Ideal wires, Ideal device models, Ideal circuits

- Ideal models for circuit elements
- Wires
  - Currents and Voltages
- Joints
- Resistors
- Voltage sources
- Current sources.
Cast of Characters

- Fundamental quantities
  - Charge
  - Current
  - Voltage
  - Power

- Fundamental concern
  - Current-Voltage Relationship

- Fundamental elements
  - Resistor
  - Voltage Source
  - Current Source
Charge

- You are already familiar with the idea of charge from chemistry or physics.
- We say a proton has a positive charge, and an electron has a negative charge.
- Charge is measured in units called Coulombs, abbreviated C.

\[ 1 \text{ proton} = 1.6 \times 10^{-19} \text{ C} \]
\[ 1 \text{ electron} = -1.6 \times 10^{-19} \text{ C} \]

1 C is a whole lot of protons!
6.25 x 10^{18} protons in 1 C.
Electric Field

- We know that opposite charges attract each other, and like charges repel.
- The presence of a charged particle creates an electric field. Other phenomena also create an electric field.
- The electric field is a lot like gravity. It can point in different directions and have different strength depending on location.

**Vector fields** are like wind maps from your weather forecast.
Voltage

- It takes energy to move a proton against the direction of an electric field (just like it takes energy to lift an object off the ground, against gravity).

- Suppose it takes (positive) energy to move a proton from point $a$ to point $b$. Then we say point $b$ is at a higher electric potential than point $a$.

- The difference in electric potential between two points is called voltage. Voltage, measured in Volts (V) indicates how much energy it takes to move a charge from point to point.
Voltage Conventions

- Voltage is always measured between two points (just like distance). We need to specify the “start” and “finish”.

- We could write saying that \( b \) is 5 V higher than \( a \).

- Or, we could write saying that \( a \) is -5 V higher than \( b \).

- When we put down a + and a – to specify a voltage, it is simply a reference frame. We are not making a statement about which point actually has the higher potential, since the voltage in between can be negative!
Voltage Conventions: Notation

- We can use subscript convention to define a voltage between two labeled points:

- Remember, this is not saying that the potential at “a” is higher than the potential at “b”. The difference could be negative.

- We can make up voltages with any names we wish, as long as we provide a reference frame (+ and -).

- Here, \( V_{\text{Fred}} \) is the potential rise from left to right (or, the potential drop from right to left, or the right potential minus the left).
Examples

The flat end of the battery is at lower potential than the “bump” end.

What is $V_{AD}$?

$-1.5\text{ V} + -1.5\text{ V} + 9\text{ V} = 6\text{ V}$

Find $V_1$ and $V_x$.

$V_1 = 1.5\text{ V}$  $V_x = -6\text{ V}$
Voltage Conventions: Ground

- Many times, a common point will be used as the starting (-) point for several voltage measurements. This common point is called common or ground.
- We may define a voltage at point “a” with respect to ground. This refers to the voltage with + reference at “a” and – reference at ground.
- Voltages with respect to ground are often denoted using a single subscript:
- Notice the symbol for ground. Also seen is
Current: Moving Charge

- An electric field (or applied energy) can cause charge to move.
- The amount of charge per time unit moving past a point is called current.
- Current is measured in coulombs per second, which are called amperes (abbreviated A and called amps for short).
- Mathematically speaking,
  \[ i = \frac{dq}{dt} \]
  where \( i \) is current in A, \( q \) is charge in C, and \( t \) is time in s.
- Even though it is usually (negative) electrons that do the moving, current is defined as the flow of positive charge.
Water Model for Electric Current

Since we can’t see electric charges moving in a wire, it is helpful to use the analogy between water flow and charge flow: electric current flowing in a wire is like water flowing in a pipe.

Electric charge (coulombs) is like quantity of water (gallons)
Current flow (coulombs per second – amperes) is like water flow rate past a point (gallons per second)
Schematic Symbol and Water Model of DC Voltage Source (assumes gravity acting downward)
Current Reference Direction

- Current also needs a reference frame. To define a current, draw an arrow:

  ![Diagram showing current direction with 5 A and -5 A arrows]

- This says “the current moving through the device from left to right is 5 A”.

- We could also say, “the current moving through the device from right to left is -5 A”.

- Drawing an arrow does not make a statement about the direction the current is actually going. It is just a reference frame. You can draw arrows however you want when you need to solve for currents.
Resistance

- Current flow results from the ability of electrons to break away from atoms and move around in a solid.

- In some materials such as metals, mobile electrons exist and can move around where an electric field exists to drive them—in such materials the **resistance** for current flow is low, and these are called **good conductors** of electricity.

- In other materials, very few mobile electrons exist and less current flows in the same electric field. These materials are said to have a higher **resistance** and be **poorer conductors**.

