2nd Order Transfer Functions

- Imaginary axis zeroes
- Tow-Thomas Biquad
- Example

Imaginary Axis Zeros

- Sharpen transition band
- “notch out” interference
- High-pass filter (HPF)
- Band-reject filter

\[ H(s) = K \left\{ \frac{1 + \left( \frac{s}{\omega_z} \right)^2}{1 + \frac{s}{\omega_z Q_z} + \left( \frac{s}{\omega_z} \right)^2} \right\} \]

\[ |H(j\omega)|_{\text{peak}} = K \left( \frac{\omega_z}{\omega_x} \right)^2 \]

Note: Always represent transfer functions as a product of a gain term, poles, and zeros (pairs if complex). Then all coefficients have a physical meaning, reasonable magnitude, and easily checkable unit.
Imaginary Axis Zeros

- **No finite zeros**

- **With finite zeros**

**Magnitude Response (s-plane)**

- Frequency [Hz]
- Magnitude [linear]

**Pole-Zero Map**

- Real Axis
- Imag Axis

**Imaginary Zeros**

\[ f_p = 100kHz \]
\[ Q_p = 2 \]
\[ f_z = 3f_p \]

- Zeros substantially sharpen transition band
- At the expense of reduced stop-band attenuation at high frequency
Moving the Zeros

\[ f_p = 100kHz \]
\[ Q_r = 2 \]
\[ f_z = f_p \]

Tow-Thomas Biquad

- Parasitic insensitive
- Multiple outputs

Frequency Response

\[
\frac{V_{o1}}{V_{in}} = -k_2 \left( b_2 a_1 - b_1 \right) s + \left( b_2 a_0 - b_0 \right) \frac{s^2 + a_1 s + a_0}{s^2 + a_1 s + a_0}
\]

\[
\frac{V_{o2}}{V_{in}} = b_2 s^2 + b_1 s + b_0 \frac{s^2 + a_1 s + a_0}{s^2 + a_1 s + a_0}
\]

\[
\frac{V_{o3}}{V_{in}} = -\frac{1}{k_1 \sqrt{a_0}} \left( b_2 a_0 s + \left( a_1 b_0 - a_0 b_1 \right) \right) \frac{s^2 + a_1 s + a_0}{s^2 + a_1 s + a_0}
\]

- \( V_{o2} \) implements a general biquad section with arbitrary poles and zeros
- \( V_{o1} \) and \( V_{o3} \) realize the same poles but are limited to at most one finite zero

Component Values

\[
b_1 = \frac{R_s}{R_sR_cR_cC_2}
\]

\[
b_1 = \frac{1}{R_cC_1} \left( \frac{R_s}{R_s} - \frac{R_sR_c}{R_cR_c} \right)
\]

\[
b_2 = \frac{R_s}{R_s}
\]

\[
a_0 = \frac{R_sR_c}{R_sR_cC_2}
\]

\[
a_1 = \frac{1}{R_cC_1}
\]

\[
k_1 = \frac{R_sR_c}{R_sR_cC_1}
\]

\[
k_2 = \frac{R_s}{R_s}
\]

\[
given a_i, b_i, C_i, \text{ and } R_i \]

\[
R_i = \frac{1}{a_iC_i}
\]

\[
R_2 = \frac{k_1}{\sqrt{a_0C_2}}
\]

\[
R_3 = \frac{1}{k_1k_2 \sqrt{a_0C_1}} - \frac{1}{k_1 k_1 k_2 \sqrt{a_0C_1}}
\]

\[
R_4 = \frac{1}{k_1 k_1 k_2 \sqrt{a_0C_1}} - \frac{1}{k_1 k_1 k_2 \sqrt{a_0C_1}}
\]

\[
R_5 = k_1 k_2 \sqrt{a_0C_1}
\]

\[
R_6 = \frac{R_s}{b_2}
\]

\[
R_i = k_i R_i
\]

it follows that

\[
\omega_p = \frac{R_s}{\sqrt{R_sR_cR_cC_2}}
\]

\[
Q_p = \omega_p R_c
\]
Filter Design Example

- Application: testing of ultra-linear ADC
- Problem: sinusoidal source has higher distortion than the ADC!
- Solution
  - Filter source with bandpass before converting
  - Check resulting source with spectral analyzer
    Twist: the analyzer is not sufficiently linear either
  - notch out sinusoid and look just at harmonics
- Implementation
  - Bandpass & Notch at 1kHz
  - Use $V_{o2}$ for bandpass (only possibility), $V_{o1}$ for notch

Principle: IC test circuits are useless if you can’t verify their performance!
Filter Coefficients

\[ V_{o1} = -k_1 \left( b_1 a_0 - h_0 \right) s + \left( b_2 a_0 - h_0 \right) \]
\[ V_o = \frac{b_1 s^2 + b_2 s + h_0}{s^2 + a_1 s + a_0} \]

\[ V_{o2} = \frac{1}{k_1 \sqrt{a_0}} \left( \frac{h_0 - b_2 a_0}{s} + \left( a_2 b_0 - a_0 h_0 \right) \right) \]

Design Notch:
\[ h_0 = a_0 = \omega_0^2 = \left( 2\pi \times 1kHz \right)^2 \]
\[ h_0 = 0 \]
\[ h_2 = 1 \]

Get Bandpass for "free":
\[ b_2 a_0 - h_0 = a_0 \]
\[ b_2 a_0 - h_0 = 0 \]
(just as we want in a bandpass)

Choose reasonable signal levels:
\[ k_1 = 1.05 \]
(to keep unused \( V_{o3} \) slightly below other outputs)
\[ k_2 = 1 \]

---

Final Filter

Choose:
C1=C2=112nF (large to minimize noise)
R8=1kΩ
\( f_p=1kHz \), \( Q_p=30 \) (check sensitivity!)

