Electronic Noise

- Dynamic range in the analog domain
  - Resistor noise
  - Amplifier noise
  - Maximum signal levels

- Tow-Thomas Biquad noise example

- Implications on power dissipation

Analog Dynamic Range

- Finite precision effects in digital filters are rapidly becoming negligible
  - Floating point digital filters with huge mantissas will be reduced to negligible cost
  - The only fixed-point numbers will come from ADCs

- But we will **always** have thermal noise
Analog Dynamic Range

• Let’s say you’ve selected the poles and zeroes of your analog filter transfer function

• Of the infinitely many ways to build a filter with a given transfer function, each of those ways has a different output noise!

Analog Dynamic Range

• The job of a high-performance analog filter designer is to get reasonably close to the optimal noise for a given transfer function
  – Not the absolute minimum noise, just close

• The job of a mixed-signal chip architect is to appreciate filter noise and to be able to model filters well enough to know that a given dynamic range objective is feasible
Analog Dynamic Range

- We'll begin our adventure in analog filter implementation by looking at the noise in resistors and simple RC filters…

Resistor Noise

- Capacitors are noiseless
- Resistors have thermal noise
  - This noise is uniformly distributed from dc to infinity
  - Frequency-independent noise is called “white noise”
Resistor Noise

- Resistor noise has
  - A mean value of zero
  - A mean-squared value

\[
\overline{v_n^2} = 4k_B T R \Delta f
\]

where:
- \( R \) is in ohms
- \( C \) is in farads
- \( v_{IN} \) and \( v_{OUT} \) are in Volts
- \( \Delta f \) is the measurement bandwidth (Hz)
- absolute temperature (°K)
- Boltzmann's constant = 1.38e-23 J/°K

- Resistor rms noise voltage in a 10Hz band centered at 1kHz is the same as resistor rms noise in a 10Hz band centered at 1GHz

- Resistor noise spectral density, \( N_0 \), is the rms noise per \( \sqrt{\text{Hz}} \) of bandwidth:

\[
N_0 = \frac{\sqrt{\overline{v_n^2}}}{\sqrt{\Delta f}} = \sqrt{4k_B T R}
\]
Resistor Noise

- Don’t bother to remember Boltzmann’s constant

- Instead, remember forever that $N_0$ for a 1kΩ resistor at room temperature is 4nV/√Hz

- Scaling $R$,
  - A 10MΩ resistor gives 400nV/√Hz
  - A 50Ω resistor gives 0.9nV/√Hz

- Or, remember that $k_B T_r = 4 \times 10^{-21}$ J ($T_r = 17$ °C)

Resistor Noise

- Resistor noise gives our filter a non-zero output when $v_{IN}=0$
- In this simple example, both the input signal and the resistor noise obviously have the same transfer functions to the output
- Since noise has random phase, we can use any polarity convention for a noise source (but we have to use it consistently)
Resistor Noise

- What is the thermal noise of the RC filter?

- Let’s ask SPICE!

Netlist:

```
Noise from RC LPF
vin vin 0 ac 1V
r1 vin vout 8kOhm
c1 vout 0 1nF
.ac dec 100 10Hz 1GHz
.noise V(vout) vin
.end
```

LPF1 Output Noise Density

\[ N_0 = \sqrt{4k_BT} \cdot R \]

\[ = \sqrt{8} \times 4 \frac{nV}{\sqrt{Hz}} \]

\[ = 11.3 \frac{nV}{\sqrt{Hz}} \]
Total Noise

• Suppose we want to know the value of $v_o$ “now”, what’s the standard deviation error? (E.g. on the display of a volt-meter connected to $v_o$).

• Answer:

$$v_o^2 = 4k_BT R \int_0^\infty H(2\pi f)^2 df$$

Total Noise

• Note that noise is integrated in the mean-squared domain, because noise in a bandwidth $df$ around frequency $f_1$ is uncorrelated with noise in a bandwidth $df$ around frequency $f_2$
  – Powers of uncorrelated random variables add
  – Squared transfer functions appear in the mean-squared integral
Total Noise

\[ \overline{v_o^2} = \int_{0}^{\infty} 4k_B T R |H(2\pi f)|^2 df = \int_{0}^{\infty} 4k_B T R \left| \frac{1}{1 + 2\pi i f RC} \right|^2 df = \frac{k_B T}{C} \]

- This interesting and somewhat counterintuitive result means that even though resistors provide the noise sources, capacitors set the total noise.

- For a given capacitance, as resistance goes up, the increase in noise density is balanced by a decrease in noise bandwidth.
kT/C Noise

- The rms noise voltage of the simplest possible (first order) filter is $\sqrt{k_BT/C}$

- For 1pF, $\sqrt{k_BT/C} = 64\,\mu V\text{-rms}$ (at 298°K)

- 1000pF gives 2 $\mu V\text{-rms}$

- The noise of a more complex filter is $\sqrt{K \times k_BT/C}$
  K depends on implementation and features such as filter order
LPF1 Output Noise

• Note that the integrated noise essentially stops growing above 100kHz for this 20kHz lowpass filter

• Beware of faulty intuition which might tempt you to believe that an $80\Omega$, 1000pF filter has lower integrated noise than our $8000\Omega$, 1000pF filter…
LPF1 Output Noise

- Of course, an 80Ω, 100,000pF filter has both the same bandwidth AND lower integrated noise than our 8000Ω, 1000pF filter.

