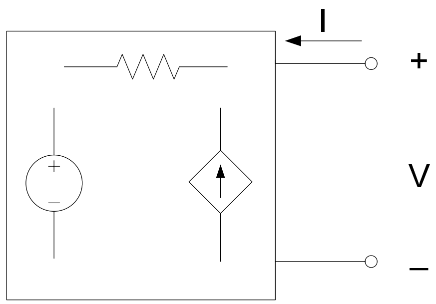
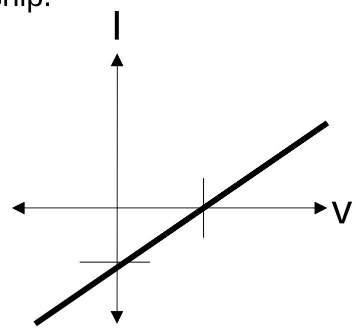
Lecture 8: Linearity and Equivalent Circuits

Every circuit which is composed of **ideal independent voltage and current sources, linear dependent sources, and resistors,** has a linear I-V relationship.

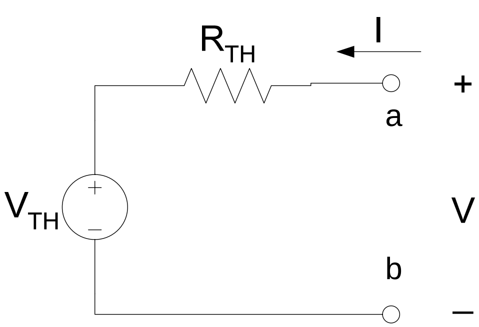


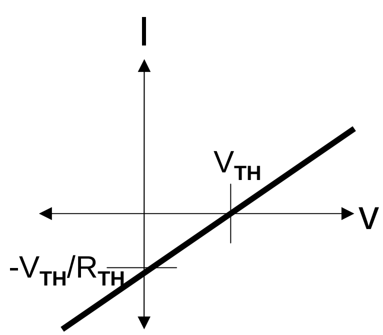


There is a simpler circuit with the same I-V relationship.

Thevenin Equivalent Circuit

The **Thevenin equivalent** circuit is composed of a voltage source in series with a resistor:

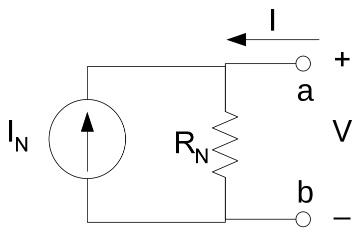


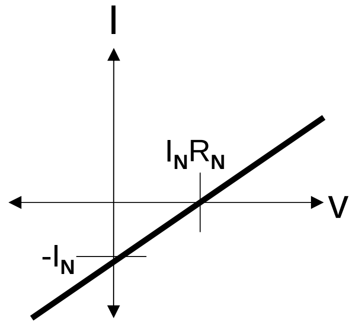


It can model any circuit except a pure independent current source, through choice of V_T and R_T.

Norton Equivalent Circuit

The **Norton equivalent** circuit is composed of a current source in parallel with a resistor:





It can model any circuit except a pure independent voltage source, through choice of I_N and R_N.

Two Points Define a Line

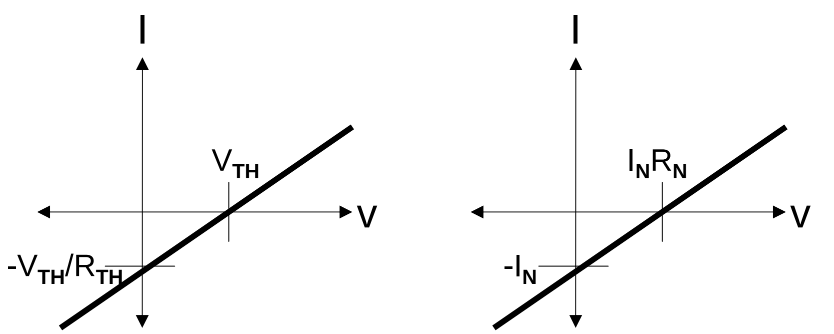
To find the Thevenin or Norton equivalent for a circuit, all we need to do is:

- Find two points on the I-V graph for the circuit.
  - Set the voltage V and find the corresponding I
  - Set the current I and find the corresponding V
- Find the x-intercept and y-intercept of the graph.
- Find the V_{TH} and R_{TH}, or the I_N and R_N that replicate this line.

