Example Problem Set 4

1a) start by noting that for all the BJT's

$I_C = 1.3\,mA$, $V_{bc} \approx 1V$ $\therefore$ $g_m = 50\,mS$

$C_\pi = 572\,fF$

$C_\mu = 26\,fF$

$r_\pi = 2\,k\Omega$

next, we find the equivalent parameters for the degenerated diff-pair:

note that $g_m R_E = 50mS \cdot 60\Omega = \underline{3}$

$$g_m' = \frac{g_m}{1+g_m R_E} = 12.5\,mS$$

$$C_\pi' = \frac{C_\pi}{1+g_m R_E} = 143\,fF$$

$C_\mu' = 26\,fF$

$r_\pi' = r_\pi \cdot (1+g_m R_E) = 8\,k\Omega$

now, start with the 2nd stage: $A_{V_2} = g_m' \cdot (R_L \;\; C_\mu)$

since $R_L << r_o$, <u>neglect r_o</u>

$$= \frac{g_m' R_L}{1 + R_L C_\mu j\omega}$$

$$= \frac{2}{1 + j\omega \cdot 4 \times 10^{-12}}$$

(pole @ 38 GHz)

this stage presents a load to the previous stage of $\frac{1}{j\omega C_\pi'} \,||\, \frac{1}{j\omega(C_\mu(1+A_{V_2}))} \,||\, r_\pi' = \frac{1}{j\omega C_2} \,||\, r_\pi'$

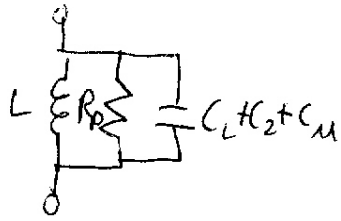
where $C_2 = C_\pi' + C_\mu(1+A_{V_2}) = (143+78)\,fF = \underline{221\,fF}$

the first stage has a gain : $A_{v1} = g_m \cdot Z_L$

where $Z_L = j\omega L \,||\, R_p \,||\, \frac{1}{j\omega C_L} \,||\, r_\pi' \,||\, \frac{1}{j\omega C_2} \,||\, \frac{1}{j\omega C_\mu} \,||\, r_o$

$R_p \ll r_\pi' \ll r_o$, so <u>neglect r_π', r_o</u>

$$Z_L = \frac{1}{\frac{1}{j\omega L} + \frac{1}{R_p} + j\omega(C_L + C_2 + C_\mu)}$$



$$= \frac{j\omega L}{1 + \frac{j\omega L}{R_p} - \omega^2\left(L(C_L + C_2 + C_\mu)\right)}$$

So, $\omega_n = \frac{1}{\sqrt{L \cdot (C_L + C_2 + C_\mu)}} = 6.28 \times 10^9 \Rightarrow \boxed{f_c = 1\,GHz}$

Now find R_p for this center frequency!

$= Q \cdot L \cdot \omega = 4 \cdot 4nH \cdot 6.28 \times 10^9 \approx 100\Omega$