- In the analog filter dynamic range game, the highest capacitance wins.
Analog Circuit Dynamic Range

- The biggest signal we can ever expect at the output of a circuit is limited by the supply voltage, \( V_{DD} \) hence (for sinusoids)

\[
V_{\text{max}}(\text{rms}) = \frac{1}{\sqrt{2}} V_{\text{DD}}
\]

- The noise is

\[
V_n(\text{rms}) = \sqrt{k_B T C}
\]

- So the dynamic range in dB is:

\[
DR = \frac{V_{\text{max}}(\text{rms})}{V_n(\text{rms})} = \frac{V_{\text{DD}} \sqrt{C}}{\sqrt{8k_B T}} \quad [\text{V/V}]
\]

\[
= 20 \log_{10}\left( \frac{V_{\text{DD}} \sqrt{C}}{K} \right) + 75 \quad [\text{dB}] \text{ with } C \text{ in } [\text{pF}]
\]

Analog Circuit Dynamic Range

- For integrated circuits built in modern CMOS processes, \( V_{\text{DD}} < 3V \) and \( C < 1\text{nF} \) (K = 1)
  - DR < 110dB

- For PC board circuits built with “old-fashioned” 30V opamps and discrete capacitors of < 100nF
  - DR < 140dB
  - A 30dB advantage!
Dynamic Range versus Bits

- Bits and dB are related:

\[ DR = 2 + 6N \quad [\text{dB}] \]

- see “quantization noise”, later in the course

- Hence

  110 dB → 18 Bits
  140 dB → 23 Bits

Dynamic Range versus Power

- Each extra bit corresponds to 6dB
- 6dB means cutting noise power by 4!
- This translates into 4x larger capacitors
- To drive these at the same speed, \( G_m \) must increase 4x
- Power is proportional to \( G_m \) (for fixed supply and \( V_{\text{sat}} \))

  In analog circuits that are limited by thermal noise,

  1 extra bit costs 4x power

  E.g. 16Bit ADC at 200mW → 17Bit ADC at 800mW

  Do not overdesign the dynamic range of analog circuits!

P.S. What is the cost of an extra bit in a 64Bit adder?
Active Filter Example

Frequency response:

\[ H(s) = \frac{1}{1 + sRC} \]

Total noise (see EE240):

\[ \sqrt{V_n^2} = \sqrt{2 \frac{k_BT}{C}} \quad \Rightarrow \quad K = 2 \]

- Noise depends on filter topology
- Opamps contribute yet more noise …

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Behavioral Opamp Model

**Specification**

- Gain G: 100k
- Unity-gain bandwidth \( f_u \): 100 MHz
- Input ref’d thermal noise: 5 nV/√Hz

**Example**

- 100k

Beware of flicker noise and input current noise (BJTs).
SPICE Analysis

Opamp noise dominates in this example

Opamp adds significant noise above filter roll-off
Opamp Bandwidth

Minimize opamp bandwidth:

- $f_u = 1\text{MHz} \rightarrow 7\mu\text{V-rms}$
- $f_u = 10\text{MHz} \rightarrow 20\mu\text{V-rms}$

Of course, the opamp has to be fast enough to faithfully realize the 20kHz corner!

Frequency Response
Tow-Thomas Noise Analysis

Tow-Thomas Biquad

K = 1 (multiplier, C, default)
C1 = 120pF
C2 = 150pF
R2 = 1.45kΩ
R4 = 42.16Ω
R5 = 350Ω

Opamp bandwidth = 16kHz
Noise: resistor = 1.36mV
Bandpass Noise

Unfortunately the opamp adds significant additional noise at high frequency.

Noise from the passband dominates this integral.

RC Filter Reduces BP Noise

We cannot reduce the opamp noise or bandwidth ... let's filter its noise!

1kΩ / 5nF RC LPF

0.9μV rms noise from 5nF is negligible
RC provides negligible attenuation. But that’s not the point. Let’s look at the noise …

RC filter reduces total noise from 20µV to 5µV rms. (Without opamp noise is 3µV rms).
BP Dynamic Range

- Maximum sinewave input: 7.8V rms (limited by opamp)
- Noise: 5.2μV rms (with RC)
- Dynamic range: 123dB

→ No IC with integrated capacitors can get close to this dynamic range

Bandstop Noise

Opamp doubles total noise

No notch in the noise response

Much lower than at 1kHz, but much higher bandwidth! Noise above notch dominates.
Noise versus Pole Q

Noise drops by $\sqrt{30/7}$ from 2.8mV to 1.2mV rms.

- rms total noise is approximately proportional to $\sqrt{Q}$
- of course in this circuit the opamp noise swamps this effect
  (this simulation uses "noiseless" opamps)
Noise Summary

• Thermal noise is a fundamental property of (electronic) circuits
• Noise is closely related to
  – Capacitor size and
  – Power dissipation
• In filters, noise is proportional to order, \( Q \), and depends on implementation
• Operational amplifiers can contribute significantly to overall filter noise