Lecture 6

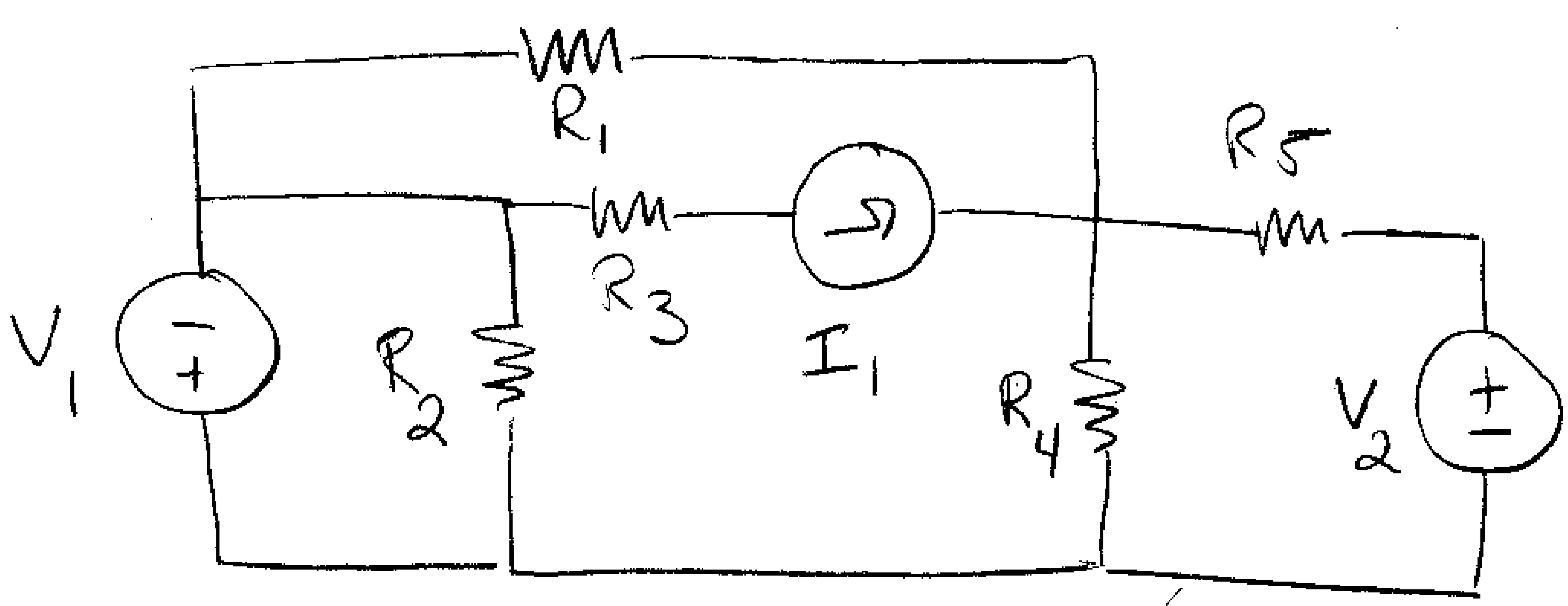
Today we will see examples using:

- Nodal Analysis
- "As needed" application of KVL + KCL, resistor combination, Voltage + current division
- Realistic sources and measuring instruments

