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EECS 142

Fall 2007
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Midterm Exam (closed book/notes)
Thursday, October 16, 2007

Guidelines: Closed book. You may use a calculator. Do not unstaple the exam. In order to maximize your score, write clearly and indicate each step of your calculations. We cannot give you partial credit if we do not understand your reasoning. Feel free to use scratch paper.

Common two-port equation:

$$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}}$$

$$Y_{out} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_S + Y_{11}}$$

$$G_p = \frac{P_L}{P_{in}} = \frac{|Y_{21}|^2}{|Y_L + Y_{22}|^2} \frac{\Re(Y_L)}{\Re(Y_{in})}$$

Simple trigonometric identify:

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \cos(y)\sin(x) + \cos(x)\sin(y)$$

$$2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$$

Distortion equations: $s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \cdots$

$$IM_2 = 2HD_2 = \frac{a_2}{a_1} S_i$$

$$IM_3 = 3HD_3 = \frac{3}{4}\frac{a_3}{a_1} S_i^2$$

Series inversion: If, $s_i = a_1 s_o + a_2 s_o^2 + a_3 s_o^3 + \cdots$, then $s_o = b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \ldots$ where

$$b_1 = \frac{1}{a_1}$$

$$b_2 = -\frac{a_2}{a_1^3}$$

$$b_3 = \frac{2a_2^2}{a_1^5} - \frac{a_3}{a_1^4}$$

Cascade of two power series:

$$c_1 = a_1 b_1$$

$$c_2 = b_1 a_2 + b_2 a_1^2$$

$$c_3 = b_1 a_3 + 2b_2 a_1 a_2 + b_3 a_1^3$$

Effect of feedback on distortion (not required for this test but useful)

$$b_1 = \frac{a_1}{1 + a_1 f} = \frac{a_1}{1+T}$$

$$b_2 = \frac{a_2}{(1+T)^3}$$

$$b_3 = \frac{a_3(1+T) - 2a_2^2 f}{(1+T)^5}$$

Taylor Series Expansion about $x = 0$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

MOS Square Law Device Physics (Saturation)

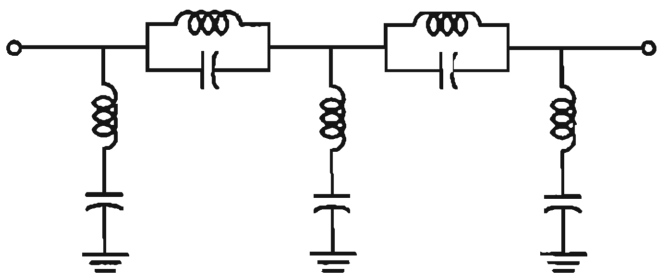
$$I_{DS} = \mu C_{ox} \frac{W}{L}\frac{1}{2}(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

$$C_{GS} = \frac{2}{3} W \cdot L C_{ox}$$

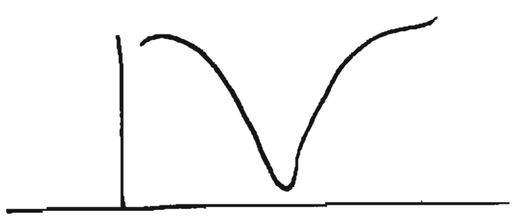
$$\omega_T = \frac{g_m}{C_{gs}} = \frac{3}{2}\frac{\mu(V_{GS} - V_T)}{L^2}$$

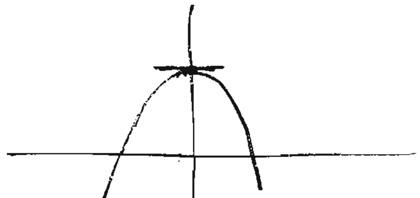
1. (*50 points*) Answer the following questions succinctly.

   (a) (*10 points*) Identify the type (LP,HP,BP,BS,AP) of the following filter and sketch the approximate magnitude response.



BS





(b) (*10 points*) A system is described by the following non-linear equation

$$V_o = K_v \cos\left(\frac{V_i}{V_x}\right)$$

where $K_v = 10\text{V}$ and $V_x = 50\text{mV}$ are constants and $|V_i| < \pi V_x/2$. Find an expression for the apparent gain of the system $G(V_i)$ as a function of the input amplitude.

