Review of Circuit Analysis

- Fundamental elements
  - Wire
  - Resistor
  - Voltage Source
  - Current Source
- Kirchhoff’s Voltage and Current Laws
- Resistors in Series
- Voltage Division
Voltage and Current

- Voltage is the difference in electric potential between two points. To express this difference, we label a voltage with a “+” and “-”:

Here, $V_1$ is the potential at “a” minus the potential at “b”, which is -1.5 V.

- Current is the flow of positive charge. Current has a value and a direction, expressed by an arrow:

Here, $i_1$ is the current that flows right; $i_1$ is negative if current actually flows left.

- These are ways to place a frame of reference in your analysis.
Basic Circuit Elements

- Wire (Short Circuit)
  - Voltage is zero, current is unknown

- Resistor
  - Current is proportional to voltage (linear): $v = iR$

- Ideal Voltage Source
  - Voltage is a given quantity, current is unknown

- Ideal Current Source
  - Current is a given quantity, voltage is unknown
Resistor

The resistor has a current-voltage relationship called Ohm’s law:

\[ v = i R \]

where \( R \) is the resistance in \( \Omega \), \( i \) is the current in A, and \( v \) is the voltage in V, with reference directions as pictured.

- If \( R \) is given, once you know \( i \), it is easy to find \( v \) and vice-versa.

- Since \( R \) is never negative, a resistor always absorbs power...
Ideal Voltage Source

- The ideal voltage source explicitly defines the voltage between its terminals.
  - Constant (DC) voltage source: $V_s = 5 \text{ V}$
  - Time-varying voltage source: $V_s = 10 \sin(\omega t) \text{ V}$
  - Examples: batteries, wall outlet, function generator, …

- The ideal voltage source does not provide any information about the current flowing through it.

- The current through the voltage source is defined by the rest of the circuit to which the source is attached. Current cannot be determined by the value of the voltage.

- **Do not assume that the current is zero!**
Wire

- Wire has a very small resistance.
- For simplicity, we will idealize wire in the following way: the potential at all points on a piece of wire is the same, regardless of the current going through it.
  - Wire is a 0 V voltage source
  - Wire is a 0 Ω resistor
- This idealization (and others) can lead to contradictions on paper—and smoke in lab.
### Ideal Current Source

- The ideal current source sets the value of the current running through it.
  - Constant (DC) current source: \( I_S = 2 \) A
  - Time-varying current source: \( I_S = -3 \sin(\omega t) \) A
  - Examples: few in real life!

- The ideal current source has known current, but unknown voltage.

- The voltage across the voltage source is defined by the rest of the circuit to which the source is attached.

- Voltage cannot be determined by the value of the current.

- Do not assume that the voltage is zero!
I-V Relationships Graphically

**Resistor:** Line through origin with slope $1/R$

**Ideal Voltage Source:** Vertical line

**Ideal Current Source:** Horizontal line

**Wire:** Vertical line through origin: any current, no voltage across wire

$v = iR$

$i = I_S$
CIRCUIT ANALYSIS: given a circuit, to find all the currents & voltages in the circuit.

CIRCUIT SYNTHESIS: given a goal, find a circuit...
Kirchhoff’s Laws

- The I-V relationship for a device tells us how current and voltage are related within that device.

- Kirchhoff’s laws tell us how voltages relate to other voltages in a circuit, and how currents relate to other currents in a circuit.

- Kirchhoff’s Voltage Law (KVL): The sum of voltage drops around a closed path must equal zero.

- Kirchhoff’s Current Law (KCL): The sum of currents entering a node must equal zero.
Kirchhoff’s Voltage Law (KVL)

- Suppose I add up the potential drops around the closed path, from “a” to “b” to “c” and back to “a”.
- Since I end where I began, the total drop in potential I encounter along the path must be zero: \( V_{ab} + V_{bc} + V_{ca} = 0 \)
- It would not make sense to say, for example, “b” is 1 V lower than “a”, “c” is 2 V lower than “b”, and “a” is 3 V lower than “c”. I would then be saying that “a” is 6 V lower than “a”, which is nonsense!
- Alternatively, we can use potential rises throughout instead of potential drops; this is an alternative statement of KVL.
KVL Tricks

- A voltage rise is a negative voltage drop.

Along a path, I might encounter a voltage which is labeled as a voltage drop (in the direction I’m going). The sum of these voltage drops must equal zero.

I might encounter a voltage that is labeled as a voltage rise (in the direction I’m going). This rise can be viewed as a “negative drop”. Rewrite:

- Look at the first sign you encounter on each element when tracing the closed path. If it is a “-”, it is a voltage rise and you will insert a “-” to rewrite it as a drop.
Writing KVL Equations

What does KVL say about the voltages along these 3 paths?

Path 1: $-v_a + v_2 + v_b = 0$

Path 2: $-v_b - v_3 + v_c = 0$

Path 3: $-v_a + v_2 - v_3 + v_c = 0$
Elements in Parallel

- KVL tells us that any set of elements that are connected at both ends carry the same voltage.
- We say these elements are in parallel.

KVL clockwise, start at top:

\[ V_b - V_a = 0 \]

\[ V_a = V_b \]
Kirchhoff’s Current Law (KCL)

- Electrons don’t just disappear or get trapped (in our analysis).
- Therefore, the sum of all current entering a closed surface or point must equal zero—whatever goes in must come out.
- Remember that current leaving a closed surface can be interpreted as a negative current entering:

\[ \sum i_{\text{into node}} = 0 \]
KCL Equations

In order to satisfy KCL, what is the value of $i$?

KCL says:

$24 \, \mu A + (-10 \, \mu A) + (-)(-4 \, \mu A) + (-i) = 0$

$18 \, \mu A - i = 0$

$i = 18 \, \mu A$
Elements in Series

- Suppose two elements are connected with nothing (no wire) coming off in between.
- KCL says that the elements carry the same current.
- We say these elements are in series.

\[ i_1 - i_2 = 0 \quad \text{and} \quad i_1 = i_2 \]
Resistors in Series

Consider resistors in series. This means they are attached end-to-end, with nothing coming off in between (current has no choice of where to go).

Each resistor has the same current (labeled i).

Each resistor has voltage iR, given by Ohm’s law.

The total voltage drop across all 3 resistors is

\[ V_{\text{TOTAL}} = i R_1 + i R_2 + i R_3 = i (R_1 + R_2 + R_3) \]
Resistors in Series

- When we look at all three resistors together as one unit, we see that they have the same I-V relationship as one resistor whose value is the sum of the resistances:

- So we can treat these resistors as just one **equivalent resistance**, as long as we are not interested in the individual voltages. Their effect on the rest of the circuit is the same, whether lumped together or not.
Voltage Division

If we know the total voltage over a series of resistors, we can easily find the individual voltages over the individual resistors.

\[ V_{\text{TOTAL}} = \frac{R_1}{R_1 + R_2 + R_3} \cdot V_{\text{TOTAL}} \]

Since the resistors in series have the same current, the voltage divides up among the resistors in proportion to each individual resistance.
Voltage Division

- For example, we know
  \[ i = \frac{V_{\text{TOTAL}}}{(R_1 + R_2 + R_3)} \]
  so the voltage over the **first resistor** is
  \[ i \cdot R_1 = R_1 \left[ \frac{V_{\text{TOTAL}}}{(R_1 + R_2 + R_3)} \right] \]

\[ = V_{\text{TOTAL}} \cdot \frac{R_1}{R_1 + R_2 + R_3} \]

- To find the voltage over an individual resistance in series, take the total series voltage and multiply by the ratio of the individual resistance to the total resistance.