- Resistance, measured in **ohms** (Ω), indicates how much voltage is necessary to create a certain amount of current.

\[ V_R = I_R R \]
Resistor (top left), its Schematic Symbol (top right), and Two Water Models of a Resistor
Power

Power is the amount of energy absorbed or generated per unit time. It is the time derivative of energy, and it is measured in **watts (W)**.

The power absorbed (or generated) by a device is equal to the product of the current through the device with the voltage over the device:

\[ p = v \times i \]

where \( p \) is power in W, \( v \) is voltage in V and \( i \) is current in A.

Sometimes this equation gives you the power absorbed by the device, and sometimes it provides the power generated by the device.
Power: Sign Convention

- Whether \( p = v \cdot i \) provides absorbed power or generated power depends on the relationship between the current and voltage directions.

- If the current \( i \) is referenced to flow from the “+” terminal of \( v \) to the “-” terminal of \( v \), then \( p = v \cdot i \) provides the power absorbed by the device.

- When the opposite is true, \( p = v \cdot i \) provides the power generated by the device (such as a battery).

\[
\text{Power absorbed by device} = (V_{\text{device}}) (i_1)
\]

\[
\text{Power generated by device} = (V_{\text{device}}) (i_2)
\]
Power Calculations

Find the power absorbed by each element.

Element □ : \((3 \text{ V})(-3 \text{ mA}) = -9 \text{ mW}\)

Element ◆ : \((2 \text{ V})(3 \text{ mA}) = 6 \text{ mW}\)

Element × : \((1 \text{ V})(0.5 \text{ mA}) = 0.5 \text{ mW}\)

Element + : \((1 \text{ V})(2.5 \text{ mA}) = 2.5 \text{ mW}\)
Current-Voltage Relationship (I-V)

- In this course, we deal with circuits that perform computations, where the numbers are represented as voltages.
- Voltages appear at the input, and create current in the devices, which in turn changes the output voltage—and computation has taken place.
- The relationship between current and voltage in a device is fundamental. Circuit elements are characterized by their current-voltage (i-v) relationships. It is these relationships that allow us to design and analyze circuits.
- We will now present current-voltage relationships (called i-v relationships for short) for basic circuit elements.
Basic Circuit Elements

- **Wire (Short Circuit)**
  - Voltage is zero, current is unknown

- **Resistor**
  - Current is proportional to voltage (linear)

- **Ideal Voltage Source**
  - Voltage is a given quantity, current is unknown

- **Ideal Current Source**
  - Current is a given quantity, voltage is unknown

- **Air (Open Circuit)**
  - Current is zero, voltage is unknown
Wire

- Wire has a very small resistance.
- For simplicity, we will idealize wire in the following way: the potential at all points on a piece of wire is the same, regardless of the current going through it.
  - Wire is a 0 V voltage source
  - Wire is a 0 Ω resistor
- This idealization (and others) can lead to contradictions on paper—and smoke in lab.
Resistor

The resistor has a current-voltage relationship called Ohm’s law:

\[ v = i R \]

where \( R \) is the resistance in \( \Omega \), \( i \) is the current in A, and \( v \) is the voltage in V, with reference directions as pictured.

- If \( R \) is given, once you know \( i \), it is easy to find \( v \) and vice-versa.

- Since \( R \) is never negative, a resistor always absorbs power…
Ideal Voltage Source

- The ideal voltage source explicitly defines the voltage between its terminals.
  - Constant (DC) voltage source: \( V_s = 5 \text{ V} \)
  - Time-varying voltage source: \( V_s = 10 \sin(\omega t) \text{ V} \), where \( \omega \) is a constant called the angular frequency – more later on this.
  - Examples: batteries, wall outlet, function generator, …

- The ideal voltage source does not provide any information about the current flowing through it.

- The current through the voltage source is defined by the rest of the circuit to which the source is attached. Current cannot be determined from just the value of the voltage.

- Do not assume that the current is zero!
Ideal Current Source

The ideal current source sets the value of the current running through it.

- Constant (DC) current source: \( I_S = 2 \, \text{A} \)
- Time-varying current source: \( I_S = -3 \sin(\omega t) \, \text{A} \)
- Examples: few in real life – you can’t buy them at RadioShack!

The ideal current source has known current, but unknown voltage across it.

The voltage across the voltage source is defined by the rest of the circuit to which the source is attached.

Voltage cannot be determined from just the value of the current.

Do not assume that the voltage is zero!
Air

- Many of us at one time, after walking on a carpet in winter, have touched a piece of metal and seen a blue arc of light.
- That arc is current going through the air. So is a bolt of lightning during a thunderstorm.
- However, these events are unusual. Air is usually a good insulator and does not allow current to flow except when very high voltages are present.
- For simplicity, we will idealize air in the following way: current never flows through air (or a hole or gap in a circuit), regardless of the potential difference (voltage) present.
  - Air is a 0 A current source
  - Air is a very very big (infinite) resistor
- There can be nonzero voltage over air or a hole in a circuit!