(I)

$$V_o = K_v \cos\left(\frac{\hat{V}_i + v_i}{V_x}\right) = k_v \cos\Big(\overbrace{\frac{\hat{V}_i}{V_x}}^{A} + \overbrace{\frac{v_i}{V_x}}^{y}\Big)$$

$$= K_v \cos(A + y) = K_v\left[\cos A \cos y - \sin A \sin y\right] \qquad K_v \cos A \triangleq K_1, \quad K_v \sin A \triangleq K_2$$

$$= K_1\left[1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots\right] + K_2\left[y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots\right]$$

$$\Rightarrow \begin{cases} a_1 = K_2/v_x \\ a_2 = \dfrac{-K_1}{2!\, v_x^2} \\ a_3 = \dfrac{-K_3}{3!\, v_x^3} \\ a_0 = K_1 \end{cases}$$

Apparent gain =

$$a_1\left(1 + \frac{3}{4}\frac{a_3}{a_1} S_1^2\right)$$

*Since $|V_i| < \frac{\pi V_x}{2}$ its biased around ϕ. In this case there is no apparent gain

$$a_1 = a_3 = 0$$

(c) (*10 points*) For the same amplifier as the previous problem, find the maximum signal amplitude V_i which causes a 10% shift in the bias current. Only consider the effect of lowest-order non-linearity.

$$V_o = a_o + a_1 v_i + a_2 v_i^2 + \cdots \qquad v_i = \cos\omega t$$

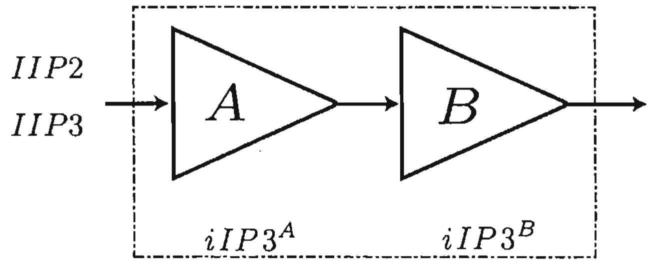
$$\Rightarrow a_2 v_i^2 = a_2 \cos^2\omega t = a_2\left[\tfrac{1}{2}(1+\cos 2\omega t\right] \Rightarrow$$

$\frac{a_2}{2}$ is added to the DC level:

and we need $v_i^2 \frac{a_2}{2} < 0.1\, a_o$

$$v_i^2 \frac{+K_v \cancel{\cos}\left(\cancel{\frac{v_i}{v_x}}\right)}{2 \times 2!\, v_x^2} < 0.1 K_v \cancel{\cos} \frac{v_i}{v_x} \Rightarrow$$

$$v_i^2 < 0.4\, v_x^2 \Rightarrow \boxed{v_i < \sqrt{0.4}\, v_x}$$

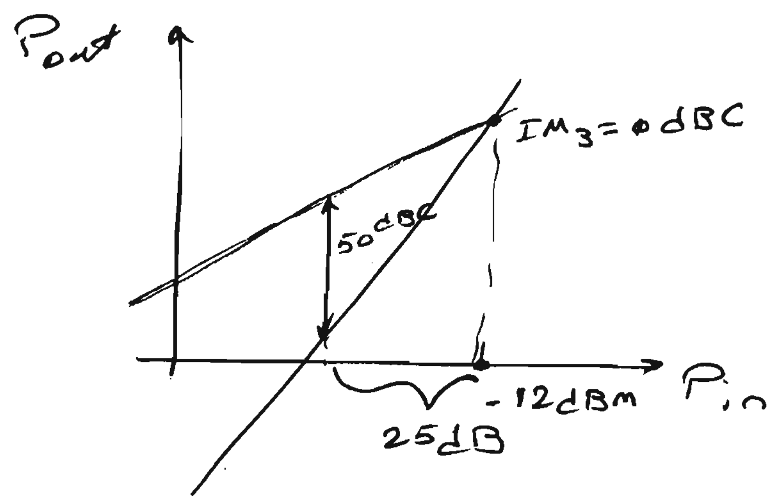


(d) (*10 points*) Calculate the iIP_3 of the overall amplifier and find the maximum input signal allowed if we desire $IM_3 > 50\text{dBc}$. The power gain of the first amplifier $G_A = 17\text{dB}$, the $iIP_3^A = -11\text{dBm}$, whereas the second amplifier has a gain $G_B = 50\text{dB}$, and an $iIP_3^B = +12\text{dBm}$. *Note:* You may neglect second-order interaction and assume all stages are matched.