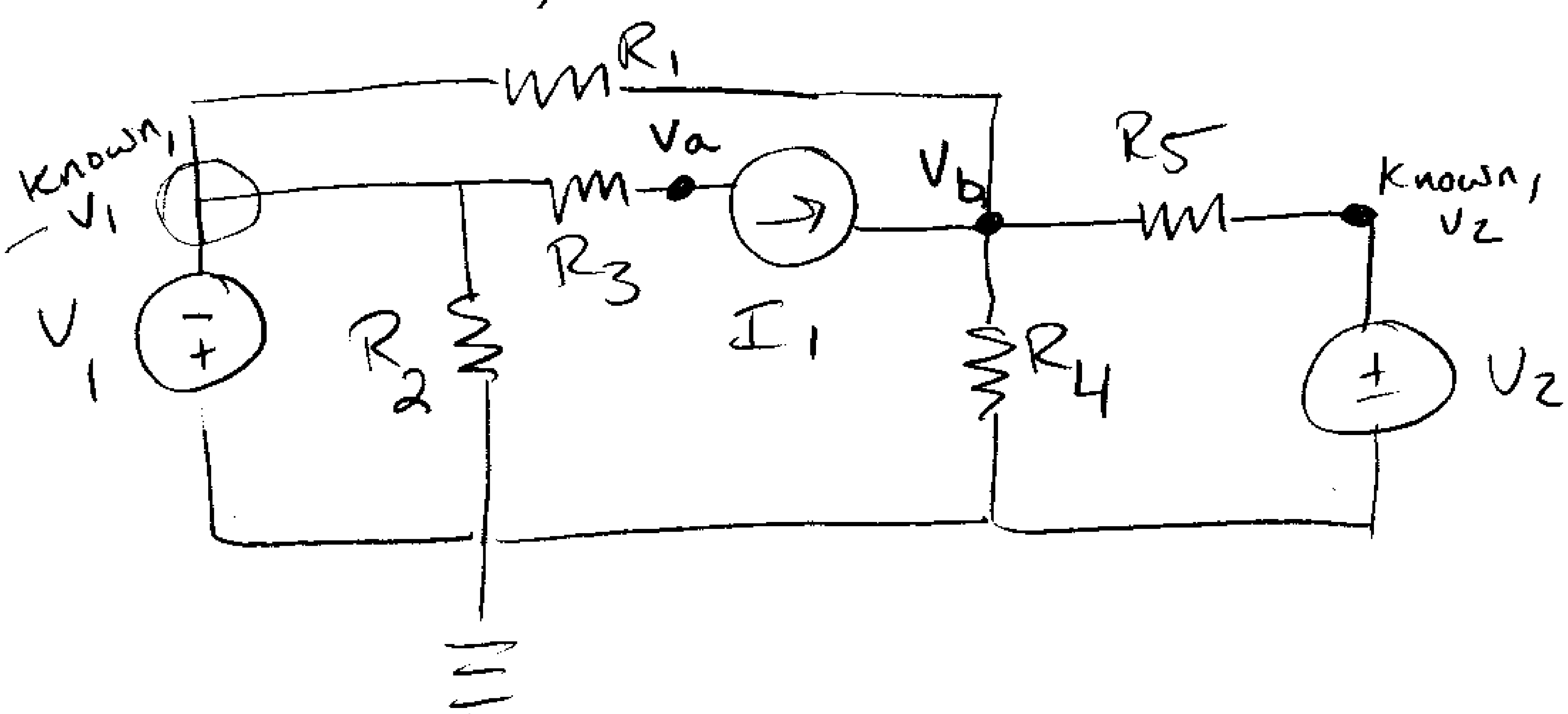
Nodal Analysis

1. Choose reference node (bottom or most connections)
2. Give names to all unknown node voltages (those not connected to ground by voltage source)
3. Write KCL equation at each unknown node
   - Add up all currents that could leave node
   - If current goes thru resistor, use Ohm's law to specify current.
   - If current given by current source, just use value.
   - If current thru voltage source, draw surface around source and KCL it. Also write down relationship between voltage source terminals.
4. Solve equations for node voltages.

Perform nodal analysis:



1. Choose ground. I will choose the bottom node.
2. Identify unknown node voltages:

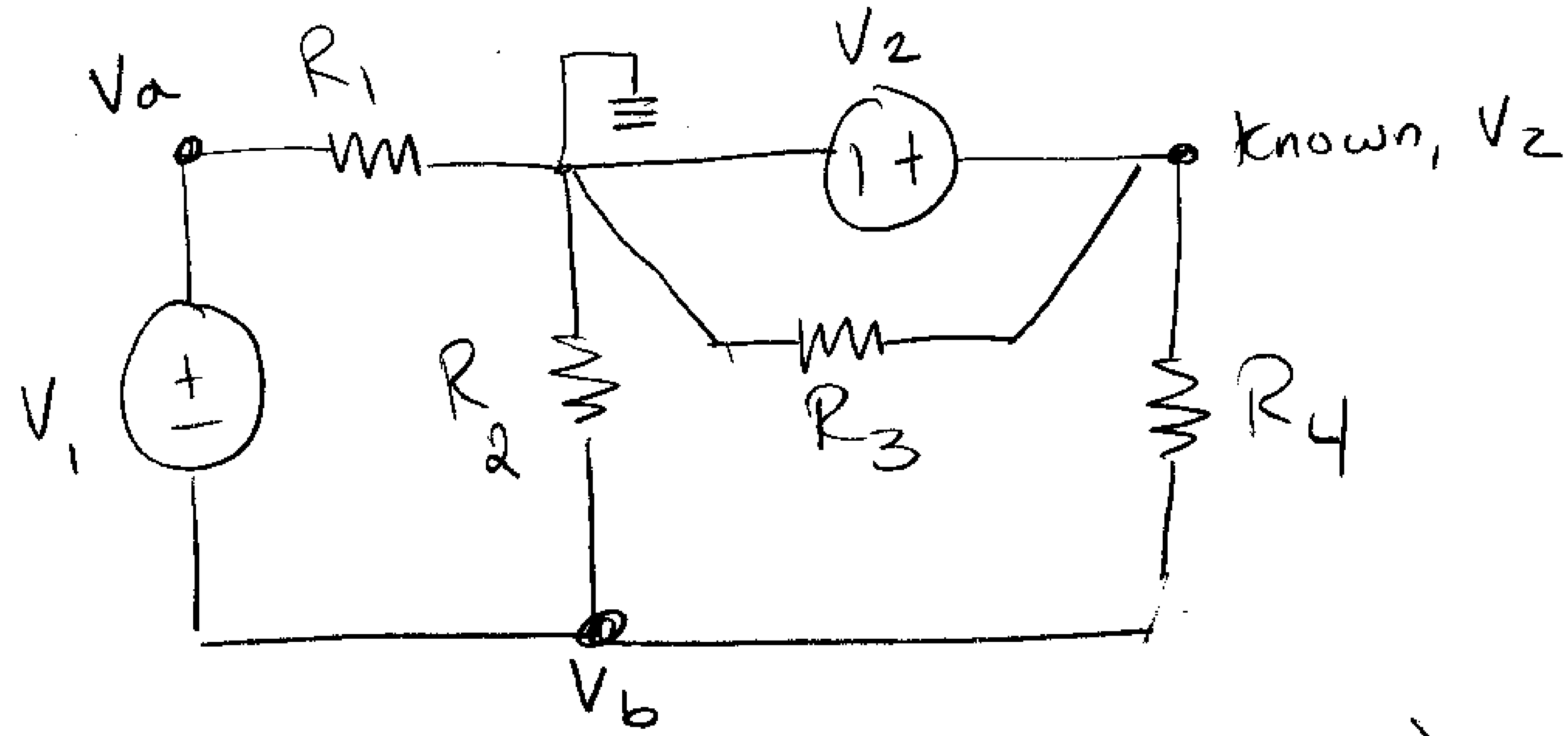


3. Write KCL equations:

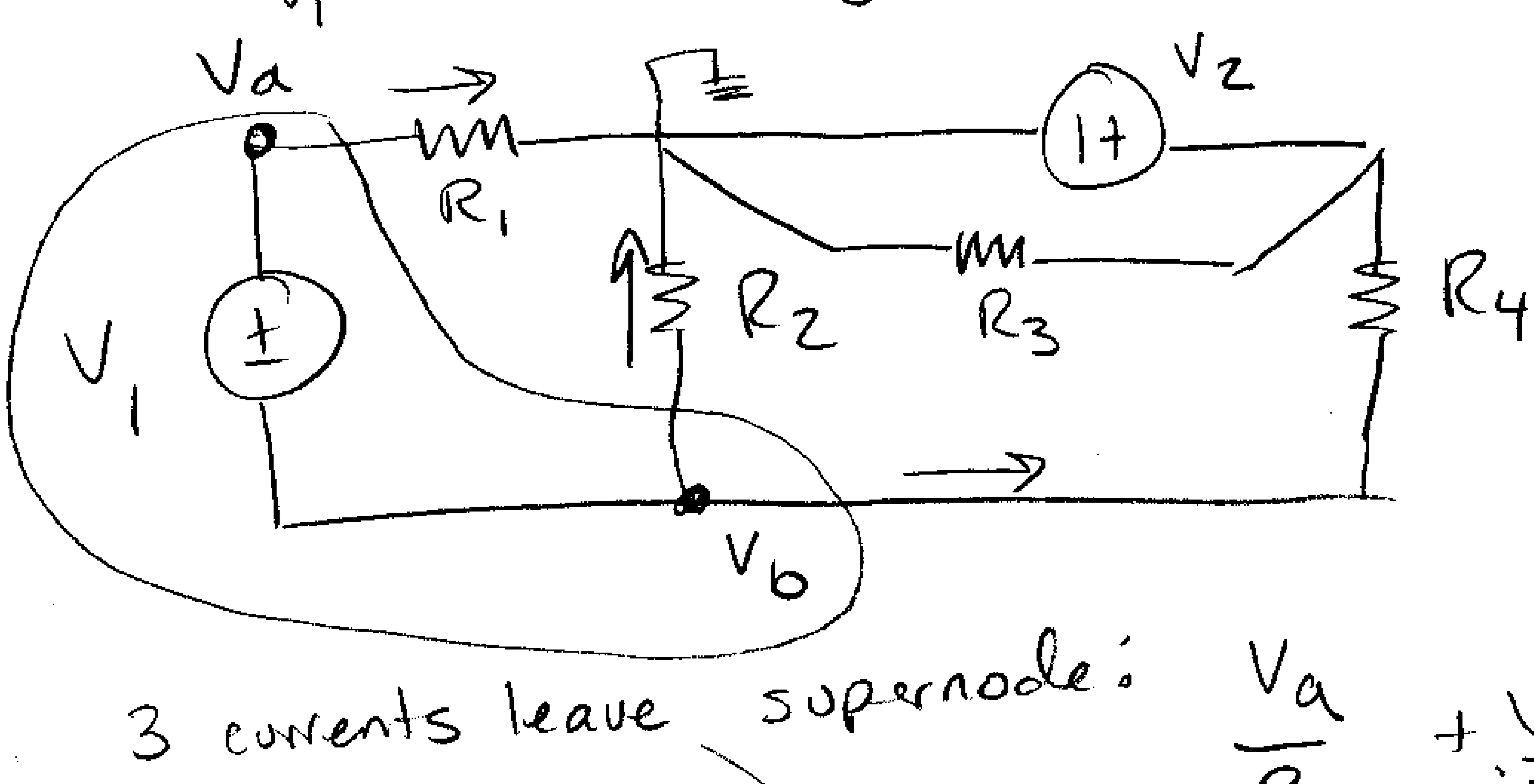
@ V_a: $\underbrace{\frac{V_a - -V_1}{R_3}}_{\text{current going left thru } R_3} + I_1 = 0$

