Example Problem Set 4

1a) start by noting that for all the BJTs

\[ I_c = 1.3 \text{ mA}, \quad V_{bc} \approx 1 \text{ V} \]

\[ g_m = 50 \text{ mS} \]
\[ C_T = 572 \text{ fF} \]
\[ C_m = 26 \text{ fF} \]
\[ R_i = 2 \text{ k}\Omega \]

Next, we find the equivalent parameters for the degenerated diff-pair:

note that \[ g_m R_E = 50 \times 0.6 \Omega = 3 \]

\[ g_m' = \frac{g_m}{1 + g_m R_E} = 12.5 \text{ mS} \]
\[ C_T' = \frac{C_T}{1 + g_m R_E} = 143 \text{ fF} \]

\[ C_m' = 26 \text{ fF} \]
\[ R_i' = R_i \cdot (1 + g_m R_E) = 8 \text{ k}\Omega \]

Now, start with the 2nd stage: \[ A_{V_2} = g_m' e(R_L, C_m) \]

\[ \frac{g_m' R_L}{1 + R_c C_m' j\omega} \]
\[ = \frac{2}{1 + j\omega \cdot 4 \times 10^{-12}} \]

(pole @ 38.6 Hz)

This stage presents a load to the previous stage of \[ \frac{j\omega C_m'}{1 + j\omega (C_m' A_{V_2})} \]

\[ \frac{j\omega C_2}{1 \cdot R_i'} \]

where \[ C_2 = C_m' + C_m (1 + A_{V_2}) = (143 + 78) \text{ fF} = 221 \text{ fF} \]
the first stage has a gain: \( A_v1 = g_m \cdot Z_L \)

where \( Z_L = \frac{1}{j\omega L} + \frac{1}{R_p} + j\omega \left( C_2 + C_3 + C_{du} \right) \)

\( R_p \ll R_{in} \ll R_o \), so neglect \( R_{in}, R_o \)

\[
Z_L = \frac{1}{j\omega L + \frac{1}{R_p} + j\omega \left( C_2 + C_3 + C_{du} \right)}
\]

\[
= j\omega L

\]

\[
1 + j\omega L \frac{R_p}{R_o} - \omega^2 \left( L \left( C_2 + C_3 + C_{du} \right) \right)
\]

So, \( \omega_n = \frac{1}{\sqrt{L \left( C_2 + C_3 + C_{du} \right)}} = 6.28 \times 10^9 \Rightarrow f_c = 16 \text{ Hz} \)

Now find \( R_p \) for this center frequency:

\( Q \cdot L \cdot \omega = 4 \cdot 4\pi H \cdot 6.28 \times 10^9 \approx 100 \Omega \)

\[
Z_L = \frac{j\omega \cdot 4\pi H}{1 + \frac{j\omega \cdot 4\pi H}{100 \Omega} - \omega^2 \left( \frac{6.28 \times 10^9}{2} \right)^2}
\]

\[
A_v1 = g_m \cdot Z_L = 5 \cdot \frac{j\omega \cdot 40 \text{ pF}/\Omega}{1 + \frac{j\omega \cdot 40 \text{ pF}/\Omega - \left( \frac{\omega}{6.28 \times 10^9} \right)^2}{100 \Omega}
\]

So peak gain = \( 5 \cdot 2 = 10 \text{ at } 1 \text{ GHz} \)

\[
A_{v2} = A_v1 \cdot A_{v2} = 10 \cdot \frac{j\omega \cdot 40 \text{ pF}/\Omega}{1 + \frac{j\omega \cdot 40 \text{ pF}/\Omega - \left( \frac{\omega}{6.28 \times 10^9} \right)^2}{100 \Omega}
\]
Schematic for all simulations in problem 1:

AC simulation shows gain peaking at 1GHz, with a gain of 9.5 (slightly lower than calculated due to slight resistive loading from 2nd stage)
\[ (i_1 - i_2) = I_{EE} \tanh \left( \frac{V_{in}}{2V_t} \right) \]

so find Taylor coefficients:

\[
\frac{d}{dx} \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}
\]

\[
\frac{d^2}{dx^2} \tanh(x) = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}
\]

\[
= 1 - \tanh^2(x)
\]

\[
\frac{d^3}{dx^3} \tanh(x) = -2 \tanh(x) \left( \frac{d}{dx} (\tanh(x)) \right)
\]

\[
= -2 \tanh(x) (1 - \tanh^2(x))
\]

\[
= -2 \tanh(x) (1 - \tanh^2(x)) + 6 \tanh^3(x) (1 - \tanh^2(x))
\]

so

\[
a_1 = \frac{I_{EE}}{2V_t} \]

\[
a_2 = 0
\]

\[
a_3 = \frac{-P}{(2V_t)^3} = \frac{-I_{EE}}{24V_t^3}
\]

\[
= -6160 \text{ ms} \text{ s}^{-2}
\]
c) use feed-back:

\[ V_1 \approx \frac{R_E}{1 + \frac{1}{g_m}} V_2 \]

\[ + \left( V_{E-} - V_{E+} \right) \]

(Note we neglect fb effects from \( z_{mid} \), since \( z_{mid} \leq 100 \Omega \), and \( \frac{100 \Omega}{g_m} \ll R_E \))

We know, with no feed back

\[ \left( V_1 - V_2 \right) \approx F \left( V_1 - V_2 \right) \approx \frac{1}{T} \frac{V_1 - V_2}{V_T} \frac{1}{2} \left( \frac{V_1 - V_2}{V_T} \right)^2 \]

because \( V_1 - V_2 = V_{be1} - V_{be2} \)

but feed back means

\[ V_1 - V_2 = V_{be1} + \left( R_E \right) \left( i_z R_E - i_z R_E - V_{be2} \right) \]

\[ = \left( V_{be1} - V_{be2} \right) + R_E \left( i_z - i_z \right) \]

