Midterm Exam (closed book/notes)
Thursday, October 16, 2007

Guidelines: Closed book. You may use a calculator. Do not un staple the exam. In order to maximize your score, write clearly and indicate each step of your calculations. We cannot give you partial credit if we do not understand your reasoning. Feel free to use scratch paper.

Common two-port equation:

\[ Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}} \]
\[ Y_{out} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_S + Y_{11}} \]
\[ G_p = \frac{P_L}{P_{in}} = \frac{|Y_{21}|^2}{|Y_L + Y_{22}|^2} \frac{\Re(Y_L)}{\Re(Y_{in})} \]

Simple trigonometric identify:

\[ \cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y) \]
\[ \sin(x + y) = \cos(y) \sin(x) + \cos(x) \sin(y) \]
\[ 2 \cos(x) \cos(y) = \cos(x + y) + \cos(x - y) \]

Distortion equations: \( s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \cdots \)

\[ IM_2 = 2HD_2 = \frac{a_2}{a_1} S_i \]
\[ IM_3 = 3HD_3 = \frac{3a_3}{4a_1} S_i^2 \]

Series inversion: If \( s_i = a_1 s_o + a_2 s_o^2 + a_3 s_o^3 + \cdots \), then \( s_o = b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \cdots \) where

\[ b_1 = \frac{1}{a_1} \]
\[ b_2 = -\frac{a_2}{a_1^3} \]
\[ b_3 = \frac{2a_3}{a_1^5} - \frac{a_3}{a_1^4} \]

Cascade of two power series:

\[ c_1 = a_1 b_1 \]
\[ c_2 = b_1 a_2 + b_2 a_1^2 \]
\[ c_3 = b_1 a_3 + 2b_2 a_1 a_2 + b_3 a_1^3 \]
Effect of feedback on distortion (not required for this test but useful)

\[ b_1 = \frac{a_1}{1 + a_1 f} = \frac{a_1}{1 + T} \]

\[ b_2 = \frac{a_2}{(1 + T)^3} \]

\[ b_3 = \frac{a_3 (1 + T) - 2a_2^2 f}{(1 + T)^5} \]

Taylor Series Expansion about \( x = 0 \)

\[ f(x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \cdots \]

Binomial Theorem

\[ (x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \]

MOS Square Law Device Physics (Saturation)

\[ I_{DS} = \mu C_{ox} \frac{W}{L} \frac{1}{2} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

\[ C_{GS} = \frac{2}{3} W \cdot L C_{ox} \]

\[ \omega_T = \frac{g_m}{C_{gs}} = \frac{3 \mu (V_{GS} - V_T)}{2 L^2} \]
1. (50 points) Answer the following questions succinctly.

(e) (10 points) Identify the type (LP, HP, BP, BS, AP) of the following filter and sketch the approximate magnitude response.
(b) (10 points) A system is described by the following non-linear equation

\[ V_o = K_v \cos \left( \frac{V_i}{V_x} \right) \]

where \( K_v = 10V \) and \( V_x = 50mV \) are constants and \( |V_i| < \pi V_x/2 \). Find an expression for the apparent gain of the system \( G(V_i) \) as a function of the input amplitude.

\[ V_o = K_v \cos \left( \frac{\hat{V}_c - \hat{V}_i}{\hat{V}_x} \right) = K_v \cos \left( \frac{\hat{V}_c - \hat{V}_i}{\hat{V}_x} \right) = K_v \cos (A + y) = K_v \left[ \cos A \cos y - \sin A \sin y \right] \]

\[ \frac{K_v \cos A}{2} = k_1 \]

\[ \frac{K_v \sin A}{2} = k_2 \]

\[ y = \frac{1}{2} \left( \frac{y^2}{2!} + \frac{y^4}{4!} + \cdots \right) \]

\[ \Rightarrow \]

\[ \alpha_1 = \frac{k_2}{V_x} \]

\[ \alpha_2 = -k_1 \]

\[ \alpha_3 = -\frac{k_2}{3! \cdot V_x^3} \]

\[ \alpha_0 = \frac{k_1}{2! \cdot V_x^2} \]

