Electric Current

**Definition:** rate of positive charge flow

**Symbol:** $i$

**Units:** Coulombs per second $\equiv$ Amperes (A)

$$i = \frac{dq}{dt}$$

where $q =$ charge (in Coulombs), $t =$ time (in seconds)

**Note:** Current has polarity.
Electric Potential (Voltage)

• **Definition**: energy per unit charge
• **Symbol**: $\nu$
• **Units**: Joules/Coulomb $\equiv$ Volts (V)

\[ \nu = \frac{dw}{dq} \]

where $w =$ energy (in Joules), $q =$ charge (in Coulombs)

**Note:** Potential is always referenced to some point.

Subscript convention:

$v_{ab}$ means the potential at $a$ minus the potential at $b$.

\[ v_{ab} \equiv v_a - v_b \]
Electric Power

- **Definition**: transfer of energy per unit time
- **Symbol**: $p$
- **Units**: Joules per second $\equiv$ Watts (W)

$$p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right) = vi$$

- **Concept**: As a positive charge $q$ moves through a drop in voltage $v$, it loses energy
  - energy change $= qv$
  - rate is proportional to # charges/sec
The Ideal Basic Circuit Element

Attributes:

• Two terminals (points of connection)
• Mathematically described in terms of current and/or voltage
• Cannot be subdivided into other elements

Polarity reference for voltage can be indicated by plus and minus signs
Reference direction for the current is indicated by an arrow
A Note about Reference Directions

A problem like “Find the current” or “Find the voltage” is always accompanied by a definition of the direction:

In this case, if the current turns out to be 1 mA flowing to the left, we would say $i = -1$ mA.

In order to perform circuit analysis to determine the voltages and currents in an electric circuit, you need to specify reference directions. There is no need to guess the reference direction so that the answers come out positive, however.
Suppose you have an unlabelled battery and you measure its voltage with a digital voltmeter (DVM). It will tell you the magnitude and sign of the voltage.

With this circuit, you are measuring $v_{ab}$. The DVM indicates $-1.401$, so $v_a$ is lower than $v_b$ by 1.401 V.

Which is the positive battery terminal?

Note that we have used the “ground” symbol (ground) for the reference node on the DVM. Often it is labeled “C” for “common.”
Sign Convention for Power

Passive sign convention

- If $p > 0$, power is being delivered to the box.
- If $p < 0$, power is being extracted from the box.
Power Calculation Example

Find the power absorbed by each element:

Conservation of energy \[ \text{total power delivered} \Rightarrow \text{total power absorbed} \]

Aside: For electronics these are unrealistically large currents – mA is more typical than A

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>VOLTAGE (V)</th>
<th>CURRENT (A)</th>
<th>( vi ) (W)</th>
<th>( p ) (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-18</td>
<td>-51</td>
<td>918</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>-18</td>
<td>45</td>
<td>-810</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>2</td>
<td>-6</td>
<td>-12</td>
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</tr>
<tr>
<td>d</td>
<td>20</td>
<td>-20</td>
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<td></td>
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<tr>
<td>e</td>
<td>16</td>
<td>-14</td>
<td>-224</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>36</td>
<td>31</td>
<td>1116</td>
<td></td>
</tr>
</tbody>
</table>
Circuit Elements

- 5 ideal basic circuit elements:
  - voltage source
  - current source
  - resistor
  - inductor
  - capacitor

- Many practical systems can be modeled with just sources and resistors

- The basic analytical techniques for solving circuits with inductors and capacitors are the same as those for resistive circuits
Electrical Sources

• An *electrical source* is a device that is capable of converting non-electric energy to electric energy and *vice versa*.

  Examples:
  – battery: chemical $\leftrightarrow$ electric
  – dynamo (generator/motor): mechanical $\leftrightarrow$ electric

→ Electrical sources can either deliver or absorb power
Ideal Independent and Dependent Voltage Sources

- Circuit element that maintains a prescribed voltage across its terminals, **regardless of the current flowing in those terminals**.
  - Voltage is known, but current is determined by the circuit to which the source is connected.

- The voltage can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

**Circuit symbols:**

- Independent: $v_s$
- Voltage-controlled: $v_s = \mu v_x$
- Current-controlled: $v_s = \rho i_x$
Dependent Voltage Source

Application: Amplifier

\[ V_{\text{out}} = \mu V_{\text{in}} \]
Other Independent Voltage Source Symbols

Sinusoidal AC source

\[ v(t) = V_{\text{peak}} \sin(\omega t) \]

\[ V_{\text{effective}} = V_{\text{peak}} \sqrt{2} \]

(In US, 120 V, so

\[ V_{\text{peak}} = 170 \text{ V} \])

Battery (realistic source)

\[ + \quad \quad V_S \]

\[ - \]
Realistic Voltage Source

- A real-life voltage source, like a battery or the function generator in lab, cannot sustain a very high current. Either a fuse blows to shut off the device, or something melts…

- Additionally, the voltage output of a realistic source is not constant. The voltage decreases slightly as the current increases.