Solve equations ...
R1=42.631kΩ
R2=1.4921kΩ
R3=1.3534kΩ
R4=42.631kΩ
R5=1.4921kΩ
R6=R7=R8

Let's order the parts ...
Capacitors

• C0G capacitors
  – Negligible voltage coefficient (for linearity)
  – Excellent tempco (30ppm/°C)
  – 2% initial accuracy is easy to get
• No high-value capacitors are trimmable
• Resistors will be trimmed to compensate for capacitor variations

Resistors

• Trimmed resistors combine fixed metal film resistors and precision trim potentiometers in series
  – 1%-accurate, 5ppm/°C, lab grade metal film resistors provide ~90% of the nominal resistance
    Ref: Caddock Electronics, Type TN Lab Grade Low TC Precision Film Resistor datasheet, 1999.
  – 50ppm/°C trim pots provide between 0% and ~20% of the nominal resistance
    Ref: Vishay Foil Resistors, Model 1268 Precision Trimming Potentiometers datasheet
  – Use two fixed resistors in series with the trimpot to minimize trimpot value and optimize overall tempco

• R6-R8 are 0.1%-accurate, 5ppm/°C metal film
Opamps

• For opamps, we’ll use the Burr-Brown OPA627
  
  – The finest audio opamp in the world, and, at $15/each, priced accordingly!
  – But money is no object when designing IC test fixtures (only a few are ever built)
  – Adequate speed for this application

Bandpass/Bandstop Responses
Filter Design Example (cont.)

• Note that the bandpass output $H_1$ provides >30dB attenuation to all harmonics present in the 1kHz generator output
• Opamp outputs have 0.0±0.5dB peak gain
  – This maximizes each opamp’s output swing for best dynamic range
• Let’s magnify the frequency axis for the two responses of interest…

Bandpass/Bandstop Responses
Filter Design Example

- Temperature changes won’t change these responses too much
  - Lab temperatures are stable to 25±3°C
  - Our lab-grade RC products move <100ppm/°C

- Initial component values are another story
  - What if C1=114nF and C2=113nF?
  - That’s within their ±2% accuracy specifications
  - What’s $S_{C_1}^{\omega_p}$?

Bandpass/Bandstop Responses

- Frequency (kHz): 0.9, 1.1
- Gain (dB): -60, -40, -20, 0, 20
- $H_1$, $H_2$
- $C_1=.114\mu F$, $C_2=.113\mu F$
- R’s nominal
Filter Design Example

• Obviously, we’ve got to tune the filter back to its original specification
• How is that tuning done?
  – Do you tell your technician to twiddle pots randomly until it works?
  – Or do you document a robust tuning procedure?

RC Filter Tuning Strategy

• Famous biquads like the Tow-Thomas come complete with their own tuning strategies
  – The circuit topologies allow 1 trim operation to adjust 1 design parameter (such as $f_p$, $f_z$, $Q_p$, $Q_z$, gain) without changing the others

• Rationale for a biquad’s tuning strategy becomes apparent when studying design equations such as the Tow-Thomas equations on slide 6
Tow-Thomas Tuning Strategy

• R3 will be set to a fixed value to keep the unused OPAMP3 output below 0dB

• Tuning involves the following steps performed in the specified sequence:
  – Adjust R2 to center the bandpass at 1kHz
  – Adjust R5 to center the notch at 1kHz
  – Adjust R1 to set the bandpass Q to 30
  – Adjust R4 to deepen the notch

• The design equations also provide the range of adjustment required for a given resistor
  – Remember that an excessively large adjustment range translates into excessively large tempco

• R1 tuning range (from slide 7):

\[ a_1 \equiv \frac{1}{R_1 C_1} \Rightarrow \frac{1}{a_1 C_{1\text{MAX}}} < R_1 < \frac{1}{a_1 C_{1\text{MIN}}} \]

known set by capacitor tolerances
Tow-Thomas Tuning Strategy

• An even simpler way to determine resistor ranges is to:
  – Set all capacitors to their high tolerance limit (nominal+2% in this case)
  – Calculate R’s for these capacitances (these will be the minimum resistance values)
  – Set capacitors to their low tolerance limit
  – Calculate maximum R’s

Tow-Thomas Biquad

resistors: metal film + trimpot
Tow-Thomas Tuning Strategy

• If you’ve left your filter unattended for a while, assume that its trim potentiometers are completely misadjusted

• Adjust all trimpots to 0Ω and start over
  – Let’s return to our $C_1=114\,\text{nF}$, $C_2=113\,\text{nF}$ example

Bandpass/Bandstop Responses

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>Gain (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>$H_1$</td>
</tr>
<tr>
<td>1.1</td>
<td>$H_2$</td>
</tr>
</tbody>
</table>

C1=114nF
C2=113nF
R’s nominal
Bandpass/Bandstop Responses

Tow-Thomas Tuning Strategy

- For most R's and C's in this biquad: \( f_p \sim \frac{1}{\sqrt{x}} \)
- Hence, \( S_x^{f_p} = -\frac{1}{2} \)
- This means a +2% change in \( R_2 \) will cause a – 1% change in \( f_p \)
- Note that \( f_z \) sensitivities are also –1/2
  - A 4% increase in \( R_5 \) will shift our notch (currently at 1.02kHz) back to the right place
Summary

- General 2\textsuperscript{nd} order transfer function
  - Imaginary axis zeros

- General purpose biquad
  - Large selection in literature
  - Tow-Thomas biquad:
    - 3 opamps
    - Parasitic insensitive
    - Multiple outputs
    - Tuning strategy