In the paper by Akebe et al. (2020), we consider a system with \( n \) agents (i.e., families we would like to assist) and allocate resources to them. Each agent \( i \) has an "income" \( C_i \) per unit time (i.e., earned income minus expenses). The initial wealth of agent \( i \) is \( U_i \), which includes the "reserve" (i.e., initial wealth).

The income shocks can be modeled using the Random Walk model, where the probability of ruin is given by:

\[
P(\text{ruin}) = P(\text{hit 0})
\]

An income shock is something that causes a sudden expense or loss of income, e.g., illness, health bill, parking ticket, job loss, delayed paycheck, etc. Say each agent experiences income shocks at random, and agent \( i \) experiences a shock of magnitude \( \mu_i \), and there are \( B_i \) shocks per unit time.

Now, per unit time, agent \( i \)'s income grows as \( C_i - B_i \mu_i \) on average. If \( C_i - B_i \mu_i < 0 \), the agent will go negative, i.e., my ruin probability is 1.

This process is a random walk. What is \( P(\text{hit 0}) \)? This is called the ruin probability.

Say we can give subsidy \( x_i \) to each person. The ruin probability can be expressed as:

\[
P(\text{ruin}) = \frac{\sum B_i U_i}{C_i + \sum x_i}
\]

Minimize the total cost:

\[
\sum \frac{B_i U_i}{C_i + x_i} = \leq B_i
\]

"Water-filling" technique is applied to find the optimal subsidy allocation.