Switched-Capacitor Filters

• “Analog” sampled-data filters:
  – Continuous amplitude
  – Quantized time

• Applications:
  – Oversampled A/D and D/A converters
  – Analog front-ends (CDS, etc)
  – Standalone filters
    E.g. National Semiconductor LMF100
  – Replaced by ADC + DSP in many cases
Switched-Capacitor Resistor

- Capacitor C is the “switched capacitor”
- Non-overlapping clocks $\phi_1$ and $\phi_2$ control switches S1 and S2, respectively
- $v_{\text{IN}}$ is sampled at the falling edge of $\phi_1$
  - Sampling frequency $f_S$
- Why is this a resistor?
Switched-Capacitor Resistors

- The charge transferred from $v_{\text{IN}}$ to $v_{\text{OUT}}$ each sample period is:
  $$Q = C(v_{\text{IN}} - v_{\text{OUT}})$$

- The average current flowing from $v_{\text{IN}}$ to $v_{\text{OUT}}$ is:
  $$i = f_s C(v_{\text{IN}} - v_{\text{OUT}})$$
Switched-Capacitor Resistors

\[ i = f_s C(v_{\text{IN}} - v_{\text{OUT}}) \]

With the current through the switched capacitor resistor proportional to the voltage across it, the equivalent “switched capacitor resistance” is:

\[ R_{\text{EQ}} = \frac{1}{f_s C} \]

Of course this current flows in “bursts”—think of “big electrons”.

\[ T = \frac{1}{f_s} \]
Switched-Capacitor Filter

- Let’s build an “SC” filter …
- We’ll start with a simple RC LPF
- Replace the physical resistor by an equivalent SC resistor
- 3-dB bandwidth:

\[ \omega_0 = \frac{1}{R_{EQ}C_2} = f_S \frac{C_1}{C_2} \]
Switched-Capacitor Filters

- In SCFs, all critical frequencies track the sampling frequency
  - Crystal oscillators for $f_S$ are stable to $\sim 10$ppm/$^\circ$C
  - RC products used in active-RC filters can be tuned, but RCs in active-RC filters don’t track together nearly as well

- Capacitor ratios in monolithic filters are perfectly stable over time and temperature
  - Capacitor ratios can’t be trimmed easily
  - The trick is to achieve initial ratio accuracies of $\sim 1000$ppm out of double-poly CMOS processes
Transient Analysis

1st Order RC / SC LPF

Transmission Analysis to 99us

No problem

Impractical

SC response: extra delay and steps with finite rise time.

exaggerated
Transient Analysis

- ZOH: pick signal after settling (usually at end of clock phase)
- Adds delay and sin(x)/x distortion
- When in doubt, use a ZOH in periodic ac simulations
Periodic AC Analysis

1st Order RC / SC LPF

\[ fs = 1 \text{MHz} \]
\[ fc = 500 \text{kHz} \]
\[ fr = 3.571 \text{kHz} \]
Magnitude Response

1. RC filter output
2. SC output after ZOH
3. Input after ZOH
4. Corrected output
   - (2) over (3)
   - periodic with $f_s$
   - Identical to RC for $f << f_s/2$
Periodic AC Analysis

• SPICE frequency analysis
  – ac linear, time-invariant circuits
  – pac linear, time-variant circuits

• SpectreRF statements
  \[ V1 ( \text{Vi 0}) \text{ vs} \text{source type=} \text{dc} \text{ dc=} 0 \text{ mag=} 1 \text{ pacmag=} 1 \]
  \[ \text{PSS1 pss period=} 1u \text{ errpreset=} \text{conservative} \]
  \[ \text{PAC1 pac start=} 1 \text{ stop=} 1M \text{ lin=} 1001 \]

• Output
  – Divide results by sinc\( (f/f_s) \) to correct for ZOH distortion
A/D  DSP

Spectre Circuit File

rc_pac
simulator lang=spectre
ahdl_include "zoh.def"

S1 ( Vi c1 phi1 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
S2 ( c1 Vo_sc phi2 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
C1 ( c1 0 ) capacitor c=314.159f
C2 ( Vo_sc 0 ) capacitor c=lp
R1 ( Vi Vo_rc ) resistor r=3.1831M
C2rc ( Vo_rc 0 ) capacitor c=lp
CLK1_Vphi1 ( phi1 0 ) vsource type=pulse val0=-1 val1=1 period=1u
  width=450n delay=50n rise=10n fall=10n
CLK1_Vphi2 ( phi2 0 ) vsource type=pulse val0=-1 val1=1 period=1u
  width=450n delay=550n rise=10n fall=10n
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1 pacmag=1
PSS1 pss period=1u errpreset=conservative
PAC1 pac start=1 stop=3.1M log=1001
ZOH1 ( Vo_sc_zoh 0 Vo_sc 0 ) zoh period=1u delay=500n aperture=1n tc=10p
ZOH2 ( Vi_zoh 0 Vi 0 ) zoh period=1u delay=0 aperture=1n tc=10p
module zoh (Pout, Nout, Pin, Nin) (period, delay, aperture, tc)

node [V,I] Pin, Nin, Pout, Nout;
parameter real period=1 from (0:inf);
parameter real delay=0 from [0:inf);
parameter real aperture=1/100 from (0:inf);
parameter real tc=1/500 from (0:inf);
{
  integer n; real start, stop;
  node [V,I] hold;
  analog 
  // determine the point when aperture begins
  n = ($time() - delay + aperture) / period + 0.5;
  start = n*period + delay - aperture;
  $break_point(start);

