Lecture 8: Linearity and Equivalent Circuits

Every circuit which is composed of ideal independent voltage and current sources, linear dependent sources, and resistors, has a linear I-V relationship.

There is a simpler circuit with the same I-V relationship.

Thevenin Equivalent Circuit

The Thevenin equivalent circuit is composed of a voltage source in series with a resistor:

It can model any circuit except a pure independent current source, through choice of $V_T$ and $R_T$. 
**Norton Equivalent Circuit**

The **Norton equivalent** circuit is composed of a current source in parallel with a resistor:

\[ I = I_N \left( \frac{1}{R_N} \right) + \frac{V}{R_N} \]

It can model any circuit except a pure independent voltage source, through choice of \( I_N \) and \( R_N \).

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**Two Points Define a Line**

To find the Thevenin or Norton equivalent for a circuit, all we need to do is:

- Find two points on the I-V graph for the circuit.
  - Set the voltage \( V \) and find the corresponding \( I \)
  - Set the current \( I \) and find the corresponding \( V \)
- Find the x-intercept and y-intercept of the graph.
- Find the \( V_{TH} \) and \( R_{TH} \), or the \( I_N \) and \( R_N \) that replicate this line.
Our Favorite Two Points on the I-V Graph

- We can find the x-intercept directly by finding the \( V \) that occurs when \( I = 0 \).
  - This means finding the \( V \) that occurs when there is air between the circuit terminals.
  - This voltage is called the open-circuit voltage, \( V_{OC} \).
  - \( V_{TH} = I_N R_N = V_{OC} \)
- We can find the y-intercept directly by finding the \( I \) that occurs when \( V = 0 \).
  - This means finding the \( I \) that occurs when there is a wire between the circuit terminals.
  - This current is called the short-circuit current, \( I_{SC} \).
  - \( I_N = V_{TH} / R_{TH} = -I_{SC} \)

Useful Identities

\[
V_{TH} = I_N R_N \\
R_N = V_{TH} / I_N \\
R_{TH} = R_N \\
I_N = V_{TH} / R_{TH} \\
R_{TH} = V_{TH} / I_N
\]
Example (Nilsson & Riedel text)

Find the Thevenin and Norton circuits.

Example (Nilsson & Riedel text)

Find the Thevenin and Norton circuits.
VTHand INCome From Independent Sources

- If there are no independent voltage or current sources in a circuit, $V_{TH} = 0$ V and $I_N = 0$ A.
- If there is no independent voltage or current present in a circuit (only resistors and linear dependent sources), all currents and voltages in the circuit are zero.
- In this situation, you know that the I-V graph goes through the origin.
- However, the slope of the graph, $1/R_{TH}$, still must be determined. It cannot be found using $R_{TH} = V_{TH} / I_N$.

No Independent Sources? Test for $R_{TH}$

- A simple example of a circuit with no independent sources is a resistor.
- One cannot determine the resistance by measuring voltage and current—a resistor has no voltage or current on its own.
- An ohmmeter applies a test voltage and measures the resulting current to find resistance.
- Do the same to find $R_{TH}$: Set V using an independent voltage source, and measure I.
- Or, set I using an independent current source, and measure V.
- $R_{TH} = V / I$
- Here, you are finding an additional point on the I-V graph.
Example

Find the Thevenin and Norton circuits.

\[ \begin{array}{c}
5 \ \Omega \\
\downarrow \ x \\
3 \ x \\
\downarrow \\
10 \ \Omega \\
\end{array} \]

\[ \text{a} \quad \text{b} \]

- \( R_{TH} \) comes from resistors and linear dependent sources.

- The value of \( R_{TH} \) does not depend on the values of independent voltage and current sources in a circuit.

- I can turn a 12 V source into a -12 V source, or a 0 V source, and the value of \( R_{TH} \) remains the same.

- When looking for \( R_{TH} \) in a circuit that has no dependent sources, it is often easier to:
  - Turn off all independent sources (change voltage sources to 0 V wire and current sources to 0 A air)
  - Simplify remaining resistors using series/parallel combinations to find \( R_{TH} \)
Example

Find $R_{TH}$.

\[ R = \frac{20 \Omega \times 5 \Omega}{20 \Omega + 5 \Omega} = \frac{100}{25} = 4 \Omega \]

Source Transformations

One can change back and forth between Thevenin and Norton:

\[ V = \frac{V_s}{R_s} \quad \text{and} \quad I = \frac{I_s}{R_s} \]
Source Transformations

One can use source transformations to simplify a circuit just like using series/parallel rules to simplify resistors. Remember that:

\[ V_1 + V_2 \]

\[ I_1 + I_2 \]

Example (Nilsson & Riedel text)

Find the Thevenin and Norton circuits.