Sampling and Aliasing

• Any continuous time signal can be sampled and processed in the sampled-data domain
  – A/D converters “look” at signals at only discrete points in time
  – Computer models of continuous systems must operate at discrete time intervals
  – Even “analog” filters can use continuous or sampled time signal representations

• Multiple continuous time signals can yield exactly the same sampled data signal \(\rightarrow\) aliasing
  – Let’s look at samples taken at 1\(\mu\)sec intervals of several sinusoidal waveforms …

\[
v(t) = \sin [2\pi(101000)t]
\]

\[
T = 1\mu s
f_s = 1/T = 1MHz
f_x = 101kHz
\]
Sampling Sinewaves

\[ v(t) = \sin [2\pi(1101000)t] \]

\[ T = 1\mu s \]
\[ f_s = 1\text{MHz} \]
\[ f_x = 1101\text{kHz} \]

Sampling Sinewaves

\[ v(t) = -\sin [2\pi(899000)t] \]

\[ T = 1\mu s \]
\[ f_s = 1\text{MHz} \]
\[ f_x = 899\text{kHz} \]
Aliasing

- Multiple continuous time signals can produce identical series of sampled voltages.
- The translation of signals from $Nf_S \pm f_{IN}$ down to $f_{IN}$ is called aliasing.
  - Sampling theorem: $f_s > 2f_B$
- If aliasing occurs, no signal processing operation downstream of the sampling process can recover the original continuous time signal.
  - If you don’t like it, sample faster!

Time vs Frequency Domain

- Time: $T$  
- Frequency: $f_B$
- Frequency: $f_c = 1/T$
- Multiply
- Convolve
Nomenclature

Continuous time signal \( x(t) \)
Sampling interval \( T \)
Sampling frequency \( f_s = 1/T \)
Sampled signal \( x(kT) = x(k) \)

Sampled Signal Properties

- It's obvious from the preceding slides that the frequency content of a continuous signal can be grossly distorted by sampling.
- What do we know about the relationships between \( x(t) \) and \( x(k) \)?
  - We'll assume that \( x(t) \) is stationary; that is, its mean is constant and its autocorrelation \( R(t_1,t_2) \) depends only on the time difference \( t_1-t_2 \).
Sampled Signal Properties

- If $x(t)$ is stationary, then
  (Athanasios Papoulis, Signal Analysis, 1977, Section 9.4.):
  - $x(k)$ is also stationary
  - Sampling doesn’t change the mean: $E\{x(k)\}=E\{x(t)\}$
    - $E()$ is the expectation operator
  - Sampling doesn’t change energy:
    $E\{(x(k))^2\}=E\{(x(t))^2\}$
- Continuous time energy will show up someplace in the frequency domain after sampling
  - Aliasing may move continuous signal frequency components to “wrong” frequencies

Anti-Aliasing Filter

Anti-Aliasing Filter

Filtered Spectrum

Aliasing

$f_B$

$f_B'$

$f_s$

$f_s$

$f_s$

$f_s$

$f_s$
Sampled Signals

- Sampled data signals are valid only at sampling instances.
- In an actual circuit, the signal transitions to the new value between sampling instances.
- The value between sampling instances is insignificant and often erroneous (e.g., amplifier slewing distortion).

Zero-Order Hold

- Reconstructs CT signal from SD signal.
- Frequency response?
Zero-Order Hold

Step response

Laplace transform

\[
\begin{align*}
\frac{1}{sT_d} - e^{-sT_d} & = \frac{1 - e^{-sT_d}}{sT_d} = \frac{\sin(2\pi f T_d)}{2\pi f T_d} e^{-j\pi f T_d}
\end{align*}
\]

Spectrum of Reconstructed Signal

Reconstructed signal

SD signal: periodic spectrum

Sinc

T_d = T = 1/f_s
Summary

- “Quantization in Time”: Continuous time (CT) vs Sampled Data (SD)
- Sampling theorem: $f_s > 2f_B$
- IF sampling theorem is met, CT signal can be recovered from SD signal without loss of information
- ZOH reconstructs CT from SD signal