@ V_b: $\underbrace{\frac{V_b - -V_1}{R_1}}_{\text{current going left thru } R_1} + \underbrace{\frac{V_b}{R_4}}_{\text{current down thru } R_4} + \underbrace{\frac{V_b - V_2}{R_5}}_{\text{current going right thru } R_5} - I_1 = 0$

Perform Nodal Analysis:



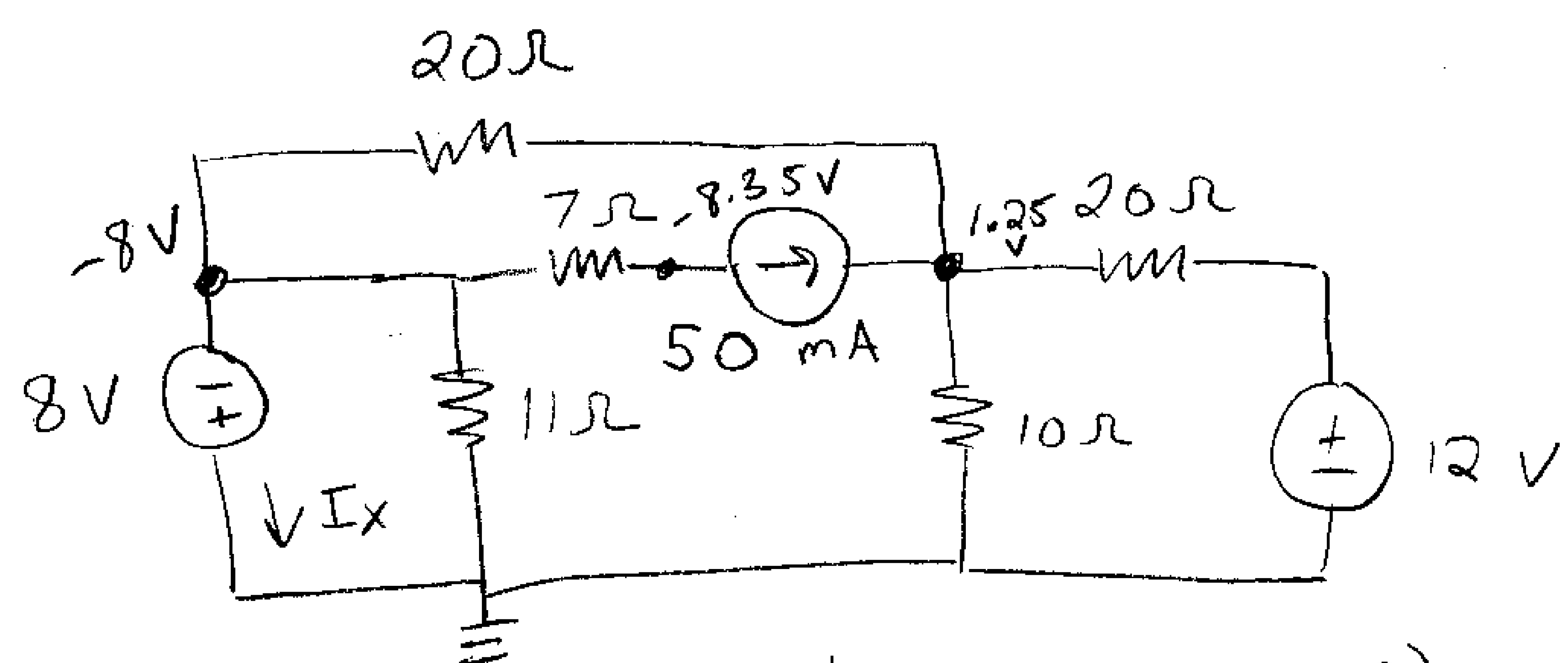
1. Choose ground (I did above).
2. Identify unknown node voltages.
   Notice that neither Va nor Vb is directly connected to ground via voltage source - hence voltages unknown. There are resistors between Va / Vb & ground.
3. Write KCL equations.
   V_1 is "floating". Supernode around it.