- We usually model realistic sources considering the second of these two phenomena. A realistic source is modeled by an ideal voltage source in series with an “internal resistance”, $R_S$. 

\[
\begin{align*}
V_s & \quad \text{+} \\
& \quad \text{−}
\end{align*}
\]
I-V Plot for a Real Battery

\[
\text{KVL: } -V_s + I R_s + V = 0 \\
V = V_s - I R_s
\]
Ideal Independent and Dependent Current Sources

- Circuit element that maintains a prescribed current through its terminals, **regardless of the voltage across those terminals**.
  - Current is known, but voltage is determined by the circuit to which the source is connected.
- The current can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

**Circuit symbols:**

- $i_s$ (independent)
- $i_s = \alpha v_x$ (voltage-controlled)
- $i_s = \beta i_x$ (current-controlled)
Electrical Resistance

• **Resistance**: Electric field is proportional to current density, within a resistive material. Thus, voltage is proportional to current. The circuit element used to model this behavior is the *resistor*.

  **Circuit symbol**: 

  **Units**: Volts per Ampere $\equiv$ ohms ($\Omega$)

• The current flowing in the resistor is proportional to the voltage across the resistor:

$$ v = i \ R $$

(Ohm’s Law)

where $v =$ voltage (V), $i =$ current (A), and $R =$ resistance ($\Omega$)
Resistance of an actual resistor

Material resistivity
= $\rho$ (Ω-cm)

Resistance = resistivity x length/(cross-sectional area)

$R = \rho \left( \frac{L}{WT} \right)$
Electrical Conductance

• **Conductance** is the reciprocal of resistance.

  **Symbol:** $G$

  **Units:** siemens (S) or mhos ( MΩ)

**Example:**

Consider an 8 $\Omega$ resistor. *What is its conductance?*
Short Circuit and Open Circuit

Wire (“short circuit”):
• $R = 0 \rightarrow \text{no voltage difference exists}$
  (all points on the wire are at the same potential)
• Current can flow, as determined by the circuit

Air (“open circuit”):
• $R = \infty \rightarrow \text{no current flows}$
• Voltage difference can exist,
  as determined by the circuit
Circuit Nodes and Loops

- A **node** is a point where two or more circuit elements are connected.
- A **loop** is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once

Example:
Kirchhoff’s Laws

• **Kirchhoff’s Current Law (KCL):**
  – The algebraic sum of all the currents entering any node in a circuit equals zero.

• **Kirchhoff’s Voltage Law (KVL):**
  – The algebraic sum of all the voltages around any loop in a circuit equals zero.
Example: Power Absorbed by a Resistor

\[ p = vi = (iR)i = i^2R \]

\[ p = vi = v \left( \frac{v}{R} \right) = \frac{v^2}{R} \]

Note that \( p > 0 \) always, for a resistor.

Example:

a) Calculate the voltage \( v_g \) and current \( i_a \).

b) Determine the power dissipated in the 80Ω resistor.
“Lumped Element” Circuit Modeling
(Model = representation of a real system which simplifies analysis)

• In circuit analysis, important characteristics are grouped together in “lumps” (separate circuit elements) connected by perfect conductors (“wires”)

• An electrical system can be modeled by an electric circuit (combination of paths, each containing 1 or more circuit elements) if

$$\lambda = \frac{c}{f} >> \text{physical dimensions of system}$$

Distance travelled by a particle travelling at the speed of light in one period

Example: \( f = 60 \text{ Hz} \)

$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{60} = 5 \times 10^6 \text{ m}$$
Construction of a Circuit Model

• The electrical behavior of each physical component is of primary interest.

• We need to account for undesired as well as desired electrical effects.

• Simplifying assumptions should be made wherever reasonable.
Terminology: Nodes and Branches

**Node:** A point where two or more circuit elements are connected

**Branch:** A path that connects two nodes
Notation: Node and Branch Voltages

- Use one node as the reference (the “common” or “ground” node) – label it with a symbol
- The voltage drop from node $x$ to the reference node is called the node voltage $v_x$.
- The voltage across a circuit element is defined as the difference between the node voltages at its terminals

Example:

\[ + \quad v_a \quad + \quad v_s \quad + \quad v_b \quad + \]

\[ - \quad v_1 \quad - \quad v_c \quad - \quad v_b \quad - \]

REFERENCE NODE
Using Kirchhoff’s Current Law (KCL)

Consider a node connecting several branches:

- Use reference directions to determine whether currents are “entering” or “leaving” the node – with no concern about actual current directions
Formulations of Kirchhoff’s Current Law

(Charge stored in node is zero.)