Our Favorite Two Points on the I-V Graph

- We can find the x-intercept directly by finding the **V** that occurs when **I = 0**.
  - This means finding the V that occurs when there is air between the circuit terminals.
  - This voltage is called the **open-circuit voltage**, V_{OC}.
  - $V_{TH} = I_N R_N = V_{OC}$
- We can find the y-intercept directly by finding the **I** that occurs when **V = 0**.
  - This means finding the I that occurs when there is a wire between the circuit terminals.
  - This current is called the **short-circuit current**, I_{SC}.
  - $I_N = V_{TH} / R_{TH} = -I_{SC}$

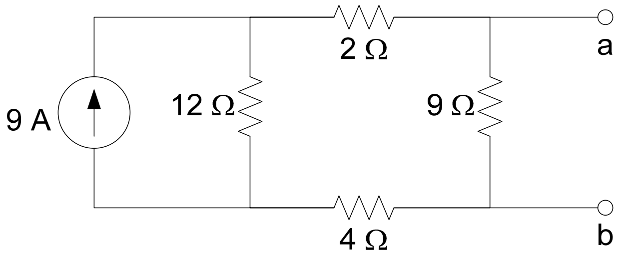
Useful Identities



$V_{TH} = I_N R_N$ $R_N = V_{TH} / I_N$ $R_{TH} = R_N$

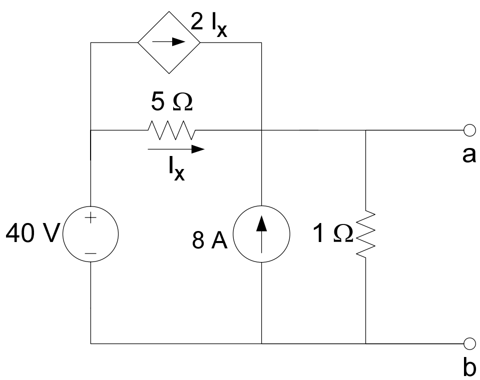
$I_N = V_{TH} / R_{TH}$ $R_{TH} = V_{TH} / I_N$

Example (Nilsson & Riedel text)



Find the Thevenin and Norton circuits.

Example (Nilsson & Riedel text)



Find the Thevenin and Norton circuits.

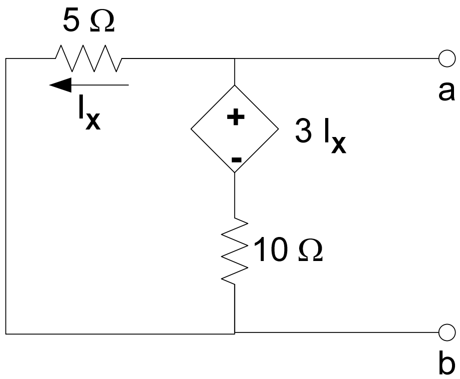
V_{TH} and I_N Come From Independent Sources

- If there are no independent voltage or current sources in a circuit, **V_{TH} = 0 V** and **I_N = 0 A**.
- If there is no independent voltage or current present in a circuit (only resistors and linear dependent sources), **all currents and voltages in the circuit are zero.**
- In this situation, you know that the I-V graph goes through the origin.
- However, the slope of the graph, $1/R_{TH}$, still must be determined. It cannot be found using $R_{TH} = V_{TH} / I_N$.

No Independent Sources? Test for R_{TH}

- A simple example of a circuit with no independent sources is a resistor.
- One cannot determine the resistance by measuring voltage and current—a resistor has no voltage or current on its own.
- An ohmmeter applies a test voltage and measures the resulting current to find resistance.
- Do the same to find R_{TH} : Set V using an independent voltage source, and measure I.
- Or, set I using an independent current source, and measure V.
- $R_{TH} = V / I$
- Here, you are finding an additional point on the I-V graph.