  // determine the time when aperture ends
  n = ($time() - delay) / period + 0.5;
  stop = n*period + delay;
  $break_point(stop);

  // Implement switch with effective series resistance of 1 Ohm
  if ( ($time() > start) && ($time() <= stop))
    I(hold) <- V(hold) - V(Pin, Nin);
  else
    I(hold) <- 1.0e-12 * (V(hold) - V(Pin, Nin));

  // Implement capacitor with an effective capacitance of tc
  I(hold) <- tc * dot(V(hold));

  // Buffer output
  V(Pout, Nout) <- V(hold);

  // Control time step tightly during aperture and loosely otherwise
  if (($time() >= start) && ($time() <= stop))
    $bound_step(tc);
  else
    $bound_step(period/5);
}

Switched-Capacitor Noise

- The resistance of switch S1 produces a noise voltage on C with variance $kT/C$

- The corresponding noise charge is $Q^2 = C^2V^2 = kTC$

- This charge is sampled when $S_1$ opens
Switched-Capacitor Noise

- The resistance of switch S2 contributes to an uncorrelated noise charge on C at the end of $\phi_2$.

- The mean-squared noise charge transferred from $v_{IN}$ to $v_{OUT}$ each sample period is $Q^2 = 2kTC$.

$$Q^2 = 2kTC$$
Switched-Capacitor Noise

- The mean-squared noise current due to S1 and S2’s kT/C noise is:

\[ \overline{i^2} = (Qf_s)^2 = 2k_BT Cf_s^2 \]

- This noise is approximately white (see next slide) and distributed between 0 and \( f_s/2 \) (noise spectra are single sided by convention). The spectral density of the noise is:

\[ \frac{\overline{i^2}}{\Delta f} = \frac{2k_BT Cf_s^2}{f_s/2} = \frac{4k_BT Cf_s}{R_{EQ}} \]
using \( R_{EQ} = \frac{1}{f_s C} \)

- The noise from an SC resistor equals the noise from a physical resistor with the same value!
SC Resistor Noise Spectrum

\[ S_y(f) = \frac{k_B T_r}{C} \frac{2}{f_s} \frac{1 - e^{-2a}}{1 + e^{-2a}(1 - \cos 2\pi f/T)} \]

\[ a = \frac{T}{R_{sw} C} \quad \text{and} \quad T = \frac{1}{f_s} \]

\[ \int_{0}^{T/2} S_y(f) df = \frac{k_B T_r}{C} \]

- Noise essentially white for \( T/t > 3 \)
- Settling constraints ensure that this condition is usually met in practice
- Note: This is the noise density of an SC resistor only. The noise density from an SC filter is usually not white.
Periodic Noise Analysis

Sampling Noise from SC S/H

Netlist
ahdlInclude "zoh.def"

Netlist
simOptions options retol=10u vabstol=1n labstol=1p

PSS pss period=100n maxacfreq=1.5G errpreset=conservative
PNOISE ( Vrc_hold 0 ) pnoise start=0 stop=20M lin=500 maxsideband=10
Sampled Noise Spectrum

Density of sampled noise with sinc distortion.

Normalized density of sampled noise, corrected for sinc distortion.
Total Noise

Sampled noise in
0 … $f_s/2$: $62.2\mu V \text{ rms}$

(expect $64\mu V$ for $1\text{pF}$)
Opamps versus OTA

- Low impedance output
- Can drive R-loads
- Good for RC filters, OK for SC filters
- Extra buffer adds complexity, power dissipation

- High impedance output
- Cannot drive R-loads
- Ideal for SC filters
- Simpler than Opamp
Opamps versus OTA Noise

Opamp and switch noise add

\[
\sqrt{\frac{v_{oT}^2}{C}} = \frac{kT}{C} \left(1 + \frac{R_{\text{noise}}}{R_{\text{switch}}}\right)
\]

OTA contributes no excess noise (actual designs can increase noise)
Amplifier Bandwidth Requirements

SC Filter:

\[ \tau \leq \frac{T}{10} = \frac{1}{10f_s} \]

for 16+ Bit settling accuracy

\[ \tau = \frac{1}{\omega_u} = \frac{1}{2\pi f_u} \]

\[ \rightarrow f_u \geq \frac{10}{2\pi} f_s \approx 2f_s \]

\[ f_s = 8 \ldots 100 \times f_{\text{corner}} \]

\[ f_u = 16 \ldots 200 \times f_{\text{corner}} \]

→ SC filters have comparable or slower amplifier bandwidth requirements than CT filters

CT Filter:

\[ f_u = 50 \ldots 1000 \times f_{\text{corner}} \]
SC Filter Summary

✓ Pole and zero frequencies proportional to
  – Sampling frequency $f_s$
  – Capacitor ratios
    ➢ High accuracy and stability in response
    ➢ Low time constants realizable without large R, C
✓ Compatible with transconductance amplifiers
  – “No” excess opamp noise
  – Reduced circuit complexity, power
✓ Amplifier bandwidth requirements comparable to CT filters
  o Catch: Sampled data filter $\rightarrow$ aliasing