Formulation 1:
Sum of currents entering node
    = sum of currents leaving node

Formulation 2:
Algebraic sum of currents entering node = 0
    • Currents leaving are included with a minus sign.

Formulation 3:
Algebraic sum of currents leaving node = 0
    • Currents entering are included with a minus sign.
A Major Implication of KCL

- KCL tells us that all of the elements in a single branch carry the same current.
- We say these elements are connected \textit{in series}.

Current entering node = Current leaving node

\[ i_1 = i_2 \]
KCL Example

Currents entering the node:

Currents leaving the node:

3 formulations of KCL:

1.

2.

3.
Generalization of KCL

• The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.*, a “black box”
Generalized KCL Examples

\[ 5 \mu A \]

\[ 2 \mu A \]

\[ 50 \text{ mA} \]
Using Kirchhoff’s Voltage Law (KVL)

Consider a branch which forms part of a loop:

- Use **reference polarities** to determine whether a voltage is dropped – with no concern about actual voltage polarities
Formulations of Kirchhoff’s Voltage Law
(Conservation of energy)

Formulation 1:
Sum of voltage drops around loop
= sum of voltage rises around loop

Formulation 2:
Algebraic sum of voltage drops around loop = 0
• Voltage rises are included with a minus sign.
(Handy trick: Look at the first sign you encounter on each element when tracing the loop.)

Formulation 3:
Algebraic sum of voltage rises around loop = 0
• Voltage drops are included with a minus sign.
A Major Implication of KVL

- KVL tells us that any set of elements that are connected at both ends carry the same voltage.
- We say these elements are connected in parallel.

Applying KVL in the clockwise direction, starting at the top:

\[ v_b - v_a = 0 \quad \Rightarrow \quad v_b = v_a \]
KVL Example

Three closed paths:

Path 1:

Path 2:

Path 3:
An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop
  Otherwise, there would be an induced voltage (Faraday’s Law)

- **Note**: Antennas are designed to “pick up” electromagnetic waves; “regular circuits” often do so undesirably.

How do we deal with antennas (EECS 117A)?

Include a voltage source as the circuit representation of the induced voltage or “noise”.

(Use a lumped circuit model rather than a distributed (wave) model.)
Resistors in Series

Consider a circuit with multiple resistors connected in series. Find their “equivalent resistance”.

• KCL tells us that the same current ($I$) flows through every resistor

• KVL tells us

Equivalent resistance of resistors in series is the sum
Voltage Divider

\[ I = \frac{V_{SS}}{R_1 + R_2 + R_3 + R_4} \]
When can the Voltage Divider Formula be Used?

\[ V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS} \]

Correct, if nothing else is connected to nodes

\[ V_2 \neq \frac{R_2}{R_1 + R_2 + R_3 + R_4} \cdot V_{SS} \]
because \( R_5 \) removes condition of resistors in series
Resistors in Parallel

Consider a circuit with two resistors connected in parallel. Find their “equivalent resistance”.

- KVL tells us that the same voltage is dropped across each resistor
  \[ V_x = I_1 R_1 = I_2 R_2 \]
- KCL tells us
General Formula for Parallel Resistors

What single resistance $R_{eq}$ is equivalent to three resistors in parallel?

Equivalent conductance of resistors in parallel is the sum
Current Divider

\[ V_x = I_1 R_1 = I_{SS} R_{eq} \]
**Generalized Current Divider Formula**

Consider a current divider circuit with >2 resistors in parallel:

\[
\begin{align*}
I_3 &= \frac{V}{R_3} = I \left[ \frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3} \right] \\
V &= \frac{I}{\left( \frac{1}{R_1} \right) + \left( \frac{1}{R_2} \right) + \left( \frac{1}{R_3} \right)}
\end{align*}
\]
## EE 42: Checklist of Terms

25 Jan. 2005

<table>
<thead>
<tr>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge, current, voltage, resistance, conductance, energy, power</td>
</tr>
<tr>
<td>Coulomb, ampere, volt, ohm, siemen (mho), joule, watt</td>
</tr>
<tr>
<td>Kirchhoff’s Current Law (KCL), Kirchhoff’s Voltage Law (KVL), Ohm’s Law</td>
</tr>
<tr>
<td>Series connection, parallel connection</td>
</tr>
<tr>
<td>DC (steady), AC (time-varying)</td>
</tr>
<tr>
<td>Independent and dependent ideal voltage and current source</td>
</tr>
<tr>
<td>Voltage divider, current divider</td>
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<tr>
<td>Analog (A/D), Digital (D/A)</td>
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