Apparent gain = \[ \alpha_1 \left( 1 + \frac{3}{4} \frac{\alpha_3 V_x^2}{\alpha_1} \right) \]

Since \( |V_c| < \pi V_x \) its biased around \( \phi \). In this case, there is no apparent gain. \( \alpha_1 = \alpha_3 = 0 \).
(c) (10 points) For the same amplifier as the previous problem, find the maximum signal amplitude $V_i$ which causes a 10% shift in the bias current. Only consider the effect of lowest-order non-linearity.

$$V_o = a_0 + a_1 V_i + a_2 V_i^2 + \cdots, \quad V_i = \cos \omega t$$

$$\Rightarrow a_2 V_i^2 = a_2 \cos^2 \omega t = a_2 \left[ \frac{1}{2} (1 + \cos 2\omega t) \right]$$

$$\frac{a_2}{2} \text{ is added to the DC level;}$$

and we need

$$\frac{\sqrt{\frac{2a_2}{2} V_i^2}}{V_i} \left( \frac{V_i^2}{2} + \frac{1}{2} \frac{\theta V_i^2}{\sqrt{V_i^2}} \right) < 0.1 a_0$$

$$\Rightarrow \frac{V_i^2}{0.4 V_i^2} \Rightarrow V_i < \sqrt{0.4} V_i$$
(d) (10 points) Calculate the \( iIP_3 \) of the overall amplifier and find the maximum input signal allowed if we desire \( IM_3 > 50 \text{dBc} \). The power gain of the first amplifier \( G_A = 17 \text{dB} \), the \( iIP_3^A = -11 \text{dBm} \), whereas the second amplifier has a gain \( G_B = 50 \text{dB} \), and an \( iIP_3^B = +12 \text{dBm} \). Note: You may neglect second-order interaction and assume all stages are matched.

\[
\frac{1}{IIIP_3} = \frac{1}{iIP_3^A} + \frac{G_A}{iIP_3^B}
\]

\[
iIP_3^A = 8 \times 10^{-5} \text{ W}
\]
\[
iIP_3^B = 1.58 \times 10^{-2} \text{ W}
\]

\( G_A = 50 \text{dB} \)

\[
\Rightarrow \frac{1}{IIIP_3} = \frac{1}{8 \times 10^{-5}} + \frac{50}{1.58 \times 10^{-2}}
\]

\[
\Rightarrow IIIP_3 = 6.38 \times 10^{-5} \approx -11.94 \approx -12 \text{ dBm}
\]

\[
\Rightarrow P_{in (map)} = -12 \text{dBm} - 25 \text{dB} = -37 \text{dBm}
\]
2. (50 points) Consider the two-stage amplifier shown above realized in a thin film resting on an insulating substrate technology. The transistor has AN $f_T = 17$GHz when biased at $V_{GS} - V_T = 2V$, and is is described by the well-known square-law behavior and the minimum channel length is $L_{min} = 0.1 \mu$. Due to the insulating substrate, the $C_{db}$ may be neglected while the $C_{gd} = 0.15C_{gs}$. Due to the particular doping profile, the threshold voltage varies with body bias as: $V_T = V_{T0} + K_1 V_{BS}^2$ ($K_1$ is positive). The channel length modulation coefficient $\lambda = 0.2$. 
(a) (10 points) What is $G_{\text{max}}$ for this two-stage amplifier at 10GHz? Derive an expression for the maximum achievable gain under the unilateral assumption. Use this result for the next problem. Compare the result to a unilateral common-gate amplifier.