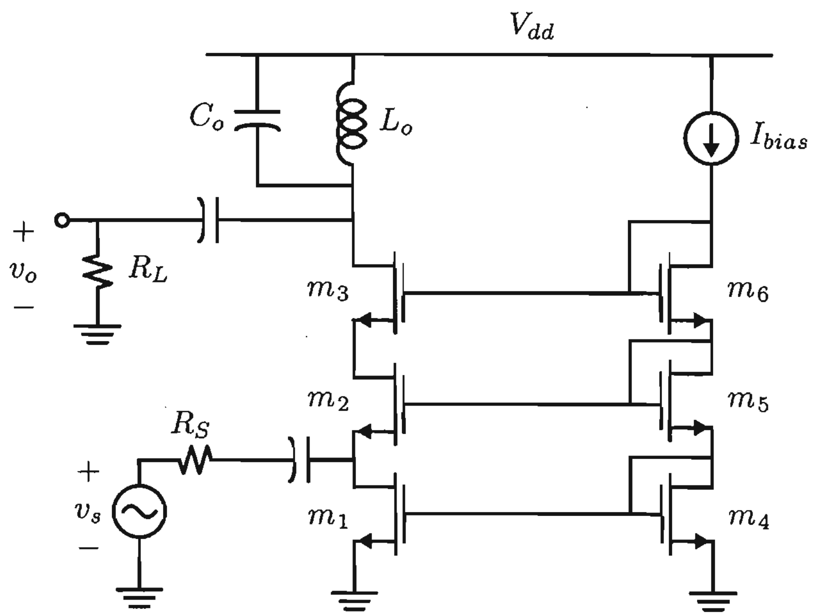
$$\frac{1}{IIP_3} = \frac{1}{iIP_3^A} + \frac{G_A}{iIP_3^B}$$

$$\left.\begin{array}{l} iIP_3^A = 8\times10^{-5}\,W \\ iIP_3^B = 1.58\times10^{-2}\,W \\ G_A \approx 50 \end{array}\right\} \Rightarrow \frac{1}{IIP_3} = \frac{1}{8\times10^{-5}} + \frac{50}{1.58\times10^{-2}}$$

$$\Rightarrow IIP_3 = 6.38\times10^{-5} \cong -11.94 \simeq -12\ \text{dBm}$$



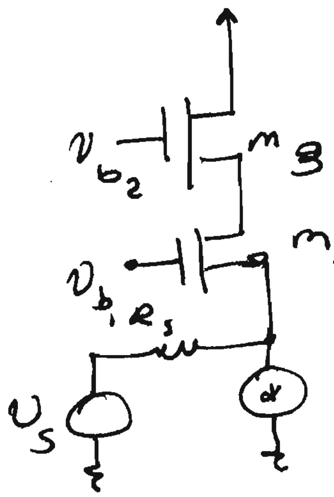
$$\Rightarrow P_{in(max)} = -12\text{dBm} - 25\text{dB} = \boxed{-37\ \text{dBm}}$$



2. (*50 points*) Consider the two-stage amplifier shown above realized in a thin film resting on an insulating substrate technology. The transistor has AN $f_T = 17$GHz when biased at $V_{GS} - V_T = 2$V, and is is described by the well-known square-law behavior and the minimum channel length is $L_{min} = 0.1\,\mu$. Due to the insulating substrate, the C_{db} may be neglected while the $C_{gd} = 0.15 C_{gs}$. Due to the particular doping profile, the threshold voltage varies with body bias as: $V_T = V_{T0} + K_1 V_{BS}^2$ (K_1 is positive). The channel length modulation coefficient $\lambda = 0.2$.

(a) (*10 points*) What is G_{max} for this two-stage amplifier at 10GHz? Derive an expression for the maximum achievable gain under the unilateral assumption. Use this result for the next problem. Compare the result to a unilateral common-gate amplifier.

$$G_{max} = \frac{|Y_{21}|^2}{4\,Re(Y_{11})\,Re(Y_{22})}$$



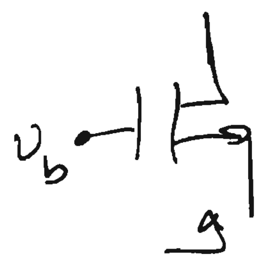
$$Y_{21} = g_{m_2}$$

$$Re(Y_{11}) \simeq g_{m_2}$$

$$R(Y_{22}) \simeq (r_{o2}\, g_{m_3}\, r_o)^{-1}$$

$$\Rightarrow G_{max} = \frac{g_{m_2}\, r_{o1}\, r_{o2}\, g_{m_3}}{4\, \cancel{g_{m_2}}}$$

$$= \frac{g_{m_2} g_{m_3} r_{o2} r_{o3}}{4}$$



It's not unilateral ? [due to r_{ds}]