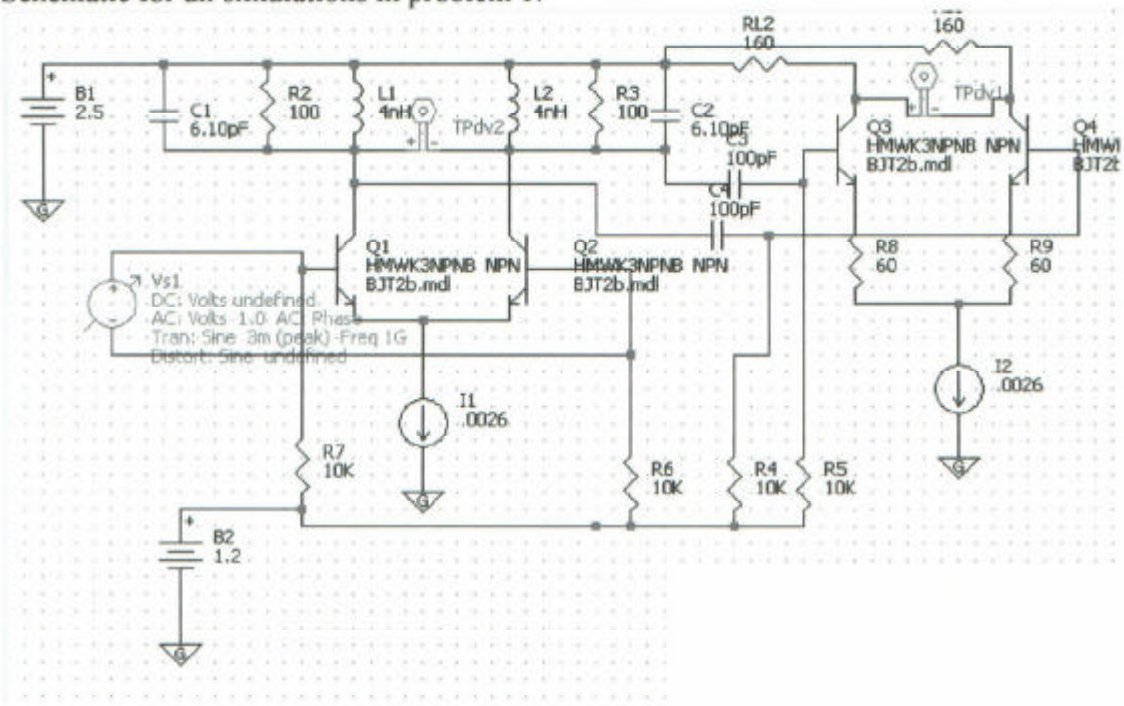
$$Z_L = \frac{j\omega \cdot 4nH}{1 + j\omega \frac{4nH}{100\Omega} - \omega^2/(6.28\times10^9)^2} = 100\Omega \cdot \frac{j\omega L \cdot \frac{4nH}{100\Omega}}{1 + j\omega \frac{4nH}{100\Omega} - \frac{\omega^2}{(6.28\times10^9)^2}}$$

$$A_{v1} = g_m \cdot Z_L = 5 \cdot \frac{j\omega\, 40pH/\Omega}{1 + j\omega \cdot 40pH/\Omega - \left(\frac{\omega}{6.28\times10^9}\right)^2}$$

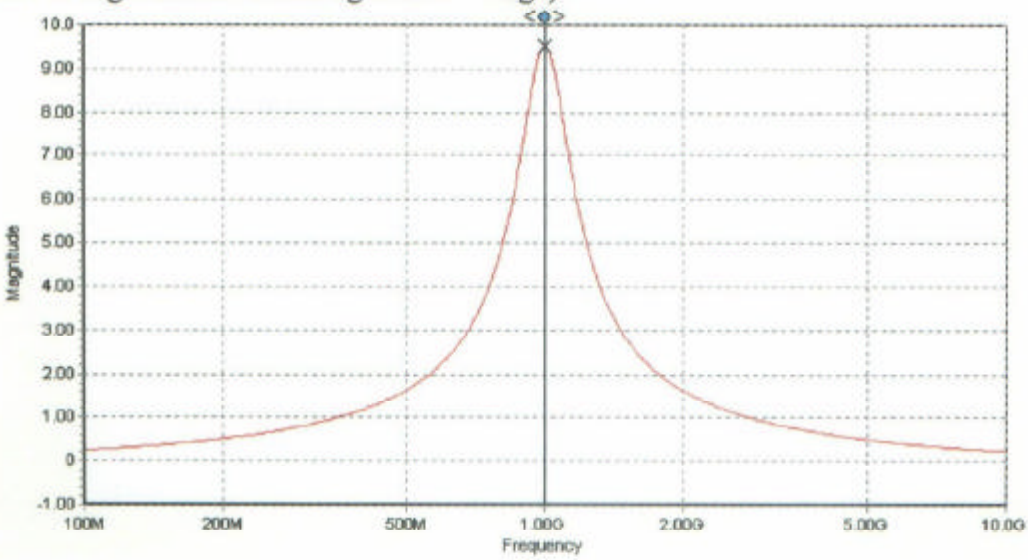
$\boxed{\text{So peak gain} = 5 \cdot 2 = 10 \text{ at } 1\,GHz}$