Example



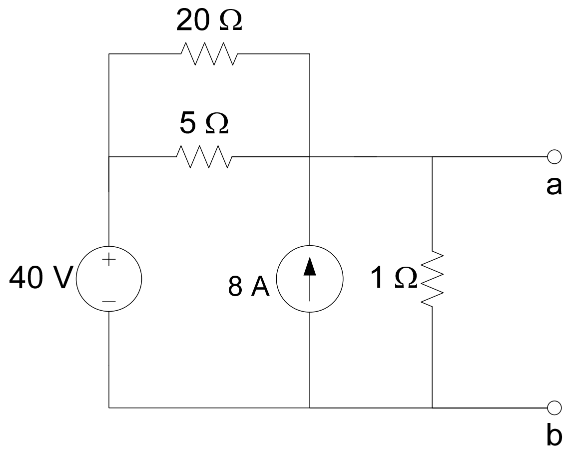
Find the Thevenin and Norton circuits.

R_{TH} Comes From Resistors and Linear Dependent Sources

- The value of R_{TH} does not depend on the values of independent voltage and current sources in a circuit.
- I can turn a 12 V source into a -12 V source, or a 0 V source, and the value of R_{TH} remains the same.
- When looking for R_{TH} in a circuit that has no dependent sources, it is often easier to:
  - Turn off all independent sources (change voltage sources to 0 V wire and current sources to 0 A air)
  - Simplify remaining resistors using series/parallel combinations to find R_{TH}

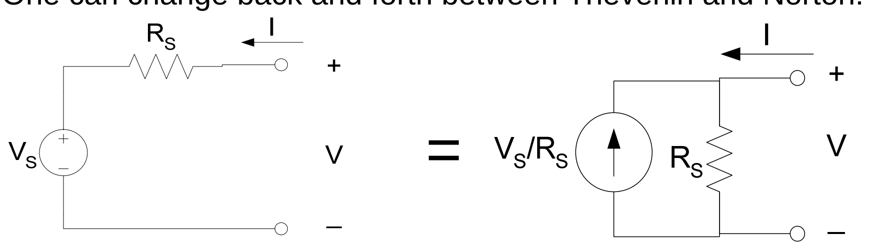
Example

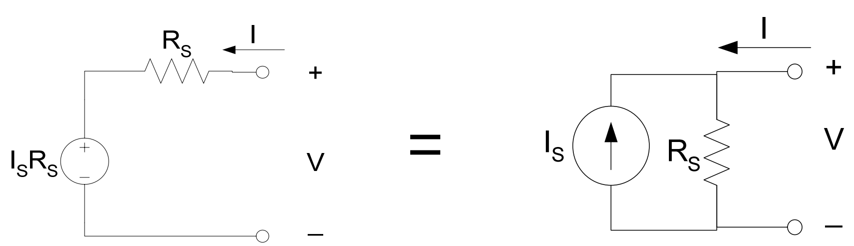
Find R_{TH}.



Source Transformations

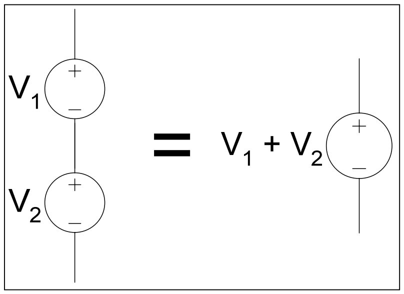
One can change back and forth between Thevenin and Norton:

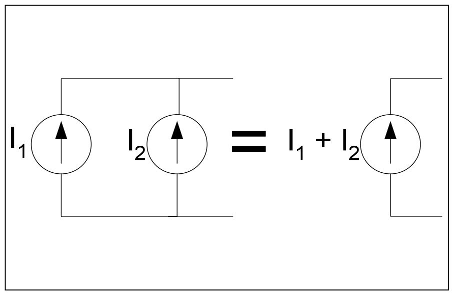




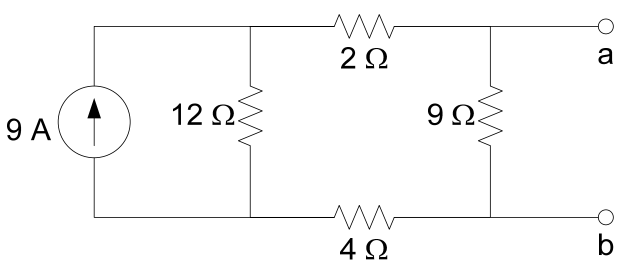
Source Transformations

One can use source transformations to simplify a circuit just like using series/parallel rules to simplify resistors. Remember that:





Example (Nilsson & Riedel text)



Find the Thevenin and Norton circuits.