So:

\[ f = R_E \]

\[ T = a_1 f = \frac{I_{EE} R_E}{2V_T} \]

\[ b_1 = \frac{a_1}{1 + T} = \frac{I_{EE}}{2V_T} \]

\[ = \frac{I_{EE}}{2V_T + I_{EE} R_E} \]

\[ = \boxed{12.5 \text{ mS}} \]

\[ b_2 = \frac{a_2}{(1 + T)^3} = 0 \]

\[ b_3 = \frac{a_3}{(1 + T)^4} = \frac{-I_{EE}}{(2V_T + I_{EE} R_E)} \]

\[ = \boxed{-24 \text{ mS} / V_T^2} \]
d) because we have set all of the frequencies
(\(w_1, w_2\) and the IM3 term, 2\(w_1 - w_2\), and \(2w_2 - w_1\)) to be in-band,
we can use the cascaded approximation;

note that we must rewrite \(a_1, a_3\) to account for
the load of the first stage:

\[
a_1' = 1000 \cdot a_1 = 1000 \cdot 0.50 \text{ ms} = 500 \\
a_2' = 100 \cdot a_1 = 100 \cdot 0 = 0 \\
a_3' = 100 \cdot a_3 = 100 \cdot (-616 \text{ ms/°}) = -616 \text{ v}^2
\]

Similarly, we must find \(b_1', b_2', b_3'\) by including the load
of the 2nd stage:

\[
b_1' = 12.5 \text{ ms} \cdot 160 \text{ n} = 2 \\
b_2' = 0 \cdot 160 \text{ n} = 0 \\
b_3' = -24 \text{ ms/°} \cdot 160 = -3.84 \text{ v}^2
\]

now we can find \(c_1, c_2, c_3\):

\[
c_1 = a_1' \cdot b_1' = 10 \\
c_2 = 0 \cdot 0 = 0 \\
c_3 = a_1'^3 \cdot b_1' + a_3' \cdot b_1' + 2b_2' a_2' = -1714 \text{ v}^2
\]

so, \(\text{IIP3} = \sqrt{\frac{4|c_1|}{3|c_3|}} = 0.088 \text{ V}\)
e) $P_{1dB}$: since compression is an in-band effect, we can use the cascaded approach, and, in fact, can use the result from part (d):

$$P_{1dB} = IP_3 - 9.6\text{ dB} = [0.02\text{ dB}]$$

HD3: now we have a problem,

$$HD3 = \frac{\text{mag(3rd harmonic)}}{\text{mag(fund)}}$$

but the 3rd harmonic is outside the pass-band of the 1st stage, so we cannot use the simple cascade approximation.

Instead, we know that 3rd harmonic output is made up of two parts, call them $s_{31}$, $s_{32}$, which are generated by the nonlinearity of the 1st and 2nd stages respectively.

$s_{out} = s_{31} \cdot G(3\text{Hz}) + s_{32}$, that is, the output 3rd harmonic is the sum of 3rd harm from the 1st stage, times the gain of the 2nd, times the 3rd harm from the 2nd stage.
\[ S_{31} = \left( \frac{1}{4} a_3 \cdot V_{in}^3 \right) \cdot Z_L \left( 2\pi \cdot 3\text{GHz} \right) \]

\[ = \frac{1}{4} \left( -6 \cdot 10^{-6} \text{m} \text{S} \cdot \text{V}^{-2} \right) \left( 3 \text{mV} \right)^3 \cdot \frac{\left( 2\pi \cdot 3 \cdot 10^9 \text{Hz} \right) \cdot 4 \pi \cdot h}{1 + \frac{4 \pi \cdot h}{100 \Omega} \left( 2\pi \cdot 3 \cdot 10^9 \text{Hz} \right) - \left( \frac{2\pi \cdot 3 \cdot 10^9 \text{Hz}}{2\Omega \cdot 1.1 \cdot 10^4} \right)^2} \]

\[ = -41.8 \text{ nA} \cdot \frac{0.754}{1 + \left( 0.754 \right) - 9} \]

\[ = -36.6 \text{ nV} + 388j \text{ nV} \]

\[ S_{32} = \left( \frac{1}{4} b_3 \cdot V_{mid}^3 \right) R_C \]

\[ = \frac{1}{4} b_3 \cdot (5 \cdot 15 \text{mV})^3 \cdot 160 \Omega \]

\[ = -3.24 \text{ mV} \]

\[ S_{3+0^+} = S_{32} + G_2 S_{31} = 3.24 \text{ mV} + 2(-36.6 \text{ nV} + 388j \text{ nV}) \]

\[ = -3.3 \text{ mV} + 0.78j \text{ mV} \]

\[ HD = \frac{1 S_{3+0^+}}{S_1} = \frac{3.4 \mu \text{V}}{10.3 \text{mV}} = \frac{3.4 \times 10^{-6} \text{V}}{30 \times 10^{-3} \text{V}} \approx 1.1 \times 10^{-4} \]
Start with a transient simulation with input of 3mV, and do a Fourier transform:

Now we see that \( f_{\text{fund}} = 28.5 \text{ mV} \), \( 3^\text{rd} = 3 \mu\text{m} \), so \( \text{HD3} = 10^{-4} \), as predicted.

Repeat the simulation for a variety of input voltages, and plot \( F_{\text{fund}} \) vs the predicted \( F_{\text{fund}} \) based on small signal gain:

It can be seen that the P1dB is at \( V_{\text{in}} = -30 \text{ dB} \), or 30mV as predicted.