Problem 1: Beam Equilibrium
A parallel plate electrostatic actuator (of area A) has a linear spring restoring force (with spring constant k). The spring is at rest at a separation \( z_0 \) when the voltage is zero. Assume that as the voltage increases the plates remain parallel.

a) Show that the last stable equilibrium point occurs at a plate separation of \( \frac{2}{3} z_0 \).
b) What is the corresponding pull-in voltage?
c) If the linear spring is made with a 2 \( \mu \)m \( \times \) 2 \( \mu \)m \( \times \) 100 \( \mu \)m polysilicon beam, and the gap is 2 \( \mu \)m \( \times \) 100 \( \mu \)m in area with an initial separation of 2 \( \mu \)m, graph the plate separation as a function of applied voltage, \( z(V) \). (Using Matlab is recommended).

Problem 2: Beam Deflection
Using what you know about simple beams, calculate a single matrix expression relating the forces and moment applied at the end of the beam to the deflection and rotation of the tip of the beam. (Here \( x \) and \( y \) are the displacement of the tip of the beam from the equilibrium position)

\[
\begin{bmatrix}
F_x \\
F_y \\
M_\theta
\end{bmatrix} =
\begin{bmatrix}
c_{xx} & 0 & 0 \\
0 & c_{yy} & c_{y\theta} \\
0 & c_{\theta y} & c_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\theta
\end{bmatrix}
\]

Problem 3: Beam Deflection using SUGAR
The structure below consists of a beam of width \( a \), thickness \( b \), and length \( L_b \). The electrostatic actuator has an initial gap \( z_0 \), a length \( L_g \), and a thickness \( b \). (Assume that this is a high-aspect-ratio process so that you may ignore fringing fields, and the actuator is wide enough to be perfectly rigid, large \( h \))

a) Ignoring the \( x \) forces and deflections, write down the simplified matrix describing the support spring.

\[
\begin{bmatrix}
y \\
\theta
\end{bmatrix} =
\begin{bmatrix}
c_{yy} & c_{y\theta} \\
c_{\theta y} & c_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
F_y \\
M_\theta
\end{bmatrix}
\]
b) Calculate the electrostatic force and torque on the tip of the spring as a function of the applied potential and position of the tip of the spring.
\[
\begin{bmatrix}
F \\
M
\end{bmatrix} = \begin{bmatrix}
F(y, \theta, V) \\
M(y, \theta, V)
\end{bmatrix}
\]

c) Assuming \( L_b = 100 \, \mu m \), \( a = 2 \, \mu m \), \( z_0 = 2 \, \mu m \), \( L_g = 100 \, \mu m \), \( b = 2 \, \mu m \), and \( h = 10 \, \mu m \), solve for the pull-in voltage and pull-in position. You may need to use numerical methods.

d) Compare to the pull-in voltage expected using the model in problem 1.
Comments?

Problem 4: Computerize the Gap-Closing Actuator

a) Design the actuator as depicted in problem 3 in the CAD tool of your choice. Does this design comply with the MUMPS design rules? In your design add the appropriate probe pads so the device can be electrically activated, and add a poly0 ground plane so the structure doesn’t just stick to the substrate when biased. Furthermore design some sort of mechanical stop (design rule correct) to prevent the plates from shorting (i.e. prevent actual pull-in)

b) Use SUGAR on the basic structure from problem 3 to calculate the pull-in voltage and to find the \( y \) and \( \theta \) deflection of the tip of the actuator plate at the last stable separation point just before pull-in. Compare this these values with problem 1b and 3c.

c) Use SUGAR to calculate the frequency of the first resonance of the system as a function of applied voltage.