3 currents leave supernode: $\frac{V_a}{R_1} + \frac{V_b}{R_2} + \frac{V_b - V_2}{R_4} = 0$

What does V_1 say about node voltages? $V_a - V_b = V_1$

Use the previous Nodal Analysis combined with other tools to find I_x:



By previous analysis (page 2),

$$\frac{V_a + 8V}{7\Omega} + 50mA = 0 \qquad V_a = -8.35\ V$$

$$\frac{V_b + 8V}{20\Omega} + \frac{V_b}{10\Omega} + \frac{V_b - 12V}{20\Omega} - 50mA = 0$$

$$V_b = 1.25\ V$$

By KCL,

$$I_x + \frac{-8V}{11\Omega} + \frac{-8V - -8.35V}{7\Omega} + \frac{-8V - 1.25V}{20\Omega} = 0$$

$$I_x = 1.14\ A$$

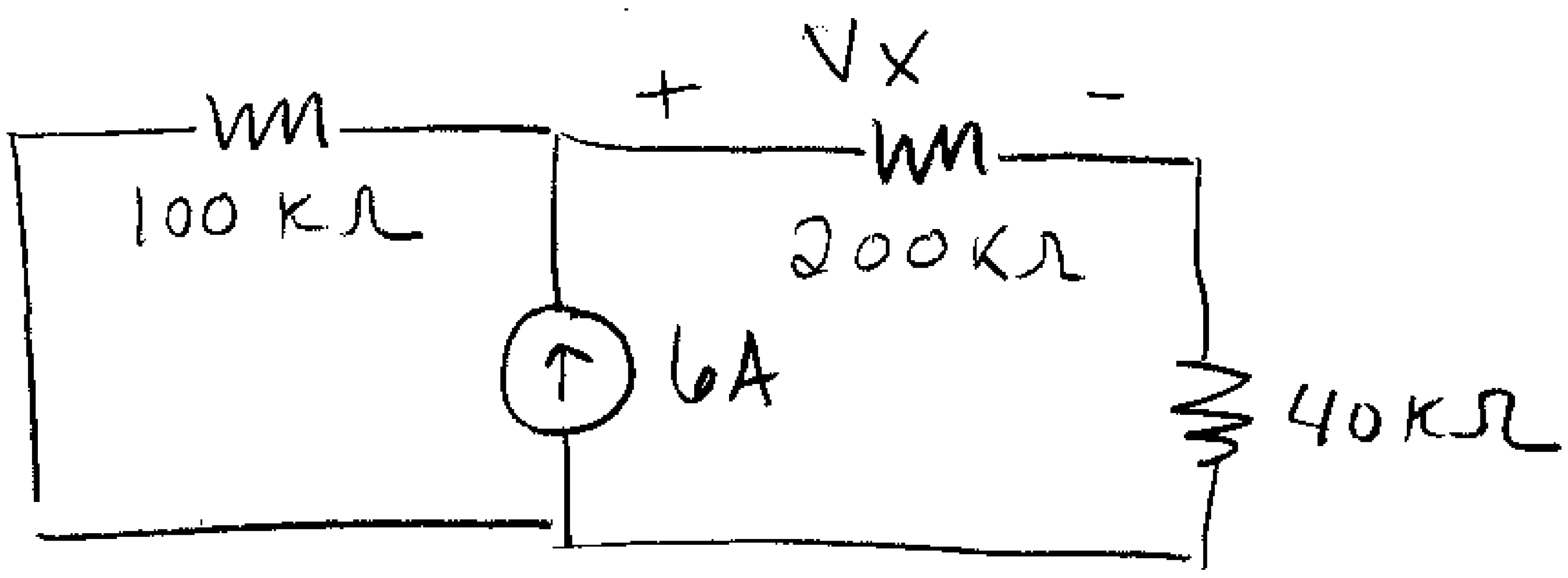
Using Other Tools

Sometimes it is easier to "intelligently" apply KVL, KCL, voltage & current division to solve a problem, rather than using nodal analysis.

With experience, you will be able to recognize the quickest path to a solution.

Some basic tips:

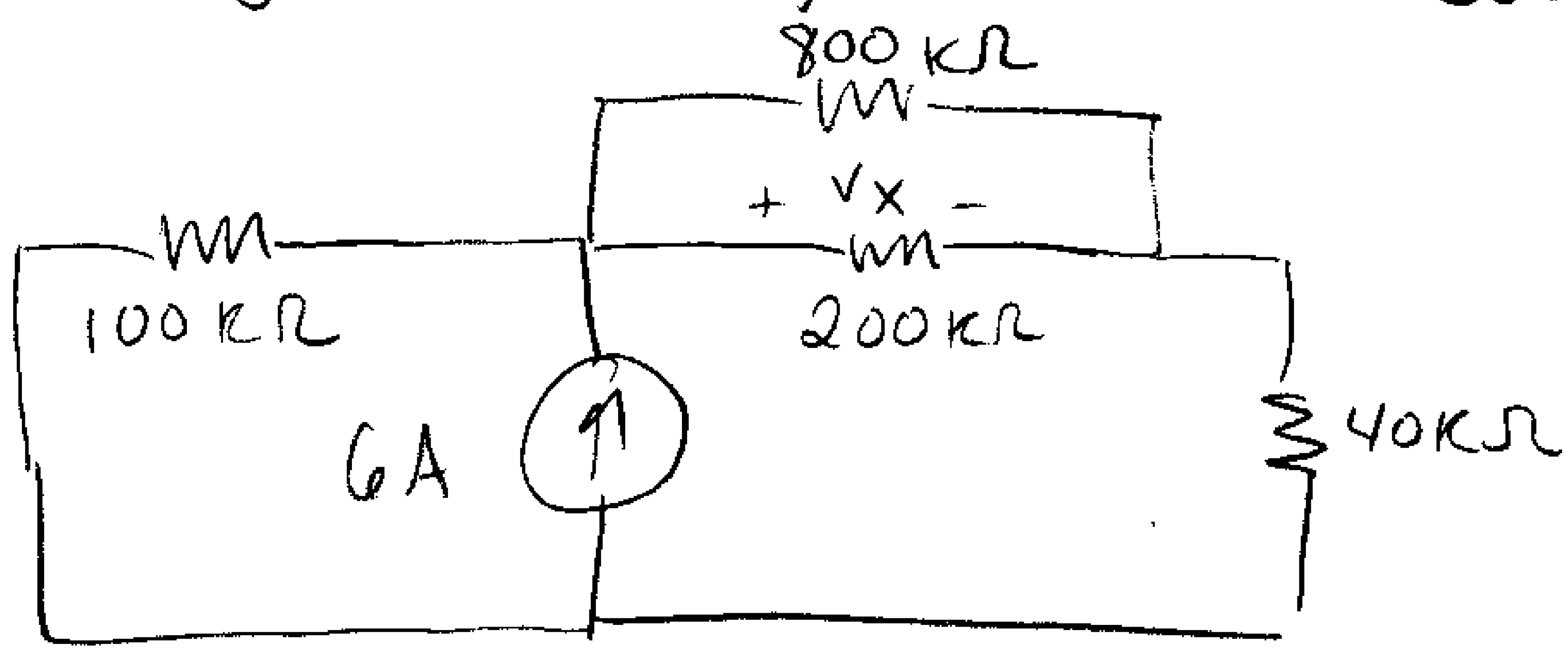
- When starting out, if you don't know what to do, write down something (anything) you do know about the circuit. See if that piece of info leads to a new piece of info. Continue detective work until finished.
- If you are looking for a voltage, try to find a closed path where most of the voltages are known, and write KVL. Remember, the path can go over air, and air & current sources can have voltage.
- Similarly, to find a current, write a KCL equation, and remember that voltage sources can have current.
- Combine resistors wherever convenient – you can always recover individual voltages & currents.

