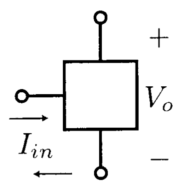
University of California, Berkeley *Fall 2004*
EECS 142 *Prof. A. Niknejad*

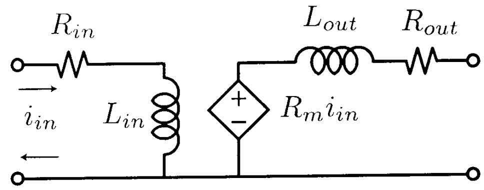
Midterm Exam (closed book)
Tuesday, October 19, 2004

Guidelines: Closed book. You may use a calculator. Do not unstaple the exam. *Warning:* Illustrations not to scale.

1. A new device has been invented with the following I-V characteristic:

$$V_o = -K_v \frac{1}{1 + \left(\frac{I_{in}}{I_p}\right)}$$





This relation holds for $I_{in} > 0$ and for any passive termination. The device parameters are as follows: $I_p = 750\,\mu$A, $K_v = .6$ V. A small-signal model including layout parasitics is shown above.

(a) (*6 points*) Calculate an expression for the maximum gain of the amplifier at 10 GHz. Note:

$$G_{Tu,max} = \frac{|z_{21}|^2}{4\Re(z_{11})\Re(z_{22})}$$

By inspection $z_{11} = R_{in} + j\omega L_{in}$

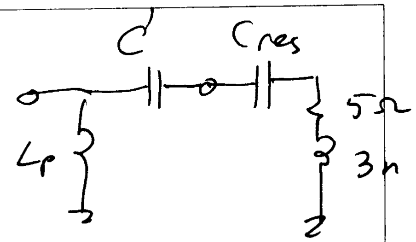
$z_{22} = R_{out} + j\omega L_{out}$

$z_{21} = R_m$

$$G_{T,max} = \frac{1}{4} \frac{R_m^2}{R_{in} R_{out}}$$

(b) (*10 points*) Design a matching network to acheive an input match with $R_S = 50\,\Omega$. Assume that $R_{in} = 5\,\Omega$ and $L_{in} = 3\,\text{nH}$. Draw the complete input network to the amplifier and specify the bandwidth of the match. Note that $f_0 = 10\,\text{GHz}$.

$$Q = \sqrt{\frac{50}{5} - 1} = 3$$



$$X_s = 3 \times 5 = 15$$

$$R_p = (1 + Q^2) \cdot 5$$

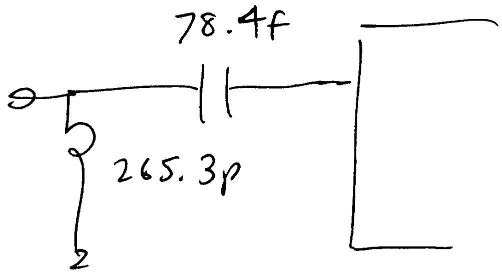
$$X_p = (1 + Q^{-2})\, X_s = 16.67\,\Omega \qquad C = 954.7\,\text{fF}$$

$C = 78.4f$ (series combination)

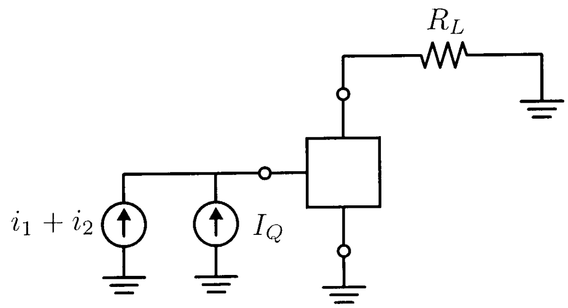
$L_p = 265.3\,pH$

$j\omega L_{in} = 188\Omega$ use cap to resonate

$C_{res} = 84.66\,f$



$$B = \frac{f_0}{Q} = \frac{10G}{3} = 3.33\,\text{GHz}$$



(c) (*10 points*) For the circuit shown above, calculate the IM_3 at low frequency for two current input signals at 100 MHz and 101 MHz of magnitude 100 μA. The circuit is biased as shown with $I_Q = 2\,\text{mA}$.

$$V_o = -K_V \frac{1}{1 + \frac{i_n + I_Q}{I_p}}$$

$$= -k_v \frac{1}{\left(1 + \frac{I_Q}{I_p}\right)\left(1 + \frac{i_n}{I_x}\right)}$$

$$I_x = \frac{1}{I_p\left(1 + \frac{I_Q}{I_p}\right)}$$

$$\boxed{I_x = \frac{1}{I_p + I_Q}}$$

$$V_o = \frac{-k_v}{1 + \frac{I_Q}{I_p}}\left(1 - \frac{i_n}{I_x} + \frac{i_n^2}{I_x^2} - \frac{i_n^3}{I_x^3} + \cdots\right)$$