$$G_{\text{max}} = \frac{\frac{1}{4}Y_{21}^2}{\text{uRe}(Y_{11})\text{Re}(Y_{22})}$$

$$Y_{21} = g_{m2}$$

$$\text{Re}(Y_{11}) = g_{m2}$$

$$\text{Re}(Y_{22}) \approx (y_{02}g_{m3} \omega_0)^{-1}$$

$$\Rightarrow G_{\text{max}} \approx \frac{g_{m2}r_{01}r_{02}g_{m3}}{4g_{m2}^2}$$

$$= \frac{g_{m2}g_{m3}r_{02}r_{03}}{4\omega_0}$$

It's not unilateral (due to $\omega_0$)

But if we assume unilateral:

$$Y_{21} = g_m$$

$$\text{Re}(Y_{11}) = g_m \quad \Rightarrow \quad G_{\text{max}} \approx \frac{g_m r_0}{4\omega_0}$$

$$\text{Re}(Y_{22}) \approx \frac{1}{\omega_0}$$
(b) (12 points) Find the appropriate bias point $V_{GS} - V_T$, device size $W_3$, and the $LC$ tank component values for operation at 10GHz under an input match while minimizing power consumption and maximizing power gain. In sizing M3 take the following constraints into account: The transistor parasitics can only account for 5% of the overall tank capacitance (for good stability and frequency tuning versus process variations). Moreover, the load inductor must be sized to meet the following practical constraints: $250pH < L < 1.5nH$. (Note: If you could not answer the previous problem, maximize the size of M3 while meeting the constraints.) Take $R_S = R_L = 50\Omega$, and $(W/L)_3 = 10\mu m/0.1\mu m$.

Assume input match so $g_{m2} = \frac{1}{50}$

Also for a common-gate device:

$$f_{-3dB} = \frac{1}{\tau} = \frac{1}{\frac{C_{gs} \times \frac{1}{2} g_{m}}{2g_{m}}} = f_{g} + \frac{g_{m}}{2}$$

we need $f_{-3dB} = 1GHz$ to maximize gain with lowest $I_D \Rightarrow f_{t} = \frac{1GHz}{2} = 5GHz$ (required $f_{t}$)

$$\frac{f_{t}}{f_{t,\text{map}}} = \frac{(V_{gs} - V_{th})}{(V_{gs} - V_{th})_{\text{map}} \Rightarrow V_{gst} = 2 \times \frac{5}{17} = 0.58}$$

Now $g_{m} = \frac{2I_{D}}{V_{gst}} \Rightarrow I_{D} = \frac{0.58 \times \frac{1}{50}}{2} = 0.058 mA$

Now we need $W_3 \Rightarrow$ we need to maximize the size of $m_3$ to increase $Re(Z_{22})$ but we should keep $C_{gd} \left< \frac{1}{2Q^2} C_{tank}$
So we go for maximum \( \frac{W}{L} \) to meet \( C_{gd} = \frac{8}{2\pi} C_{tanh} \). So we need to maximize \( C_t \) to be able to maximize \( \frac{W}{L} \). This means min \( L \). Our \( L_{min} = 25 \phi_{PH} \) then \( C_t = \frac{1}{L^2} \approx \frac{1}{(0.015\mu m)^2} \).

\[
C_{gd} = 5\phi_{71} \frac{fF}{\mu m} \]

\[
W_3 = \frac{5\phi_{71} fF}{0.15 \times \frac{2}{3} \times 0.1 \mu m \times C_{ox}}
\]

\[
W_3 C_{ox} = 5.07 \times 10^{-6}
\]

\[
\frac{f_{t_m}}{f_{m_2}} = 5 C_{U_3} = \frac{9 m_2}{2^3 W_2 L_2 C_{ox} x 2\pi}
\]

\[
\frac{f_{t_m}}{f_{m_2}} = \frac{W_3 C_{ox}}{0.15 \times \frac{2}{3} \times 2 \times 2\pi}
\]