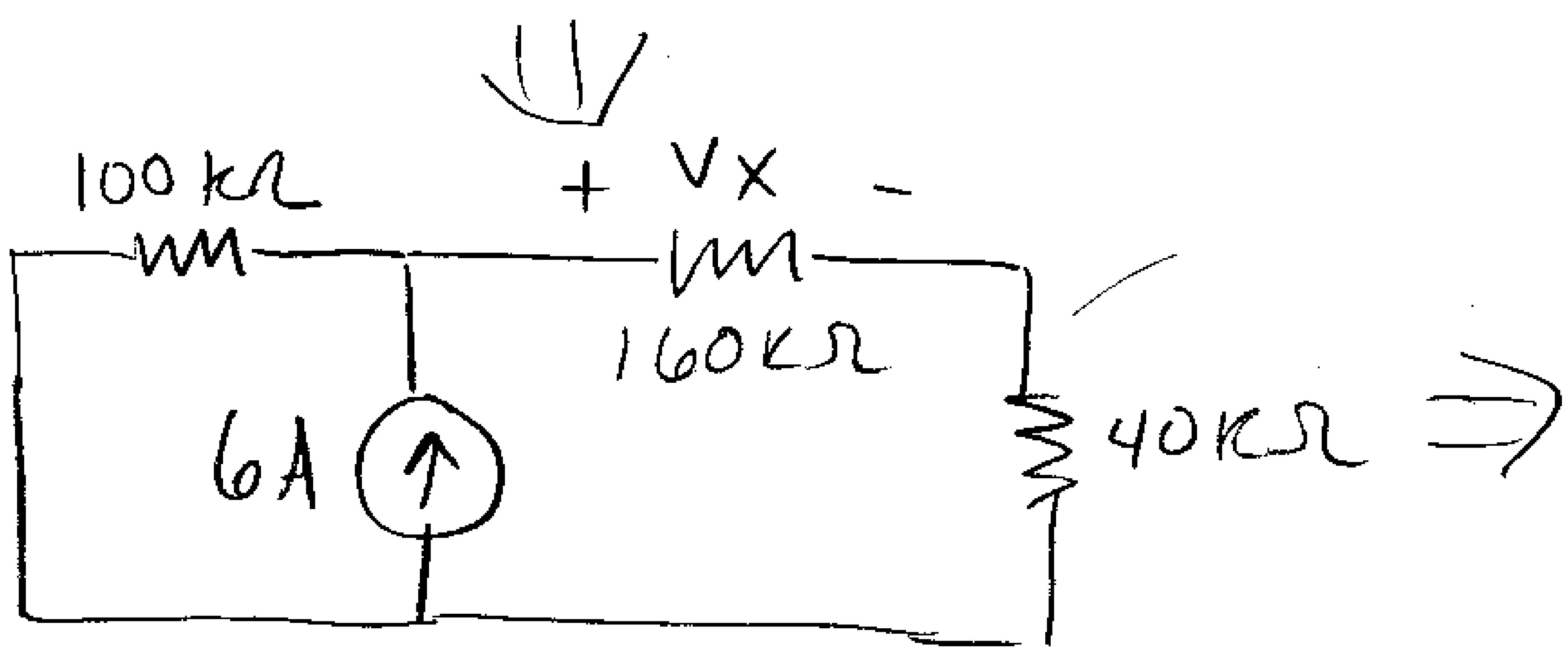


a) If we use a volt meter with internal resistance 800 kΩ to measure V_x, what does it read?

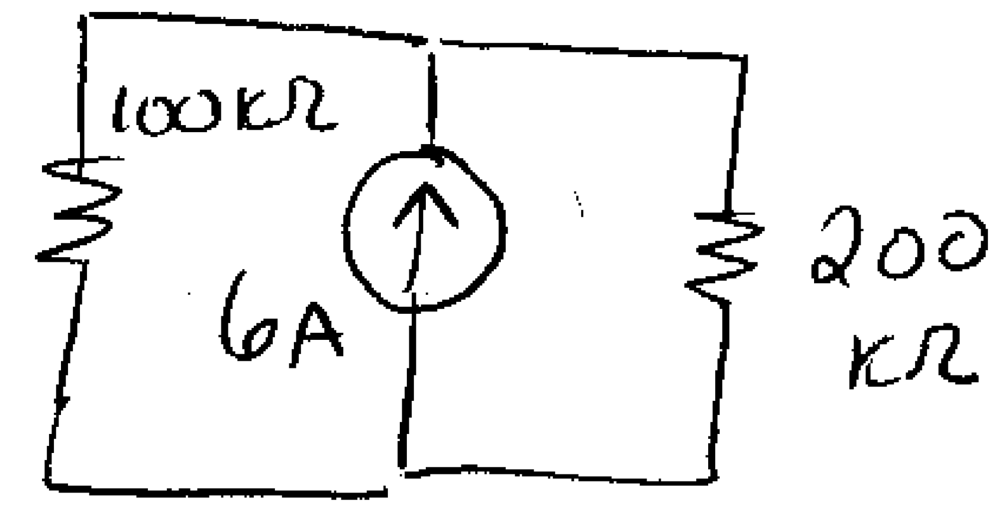
b) With the voltmeter in the circuit, what is the power generated by the current source?

a)





Vx goes away
Since resistor lost...