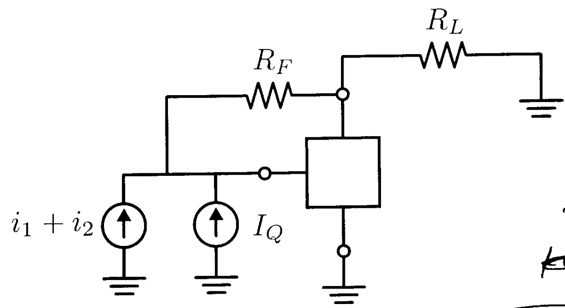
$$IM_3 = \frac{3}{4}\frac{a_3}{a_1} S_i^2 = \frac{3}{4}\frac{1}{I_x^3} \cdot I_x \, S_i^2$$

$$= \frac{3}{4}\left(\frac{i_n}{I_x}\right)^2$$

$I_x = 2\,\text{mA} + 0.75\,\text{mA}$

$i_n = 100\,\mu\text{A}$

$$= -60\,\text{dBc}$$



25
~~100~~ Ω

(d) (*10 points*) Assume a shunt feedback resistor of value $R_F = 1\,\mathrm{k}\Omega$ is added to the circuit. Calculate HD_2 under this condition. Assume $R_F \gg R_{in}$. with $V_{out} = 100\,\mathrm{mV}$

$$i_F \cong \frac{v_o}{R_F} \qquad f = \frac{1}{R_F}$$

$$T = a_1 f = \frac{R_m}{R_F}$$

$$R_m = \frac{k_v}{1 + \frac{I_Q}{I_P}} \frac{1}{I_X} = \left(\frac{0.6}{1 + \frac{2m}{0.75}}\right) \frac{1}{2.75\,m}$$

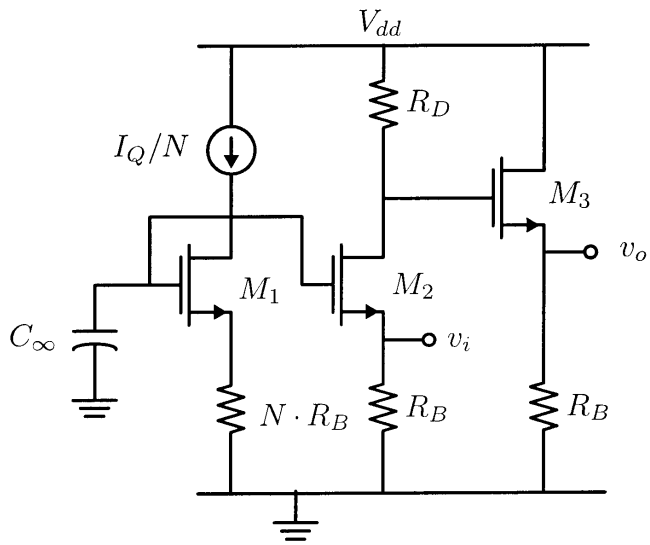
$$= 59.5\,\Omega$$

$$T = \frac{59.5}{25} = 2.38$$

$$HD_2 = \frac{1}{2} \frac{a_2}{a_1^2} \frac{1}{1+T} \quad S_o m$$

$$= \frac{1}{2} \cancel{\frac{1}{I_X^2}} \cancel{I_X^2} \left(\frac{1 + \frac{I_Q}{I_P}}{k_v}\right) \frac{1}{(1+T)} \quad S_o m \quad \times$$

$$HD_2 = \frac{1}{2} \left(\frac{1 + \frac{I_Q}{I_P}}{k_v}\right) \left(\frac{1}{1+T}\right) \quad S_o m = -21\ \mathrm{dBc}$$



2. The amplifier shown above is designed in a triple-well process (body and source are shorted). Assume that the resistor $R_B \gg \frac{1}{g_m}$ and $R_D \ll r_o$.

   (a) (*10 points*) Calculate the bandwidth of the above amplifier. Assume the amplifier is designed for an input and output match.

DRAIN OF M_2

$$C_{tot} = C_{db2} + C_{\mu 2} + \frac{C_{gs3}}{1+g_{m3}R_3} + C_{\mu 2}$$

$$R_{tot} \simeq R_D$$

$$\tau = R_D C_{tot}$$

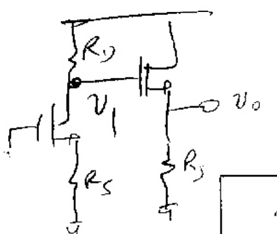
$$f = \frac{1}{2\pi\tau}$$

(b) (*10 points*) Calculate $G_{T,max}$ for this amplifier. Explain your assumptions.