\[
W_3 = 0.5 \times W_2 = 5 \mu m
\]
(c) (12 points) Calculate a power series expansion up to third order for the output signal as a function of the input signal amplitude. You should assume $R_s = 0 \Omega$ and only include the distortion generated by M2. (Note: Do not use any numerical results in this problem. Leave your answer in symbolic form)

\[
I_D = k \left( (\hat{V}_{gs} - \hat{V}_{th}) (1 + k_1 \hat{V}_{DS}) \right)
\]

\[
= k \left( (\hat{V}_{s} + \hat{V}_{o} - (\hat{V}_{th} + k_1 (\hat{V}_{gs} - \hat{V}_{s})})^2 \right)
\]

\[
(1 + \lambda (\hat{V}_{D} - \hat{V}_{S} + \hat{V}_{o})) \left( \hat{V}_{th} - k_1 \hat{V}_{s} \right)
\]

\[
= k \left[ \left( \hat{V}_{s} + \hat{V}_{o} - (\hat{V}_{th} + k_1 \hat{V}_{s}) \right)^2 \right]
\]

\[
= k \left[ (\hat{V}_{s} - \hat{V}_{o} - \hat{V}_{th}) + \hat{V}_{s} \left[ 1 - 2k_1 \hat{V}_{th} - k_2 \hat{V}_{s}^2 \right] \right]
\]

\[
\left[ (1 + \lambda \hat{V}_{DS} + \lambda \hat{V}_{S}) \right]
\]

\[
\Rightarrow a_0 = k (1 + \lambda \hat{V}_{DS}) (\hat{V}_{s} - \hat{V}_{o} - \hat{V}_{th})
\]

\[
a_1 = -k (2(1+\lambda \hat{V}_{DS}) (\hat{V}_{s} - \hat{V}_{o} - \hat{V}_{th}) + \lambda (\hat{V}_{b} - \hat{V}_{s} - \hat{V}_{th})
\]

\[
a_2 = k (1+\lambda \hat{V}_{DS}) k_1
\]
(d) (6 points) Explain why we may neglect the distortion generated by M3? How does an $R_s > 0\ \Omega$ affect the distortion.

(1) $\Rightarrow$ large $R_s = r_{02}$ degeneration

(2) $\Rightarrow$ Acts as a feedback for $M_2$

lasing, the distortion $M_2$ dominates distortion.

Even though $R_s$ increases the distortion of $M_3$, $M_3$ distortion is small.
(e) (10 points) Design an output matching network to drive an external $R_S = R_L = 50 \Omega$. Use the fewest components possible and draw the overall load and matching network schematic. If you could not answer the previous problems, you may assume that $W_2 = 2W_1$, $I_{ds} = 12mA$.

$$C_{gd3} = 50 \text{pF} \quad \text{(from previous part)}$$

$$R_{out3} = R_0 \cdot g_{m3} \cdot \frac{1}{\chi^2 \cdot I_D^2 \cdot g_{m3}} = 2.43 \text{k}\Omega \approx 2.5 \text{k}\Omega$$

$$Y_{source} = 2.5 \text{k}\Omega + j(2\pi \times 10^6 \cdot 50 \text{pF})$$

First we resonate out the capacitive part, with $L_{res}$

$$L_{res} = \frac{1}{\omega^2 C_{gd3}} = \frac{1}{(2\pi \times 10^6 \cdot 3)^2 \cdot 50 \text{pF}} \approx 50 \text{nH}$$

Now, to get the fewest components we try

$$M = \frac{2500}{50} = 50 \Rightarrow Q = \sqrt{m - 1} = 7$$

$$X_L = (1 + Q^2) \cdot X_L = 364 \Rightarrow X_C = -X_L = -364 = 43.7 \text{pF}$$

$$Q = \frac{R_{out3}}{X_L} \Rightarrow X = 3.57$$

$$\Rightarrow L = 5.68 \text{nH}$$

12