But if we assume unilateral:

$$Y_{21} = g_m$$

$$Re(Y_{11}) = g_m \Rightarrow G_{max} = \frac{g_m r_o}{4}$$

$$Re(Y_{22}) = \frac{1}{r_o}$$

(b) (*12 points*) Find the appropriate bias point $V_{GS} - V_T$, device size W_3, and the LC tank component values for operation at 10GHz under an input match while minimizing power consumption and maximizing power gain. In sizing M3 take the following constraints into account: The transistor parasitics can only account for 5% of the overall tank capacitance (for good stability and frequency tuning versus process variations). Moreover, the load inductor must be sized to meet the following practical constraints: 250pH $< L <$ 1.5nH. (*Note:* If you could not answer the previous problem, maximize the size of M3 while meeting the constraints.) Take $R_S = R_L = 50\,\Omega$, and $(W/L)_2 = 10\,\mu m/0.1\,\mu m$.

- Assume input match so $g_{m_2} = \frac{1}{50}$ 2

- Also for a common-gate device:

$$f_{-3dB} = \frac{1}{T} = \frac{1}{C_{gs}\times\frac{\cancel{2}}{2g_m}} = \frac{2g_m}{\cancel{2}C_{gs}} = 2f_t$$

We need $f_{-3dB} = 10GHz$ to maximize gain with lowest I_D $\Rightarrow$ $f_t = \frac{10GHz}{2} = 5GHz$ (required f_t)

$$\frac{f_t}{f_{t\max}} = \frac{(V_{gs}-V_{th})}{\underbrace{(V_{gs}-V_{th})_{max}}_{\text{this is}=2}} \Rightarrow \boxed{V_{gst} = 2\times\frac{5}{17} = 0.58}$$ 3

Now $g_m = \frac{2I_D}{V_{gst}} \Rightarrow I_D = \frac{0.58\times\frac{1}{50}}{2} = \boxed{5.8\ mA}$ 2

Now we need W_3: We need to maximize the size of m_3 to ~~the~~ ~~as~~ increase $Re(Z_{22})$ but we should keep $C_{gd} < \frac{1}{20}C_{tank}$ 3

- $Re(Z_{22}) \approx r_{o_3} g_{m_2} g_{m_3}$
- $g_{m_3} = \sqrt{2\mu C_{ox}\frac{W}{L}I_D}$
- I_D is constant
- $\Rightarrow (W/L)_3 \uparrow$ to $g_m \uparrow$

So we go for Maximum $\left(\frac{W}{L}\right)_3$ to meet $C_{gd} = \frac{1}{20} C_{tank}$ So we need to maximize C_{tank} to be able to maximize $\left(\frac{W}{L}\right)_3$. This means min L. Our $L_{min} = 250 pH \Rightarrow$

$$C_t = \frac{1}{L\omega^2} = 1.015 pF$$

$$\Rightarrow \boxed{C_{gd_{max}} = 50.71 fF}$$

$$C_{gd} = 0.15 C_{gs} = 0.15 \times \frac{2}{3} W L C_{ox}$$

$$\Rightarrow W_3 = \frac{50.71 fF}{0.15 \times \frac{2}{3} \times 0.1\mu \times C_{ox}}$$

$$W_3 C_{ox} = 5.07 \times 10^{-6}$$

2

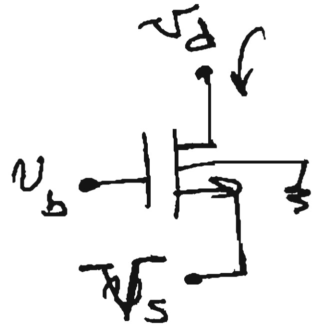
$$f_{t_{m_2}} = 5 GHz_3 = \frac{g_{m_2}}{\frac{2}{3} W_2 L_2 C_{ox} \times 2\pi}$$

$$\Rightarrow \cancel{f_{t_2} =} \quad W_2 C_{ox} = \frac{g_{m_2}}{f_{t_{m_2}} \times L_2 \times \frac{2}{3} \times 2\pi} = 9.55 \times 10^{-6}$$

$$\Rightarrow \boxed{\frac{W_3}{W_2} = \frac{5.07}{9.55}} \quad \boxed{W_3 \simeq 0.5 \times W_2 = 5 \mu m}$$