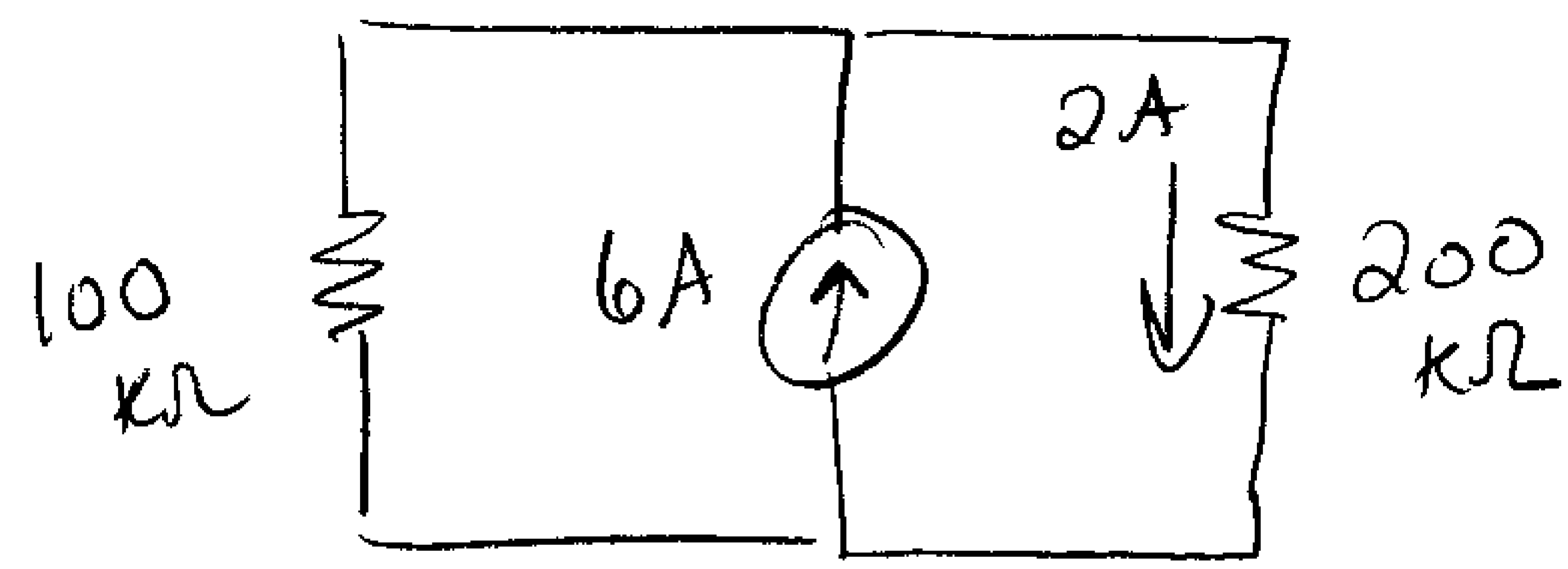


By current divison, 200 kΩ gets $6A \cdot \frac{100\,k\Omega}{100\,k\Omega + 200\,k\Omega} = 2A$

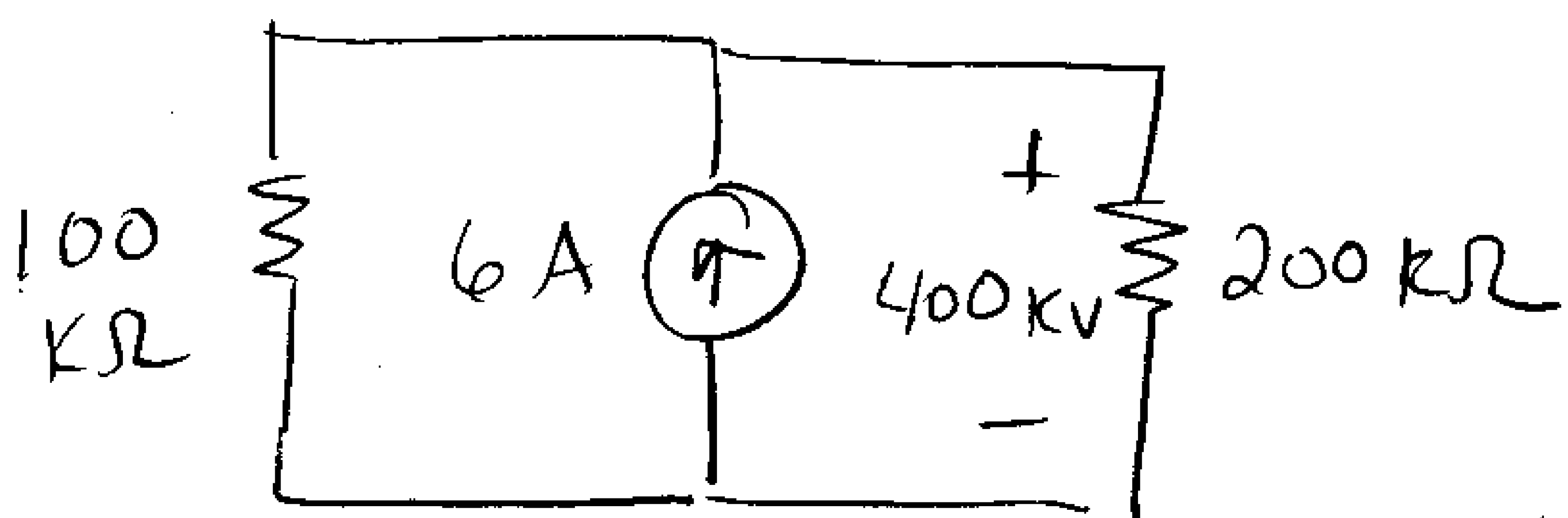
200 kΩ is the series combo of 160 kΩ and 40 kΩ. All have same current of 2A.

$V_x = 2A \cdot 160\,k\Omega = 320\ kV$ (ouch. Should have used mA.)

b) We found that



So by Ohm's law,



When I multiply $P = VI$ and I flows from − to +, then P is power generated.
That's what I have above:

$$P = 6A \cdot 400\ kV = 2.4\ MW$$