a graph, with elements as edges and nodes as nodes.

For example, when entering a circuit into PSpice via a text file, we number each node, and specify each element (edge) in the circuit with its value and endpoints.
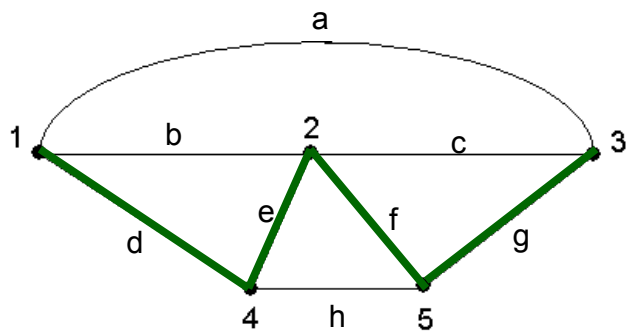
Graph of a Circuit

Here is a **graph** of the circuit. It is simply the circuit without elements.

We refer to the lines above as **edges** (and the nodes are **nodes**).

The graph provides **connectivity** information. To actually solve the circuit using this graph, the types of elements forming the edges would need to be provided.
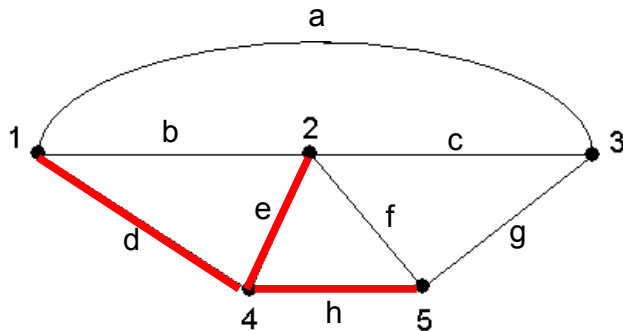
Trees and Co-Trees

A **tree** is defined as any set of edges in a graph that touches every node *without forming any closed paths.*

Also known as Hamiltonian path!

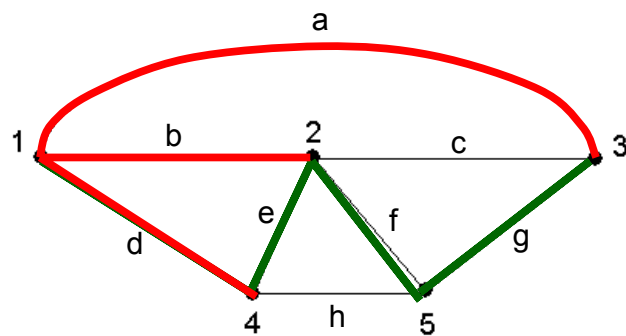Each tree has a **co-tree**, which is the set of edges not in the tree.

Cut Set

A **cut set** is a *minimal* set of edges that, when broken, breaks the graph into two completely separate parts (two groups of nodes.

Minimal means that a cut set cannot contain another smaller cut set that would break the graph into the same two parts.
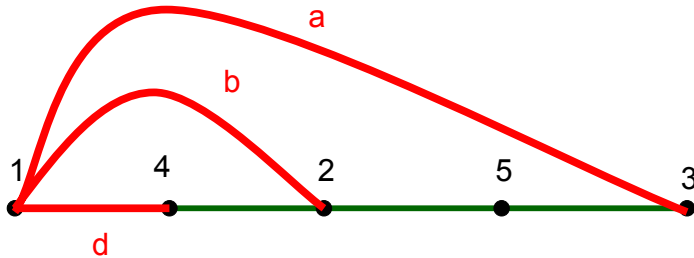
Fundamental Cut Set

Suppose I am given a tree.

A **fundamental cut set** w.r.t. that tree is a cut set that only contains one branch of the tree.

There may be many fundamental cut sets w.r.t. a given tree.

Finding Fundamental Cut Sets Systematically

1. Redraw the graph with the tree in a straight line.
2. For each tree edge, form its fundamental cut set as follows:
   - 2a) that tree edge is a member of this fundamental cut set
   - 2b) cut that edge…what two groups of nodes are separated?
   - 2c) the fundamental cut set also contains all edges in the co-tree that connect these two groups.

How is this used in circuit analysis?

- Identifying a tree for a circuit, and all of the fundamental cut sets that go with it, can be used in nodal analysis.
- Here are the steps simulation software may take to perform nodal analysis:

1. From user input, make a connectivity matrix (graph) and record the circuit element on each edge.
2. Choose a tree using the following guidelines:
   a) Place an edge in the **tree** if it contains a voltage source, or if the voltage over the edge controls a dependent source.
   b) Place an edge in the **co-tree** if it contains a current source, or if the current in the edge controls a dependent source.
3. Find all of the fundamental cut sets for this tree.
   - ■ *n* nodes yields *n-1* fundamental cut sets

How is this used in circuit analysis?

4. Each fundamental cut set breaks the circuit into two pieces: **two supernodes**. Write a KCL equation for one supernode in each fundamental cut set (in terms of node voltages).
   - The KCL equations for the two supernodes formed by a fundamental cut set will be the same.
   - This is where the circuit element info comes into play.
   - This yields $n-1$ equations in n node voltage variables.
5. Set one node voltage to zero volts (ground) and solve .

Notes

- All of this can be done computationally.
  - Graph algorithms
  - Linear equation solution
- This algorithm shows why nodal analysis always works: you get $n-1$ independent linear equations in $n-1$ unknowns.
- The fundamental cut sets ensure independence of the equations—unless the circuit has impossible elements.
  - Each fundamental cut set contains a unique element (edge) from the tree. So each KCL equation provides new info.
  - The elements themselves could destroy the independence (redundant dependent source, shorted voltage source…) but this won't happen in real life circuits.

Example
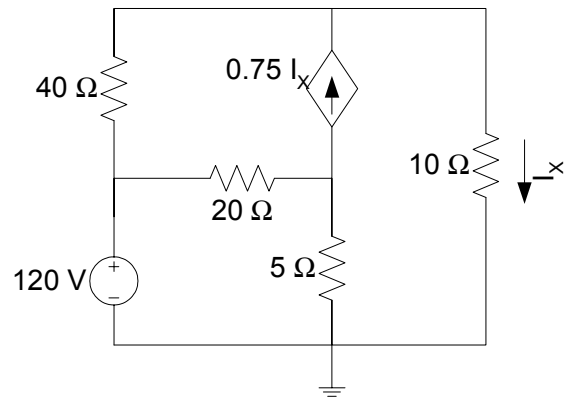
Find the node voltages using the graph method.

Circuit from Nilsson's *Electric Circuits*, Addison-Wesley, 1993.