(c) (*12 points*) Calculate a power series expansion up to third order for the output signal as a function of the input signal amplitude. You should assume $R_s = 0\,\Omega$ and only include the distortion generated by M2. (*Note:* Do not use any numerical results in this problem. Leave your answer in symbolic form)

$$I_D = k(V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$$



$$= k([\hat{V}_S + v_s - V_b - (V_{th_0} + k_1(\dot{V}_B - V_S))^2)$$

$$(1 + \lambda(V_D - V_S + v_s))$$

$(V_{th_0} + k_1 V_S)^2$

$V_{th_0} - k_1\hat{V}_S - k_1 v_s$

$\cancel{V_{th_0} + k_1 V}$

$$V_{th_0} - k_1\hat{V}_S \triangleq \hat{V}_{th}$$

$$= k[\hat{V}_S - V_b + v_s - (\hat{V}_{th}^2 + k_2 v_s^2 - 2k_2 v_s \hat{V}_{th})](1 + \lambda(V_{DS} - v_s))$$

$$= k[(\hat{V}_S - V_b - \hat{V}_{th}^2) + v_s[1 - 2k_2\hat{V}_{th}) - k_2 v_s^2]$$

$$[(1 + \lambda V_{DS}) - \lambda v_s)]$$

$$\Rightarrow a_0 = k(1 + \lambda V_{DS})(\hat{V}_S - V_b - \hat{V}_{th}^2)$$

$$a_1 = -k(2(1 + \lambda V_{DS})(\hat{V}_S - V_b - \hat{V}_{th}^2) + \lambda(V_B - \hat{V}_S - \hat{V}_{th})$$

$$a_2 = k((1 + \lambda V_{DS})k_1$$

(d) (*6 points*) Explain why we may neglect the distortion generated by M3? How does an $R_s > 0\,\Omega$ affect the distortion.

(1) $\Rightarrow$ large $R_S = r_{o2}$ degeneration

R_S

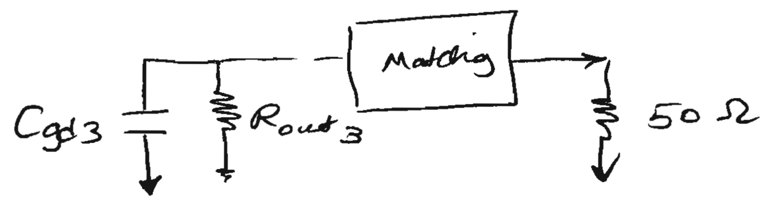
(2) $\Rightarrow$ Acts as a feed back for M_2

lowering the distortion

M_2 dominates distortion.

Even though R_S increases the distortion of M_3, M_3 distortion is small.

(e) (*10 points*) Design an output matching network to drive an external $R_S = R_L = 50\,\Omega$. Use the fewest components possible and draw the overall load and matching network schematic. If you could not answer the previous problems, you may assume that $W_2 = 2W_1$, $I_{ds} = 12\text{mA}$.



$C_{gd3} \approx 50fF$ (from previous part)

$R_{out3} \simeq r_{o3}\, g_{m_3}\, r_{o2} = \dfrac{1}{\lambda^2 I_D^{\,2}} \times g_{m_3}$

$= 2.43\,k\Omega \simeq 2.5\,k\Omega$

$g_{m_3} = \sqrt{\dfrac{W_3}{W_2}} \times g_{m_2}$

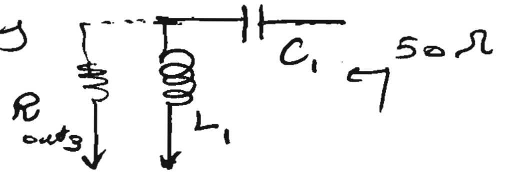
$\simeq \sqrt{0.5} \times \dfrac{1}{50} = 0.014\,℧$

$Y_{source} = 2.5k\Omega + j(2\pi \times 10G)\,50fF =$

First we resonate out the capacitive part with L_{res}

$L_{res} = \dfrac{1}{\omega^2 C_{gd3}} = \dfrac{1}{(2\pi \times 10GHz)^2 \times 50fF} = 5nH$

Now to get the fewest components we

try  this way we can combine L_1 and L_{res}.

$m = \dfrac{2500}{50} \simeq 50 \Rightarrow Q = \sqrt{m-1} = 7$

$Q = \dfrac{R_{out3}}{X_L} \Rightarrow X_L = 357$

$\Rightarrow \boxed{L = 5.68nH}$

$X_{SL} = (1 + Q^{-2}) X_L = 364$

$\Rightarrow \boxed{X_C = -X_{SL} = -364 = 43.7fF}$ $\Rightarrow$