Circuit Analysis Basics, Cont.

- Resistors in Parallel
- Current Division
- Realistic Models of Sources
- Making Measurements
- Tips and Practice Problems
Elements in Parallel

- KVL tells us that any set of elements that are **directly connected by wire at both ends** carry the **same voltage**.
- We say these elements are **in parallel**.

KVL clockwise, start at top:

\[ V_b - V_a = 0 \]

\[ V_a = V_b \]
Elements in Parallel--Examples
Which of these resistors are in parallel?

None

R_4 and R_5

R_7 and R_8
Resistors in Parallel

- Resistors in parallel have the same voltages across them. All of the resistors below have voltage $V_R$.
- The current flowing through each resistor could definitely be different. Even though they have the same voltage, the resistances could be different.

\[ i_1 = \frac{V_R}{R_1} \]
\[ i_2 = \frac{V_R}{R_2} \]
\[ i_3 = \frac{V_R}{R_3} \]
Equivalent Resistance of Resistors in Parallel

If we view the three resistors as one unit, with a current $i_{\text{TOTAL}}$ going in, and a voltage $V_R$, this unit has the following I-V relationship:

$$i_{\text{TOTAL}} = i_1 + i_2 + i_3 = V_R \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

in other words,

$$V_R = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} i_{\text{TOTAL}}$$

So to the outside world, the parallel resistors look like one:

$$\frac{1}{R_{\text{EQ}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
Case of Just Two Resistors in Parallel

We’ll often find circuits where just two resistors are connected in parallel, and it is convenient to replace them with their equivalent resistance. From our earlier general formula for any number in parallel, we see that for just $R_1$ and $R_2$ in parallel we obtain

$$R_{EQ} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$R_{EQ} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$ Equivalent resistance of two resistors in parallel
Current division between just two paralleled resistors

- If we know the total current flowing into two parallel resistors, we can easily find out how the current will divide between the two resistors:

- The expression derived on the previous page applies to the circuit below. The current through resistor \( R_1 \), for example, is just the total applied voltage \( i_{\text{total}} \times R_{\text{EQ}} \) divided by \( R_1 \), or

\[
 i_1 = i_{\text{total}} \times \left[ \frac{R_1 \times R_2}{R_1 + R_2} \right] \left( \frac{1}{R_1} \right),
\]

Thus the fraction of the currently flowing through \( R_1 \) is \( i_1 / i_{\text{total}} = \frac{R_2}{R_1 + R_2} \). Likewise, the fraction of the total current that flows through \( R_2 \) is \( i_2 / i_{\text{total}} = \frac{R_1}{R_1 + R_2} \)

- Note that this differs slightly from the voltage division formula for series resistors.
Current Division—Other Cases

- If more than two resistors are in parallel, one can:
  - Find the voltage across the resistors, $V_R$, by combining the resistors in parallel and computing $V_R = i_{\text{TOTAL}} R_{\text{EQ}}$. Then, use Ohm’s law to find $i_1 = V_R / R_1$, etc.
  - Or, leave the resistor of interest alone, and combine other resistors in parallel. Use the equation for two resistors.
Issues with Series and Parallel Combination

- Resistors in series and resistors in parallel, when considered as a group, have the same I-V relationship as a single resistor.

- If the group of resistors is part of a larger circuit, the rest of the circuit cannot tell whether there are separate resistors in series (or parallel) or just one equivalent resistor. All voltages and currents outside the group are the same whether resistors are separate or combined.

- Thus, when you want to find currents and voltages outside the group of resistors, it is good to use the simpler equivalent resistor.

- Once you simplify the resistors down to one, you (temporarily) lose the current or voltage information for the individual resistors involved.
Issues with Series and Parallel Combination

- For resistors in **series**:
  - The individual resistors have the **same current** as the single equivalent resistor.
  - The voltage across the single equivalent resistor is the **sum of the voltages** across the individual resistors.
  - Individual voltages and currents can be recovered using Ohm’s law or voltage division.
Issues with Series and Parallel Combination

- For resistors in parallel:
  - The individual resistors have the **same voltage** as the single equivalent resistor.
  - The current through the equivalent resistor is the **sum of the currents** through the individual resistors.
  - Individual voltages and currents can be recovered using Ohm’s law or current division.
Approximating Resistor Combination

- Suppose we have two resistances, $R_{SM}$ and $R_{LG}$, where $R_{LG}$ is much larger than $R_{SM}$. Then:

\[
\frac{R_{SM} R_{LG}}{R_{SM} R_{LG}} \approx R_{SM}
\]
Ideal Voltage Source

- The ideal voltage source explicitly defines the voltage between its terminals.
- The ideal voltage source could have any amount of current flowing through it—even a really large amount of current.
- This would result in high power generation or absorption (remember $P = vi$), which is unrealistic.
Realistic Voltage Source

- A real-life voltage source, like a battery or the function generator in lab, cannot sustain a very high current. Either a fuse blows to shut off the device, or something melts...

- Additionally, the voltage output of a realistic source is not constant. The voltage decreases slightly as the current increases.

- We usually model realistic sources considering the second of these two phenomena. A realistic source is modeled by an ideal voltage source in series with an “internal resistance”, \( R_S \).
Realistic Current Source

- Constant-current sources are much less common than voltage sources.
- There are a variety of circuits that can produce constant currents, and these circuits are usually composed of transistors.
- Analogous to realistic voltage sources, the current output of the realistic constant current source does depend on the voltage. (We may investigate this dependence further when we study transistors.)
Taking Measurements

- To measure voltage, we use a two-terminal device called a **voltmeter**.
- To measure current, we use a two-terminal device called a **ammeter**.
- To measure resistance, we use a two-terminal device called a **ohmmeter**.
- A **multimeter** can be set up to function as any of these three devices.
- In lab, you use a **DMM** to take measurements, which is short for **digital multimeter**.
Measuring Current

To measure current, insert the measuring instrument in series with the device you are measuring. That is, put your measuring instrument in the path of the current flow.

The measuring device will contribute a very small resistance (like wire) when used as an ammeter.

It usually does not introduce serious error into your measurement, unless the circuit resistance is small.

DMM
Measuring Voltage

- To measure **voltage**, connect the measuring instrument **in parallel** with the device you are measuring. That is, put your measuring instrument across the measured voltage.
- The measuring device will contribute a very large resistance (like air) when used as a voltmeter.
- It usually does not introduce serious error into your measurement unless the circuit resistance is large.
Measuring Resistance

- To measure resistance, connect the measuring instrument across (in parallel) with the resistor you are measuring with nothing else attached.
- The measuring device applies a voltage to the resistance and measures the current, then uses Ohm’s law to determine the resistance.
- It is important to adjust the settings of the meter for the approximate size (Ω or MΩ) of the resistance being measured so that an appropriate voltage is applied to get a reasonable current.
Example

For the above circuit, what is $i_1$?

Suppose $i_1$ was measured using an ammeter with internal resistance 1 Ω. What would the meter read?
Example

By current division, \( i_1 = -3 \text{ A} \frac{(18 \Omega)}{(9 \Omega + 18 \Omega)} = -2 \text{ A} \)

When the ammeter is placed in series with the 9 \( \Omega \),

Now, \( i_1 = -3 \text{ A} \frac{(18 \Omega)}{(10 \Omega + 18 \Omega)} = -1.93 \text{ A} \)