$$A_v(\omega) = A_{v1} \cdot A_{v2} \approx \boxed{10 \cdot \frac{j\omega\, 40pH/\Omega}{1 + j\omega\, 40pH/\Omega - \left(\frac{\omega}{6.28\times10^9}\right)^2}}$$

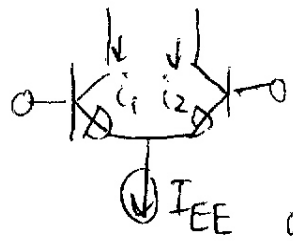
Schematic for all simulations in problem 1:



AC simulation shows gain peaking at 1GHz, with a gain of 9.5 (slightly lower than calc due to slight resistive loading from 2nd stage)



b)



$$(i_1 - i_2) = I_{EE} \tanh\left(\frac{V_{in}}{2V_T}\right)$$

we know: $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

so find taylor coefficients:

$$\frac{d\tanh(x)}{dx} = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$$

$$= 1 - \tanh^2(x)$$

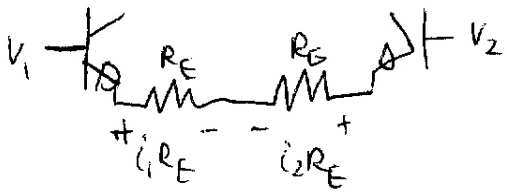
$$\frac{d^2\tanh(x)}{dx^2} = -2\tanh(x)\left(\frac{d}{dx}(\tanh(x))\right)$$

$$= -2\tanh(x)(1 - \tanh^2(x))$$

$$\frac{d^3\tanh(x)}{dx^3} = -2(1 - \tanh^2(x)) + 6\tanh^2(x)(1 - \tanh^2(x))$$

so, $\boxed{a_1 = \dfrac{I_{EE}}{2V_T} = 50\,mS}$, $\boxed{a_2 = 0}$, $\boxed{a_3 = \dfrac{-I}{(2V_T)^3} = \dfrac{-I_{EE}}{24\,V_T^3} = -6160\ mS\,V^{-2}}$

c) use feed-back:



(note we neglect fb effects from Z_{mid}, since $Z_{mid} \leq 100\Omega$, and $\frac{100\Omega}{\beta} << R_E$)

we know: with no feedback

$$(i_1 - i_2) = f(v_1 - v_2) \approx I_{EE}\frac{v_1 - v_2}{2V_T} - I_{EE}\frac{(v_1-v_2)^3}{24V_T^3}$$

because $v_1 - v_2 = V_{be1} - V_{be2}$

but feedback means

$$V_1 - V_2 = V_{be1} + i_1 R_E - i_2 R_E - V_{be2}$$

$$= (V_{be1} - V_{be2}) + \underbrace{R_E(i_1 - i_2)}_{\text{feedback}}$$

so, $f = R_E$

$$T = a_1 f = \frac{I_{EE}}{2V_T} R_E$$

$$\therefore \quad b_1 = \frac{a_1}{1+T} = \frac{\frac{I_{EE}}{2V_T}}{1 + \frac{I_{EE}R_E}{2V_T}} = \frac{I_{EE}}{2V_T + I_{EE}R_E} = \boxed{12.5\,mS}$$

$$b_2 = \frac{a_2}{(1+T)^3} = 0$$

$$b_3 = \frac{a_3}{(1+T)^4} = \frac{\frac{-I_{EE}}{24V_T^3}}{\left(1 + \frac{I_{EE}R_E}{2V_T}\right)^4} = \frac{-I_{EE} \cdot \frac{2}{3}V_T}{(2V_T + I_{EE}R_E)^4} = \boxed{-24\,mS/V^2}$$

d) because we have set all of the frequencies (w_1, w_2 and there IM3 term, $2w_1-w_2$, and $2w_2-w_1$) to be inband, we can use the cascaded aproximation;

note that we must rewrite a_1, a_3 to account for the load of the first stage:

$a_1' = 100 a_1 = 100\Omega \cdot 50\,mS = \underline{5}$

$a_2' = 100 \cdot a_1 = 100\Omega \cdot 0 = 0$

$a_3' = 100\Omega \cdot a_3 = 100\Omega \cdot (-6160\,mS/V^2) = \underline{-616\ V^{-2}}$

similarly we must find b_1', b_2' and b_3' by including the load of the 2nd stage:

$b_1' = 12.5\,mS \cdot 160\Omega = 2$

$b_2' = 0 \cdot 160\Omega = 0$

$b_3' = -24\,mS/V^2 \cdot 160 = -3.84\ V^{-2}$

now we can find C_1, C_2, C_3:

$C_1 = a_1' \cdot b_1' = 10$

$C_2 = 0 \cdot 0 = 0$

$C_3 = a_1'^3 \cdot b_3' + a_3' \cdot b_1' - \cancel{2 b_2' a_1' a_2'}^{\,0} = -1714\ V^{-2}$

so, $\boxed{IIP3 = \sqrt{\frac{4}{3}\left|\frac{C_1}{C_3}\right|} = 0.088\ V}$

e) P1dB : since compression is an in-band effect, we can use the cascaded approach, and, in fact, can use the result from part(d):

$$P_{1dBi} = IIP3 - 9.6dB = \boxed{0.029\ V}$$

HD3 : now we have a problem,

$$HD3 = \frac{mag(3rd\ harmonic)}{mag(fund)}$$

but the 3rd harmonic is outside the pass-band of the 1st stage, so we cannot use the simple cascade approximation.

instead, we know that 3rd harm output is made up of two parts, call them S_{31}, S_{32}, which are generated by the nonlinearity of the 1st and 2nd stages respectively.

$S_{3out} = S_{31} \cdot G_2(3GHz) + S_{32}$, that is, the output 3rd harmonic is the sum of 3rd harm from the 1st stage, times the gain of the 2nd, times the ~~gain~~ third harm from the 2nd stag.

$$S_{31} = \left(\frac{1}{4} d_3 \cdot V_{in}^3\right) \cdot Z_L(2\pi \cdot 3GHz)$$

$$= \frac{1}{4}\left(-6160\,mS\,V^{-2}\right)(3mV)^3 \cdot \frac{j(2\pi \cdot 3\cdot 10^9 Hz)\cdot 4nH}{1 + j\frac{4nH}{100\Omega}(2\pi\cdot 3\cdot 10^9 Hz) - \left(\frac{2\pi\cdot 3\cdot 10^9}{2\pi\cdot 1\cdot 10^9}\right)^2}$$

$$= -41.8\,nA \cdot \frac{j75.4}{1 + j(0.754) - 9}$$

$$= -36.6\,nV + 388j\,nV$$

$$S_{32} = \left(\frac{1}{4} b_3 \cdot V_{mid}^3\right) R_L$$

$$= \frac{1}{4} b_3 \cdot (5 \cdot V_{in})^3 R_L$$

(note that we are looking at the part of Vmid at the fundamental frequency)