$$Re(y_{11}) \simeq g_{m2} \qquad y_{21} = \frac{i_2}{v_1}\bigg|_{V_2=0} = g_{m2} R_D \, g_{m3}$$

$$Re(y_{11}) \simeq g_{m3}$$

$$\frac{|Y_{21}|^2}{4 Re(y_{11}) Re(y_{22})} = \frac{1}{4} \, \frac{g_{m2}^{\cancel{2}} R_D^2 g_{m3}^{\cancel{2}}}{\cancel{g_{m2}} \, \cancel{g_{m3}}} = \frac{1}{4} \left(\frac{R_D}{R_S}\right)^2$$



(c) (*16 points*) Calculate the noise figure for the amplifier. Assume the amplifier is driven by a source with resistance R_S and loaded by a matched load $R_L = R_S$. Assume the amplifier is designed for an input and output match. Ignore gate noise.

$$v_1 = v_{R_D} + i_{d1}\left(\frac{1}{1+g_m R_s}\right) R_D + i_s \left(\frac{g_m R_s}{1+g_m R_s}\right) R_D$$

$$g_m R_s = 1$$

$$\overline{v_1^2} = \overline{v_{R_D}^2} + \frac{1}{4} \overline{i_{d1}^2} R_D^2 + \frac{\overline{i_s^2} R_D^2}{4}$$

$$\overline{v_{oT}^2} = \overline{v_1^2} \left(\frac{g_m R_S}{1+g_m R_S}\right)^2 + \overline{i_{d2}^2} \frac{R_S^2}{(1+g_m R_S)^2}$$

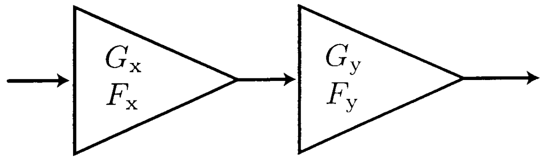
$$= \overline{v_1^2} \, \frac{1}{4} + \overline{i_{d2}^2} \, \frac{R_S^2}{4}$$

$$\overline{v_{oT}^2} = \left(\overline{v_{R_D}^2} + \frac{\overline{i_{d1}^2} R_D^2}{4} + \frac{\overline{i_s^2} R_D^2}{4}\right)\frac{1}{4} + \overline{i_{d2}^2} \frac{R_S^2}{4}$$

$$F = 1 + \frac{4 \overline{v_{R_D}^2}}{\overline{i_s^2} R_D^2} + \frac{\overline{i_{d1}^2}}{\overline{i_s^2}} + \frac{4 \overline{i_{d2}^2} R_S^2}{\overline{i_s^2} R_D^2}$$

$$= 1 + \frac{4 R_S}{R_D} + \left(\frac{\gamma}{\alpha}\right) \cancel{g_m R_S} + 4 \left(\frac{R_S}{R_D}\right)^3$$

3. You would like to design an amplifier with a power gain of 50 dB from two amplifiers, each with the following characteristics:



$$G_1 = 30\,\text{dB}$$

$$NF_1 = 1.7\,\text{dB} \quad = 1.479$$

$$G_2 = 20\,\text{dB}$$

$$NF_2 = 1.3\,\text{dB} \quad = 1.349$$

(a) (*8 points*) Find the optimal ordering of the amplifiers in cascade to acheive the lowest possible noise figure.

$$F_{12} = F_1 + \frac{F_2 - 1}{G_1} = 1.479$$

$$F_{21} = 1.349 + \frac{0.479}{100} = 1.354$$

ORDER 2—1 IS BETTER

$$M_1 = \frac{G_1 F_1 - 1}{G_1 - 1} = 1.479$$

$$M_2 = 1.353$$

(b) (*10 points*) Find the lowest possible signal power (minimum detectable signal) to maintain an $SNR_o > 10\,\mathrm{dB}$. Assume the input noise is from a source resistance $R_S = 50\,\Omega$ and the communication bandwidth is $1\,\mathrm{MHz}$.

$$SNR_o = \frac{SNR_i}{F} > 10$$

$$SNR_i > 10 \cdot F = 13.54$$

$$P_i > 13.54 \cdot P_{noise} = 13.54 \cdot kT \cdot B$$

$$= 13.54 \cdot 4 \times 10^{-21}\,W \cdot 10^{6} = 5.42 \times 10^{-14}\,W$$

$$= -103\,dBm$$

(c) (*10 points*) Assume that the system is designed to work with a signal as large as $-10\,\mathrm{dBm}$. What's the requires IIP_3 for the entire system to maintain a signal-to-distortion $SDR > 10\,\mathrm{dB}$.

$$SDR > 10$$

$$\frac{P_i}{P_d} > 10 \Rightarrow P_d < \frac{P_i}{10} = \frac{0.1\,mW}{10} = 0.01\,mW$$

IM3 DEGRADES BY 2dB/1dB INPUT

INPUT INCREASES BY 5dB $\Rightarrow$ IM3 = 0dB

$$IIP_3 = -10\,dBm + 5\,dBm = -5\,dBm$$