$$= \frac{1}{4}\left(-24\,mS\,V^{-2}\right)(15mV)^3\, 160\Omega$$

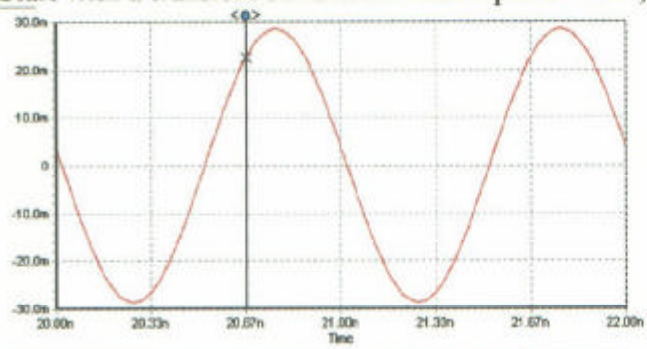
$$= -3.24\,\mu V$$

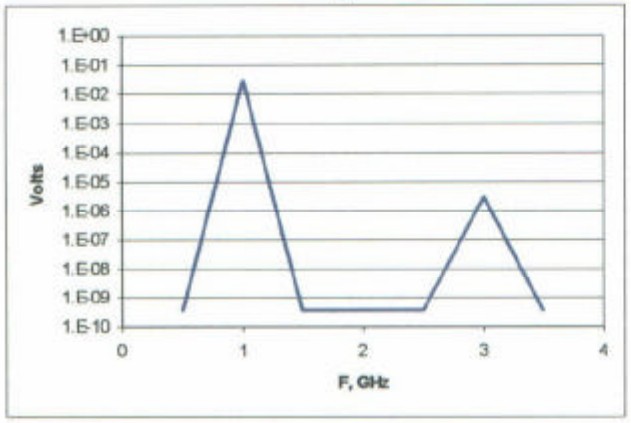
$$S_{3tot} = S_{32} + G_2 S_{31} = 3.24\,\mu V + 2(-36.6\,nV + j388\,nV)$$

$$= \underline{-3.3\,\mu V + 0.78j\,\mu V}$$

$$HD3 = \frac{|S_{3tot}|}{S_1} = \frac{3.4\,\mu V}{10.3\,mV} = \frac{3.4\times 10^{-6} V}{30\times 10^{-3} V} \boxed{= 1.1\times 10^{-4}}$$

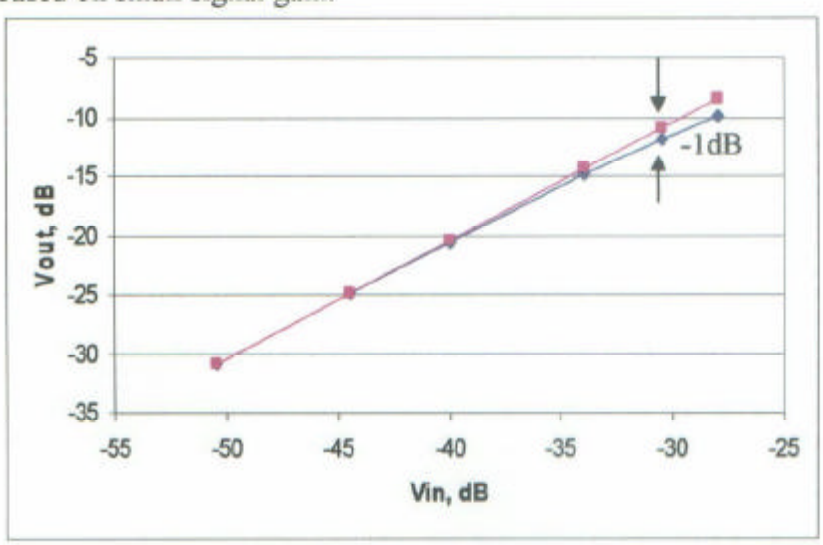
Start with a transient simulation with input of 3mV, and do a Fourier transform:





Now we see that fund = 28.5 mV, 3rd = 3μm, so **HD3 = 10^{-4}**, as predicted

Repeat the simulation for a variety of input voltages, and plot Fund vs the predicted fund based on small signal gain:



It can be seen that the P1dB is at Vin=-30dB, or **30mV** as predicted.