Tones

- 5th order $\Sigma\Delta$ modulator
  - DC inputs
  - Tones
  - Dither
  - $kT/C$ noise

5th Order Modulator

Stable input range $\sim -0.3 \ldots +0.3$
5th Order Noise Shaping

Input: 0.1V, sinusoid
2^15 point DFT
30 averages

Tones at f_s/2-Nf_m exceed input

In-Band Noise

In-Band quantization noise: ~120dB
5th Order Noise Shaping

- Input: 0.1V, sinusoid
- 215 point DFT
- 30 averages

Output Spectrum [dBWN] / Int. Noise [dBV]

150dB stopband attenuation needed to attenuate unwanted $f_s/2-N_f_{in}$ components down to the in-band quantization noise level

Out-of-Band vs In-Band Signals

- A digital (low-pass) filter with suitable coefficient precision can eliminate out-of-band quantization noise
- No filter can attenuate unwanted in-band components without attenuating the signal
- We’ll spend some time making sure the components at $f_s/2-N_f_{in}$ will not “mix” down to the signal band
- But first, let’s look at the modulator response to small DC inputs (or offset) …
**ΣΔ Tones**

Simulation technique: A random 1st input randomizes the noise and enables averaging. Without the small tones are not visible.

**Limit Cycles**

- Representing a DC term with a -1/+1 pattern ... e.g.

  \[
  \begin{bmatrix}
  \frac{1}{11} \\
  \frac{2}{11} \\
  \frac{3}{11}
  \end{bmatrix} 
  \rightarrow 
  \begin{bmatrix}
  -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1
  \end{bmatrix}
  \]

- Spectrum

  \[
  \frac{f}{11}, \frac{2f}{11}, \frac{3f}{11}, \ldots
  \]
Limit Cycles

- Fundamental
  \[ f_s = f_s \frac{V_{DC}}{V_{DAC}} \]
  \[ = 3\text{MHz} \frac{2\text{mV}}{1\text{V}} \]
  \[ = 6\text{kHz} \]

- Tone velocity
  \[ \frac{df_s}{dV_{DC}} = \frac{f_s}{V_{DAC}} \]
  \[ = 3\text{kHz/V} \]

\[ \Sigma \Delta \text{Tones} \]

Output Spectrum [dBWN] / Int. Noise [dBV]

- Output Spectrum
- Integrated Noise (30 averages)

Frequency [Hz] x 10^6

6kHz
ΣΔ Tones

• Tones follow the noise shape

• The fundamental of a tone that falls into a “quantization noise null” disappears ...

\[ V_{DC} = V_{FB} \frac{f_s}{f_B} \]

\[ = 1V \frac{10.5kHz}{3MHz} \]

\[ = 3.5mV \]
**ΣΔ Tones**

- In-band tones look like signals

- Can be a big problems in some applications
  - E.g. audio → even tones with power below the quantization noise floor can be audible

- Tones near \( f_s / 2 \) can be aliased down into the signal band
  - Since they are often strong, even a small alias can be a big problem
  - We will look at mechanisms that alias tones in the next lecture

- First let’s look at dither as a means to reduce or eliminate in-band tones

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**Dither**

- DC inputs can of course be represented by many possible bit patterns

- Including some that are random but still average to the DC input

- The spectrum of such a sequence has no tones

- How can we get a ΣΔ modulator to produce such “randomized” sequences?
Dither

- The target DR for our audio SD is 16 Bits, or 98dB
- Let's choose the sampling capacitor such that it limits the dynamic range:

\[
DR = \frac{\frac{1}{2} (V_{fs})^2}{k_b T C}
\]

\[
C = DR \frac{k_b T}{\frac{1}{2} (V_{fs})^2}
\]

\[
= 10^{9.8} \frac{k_b T}{\frac{1}{2} (1V)^2} = 50.5pF \quad \rightarrow \quad \sqrt{\frac{V^2}{C}} = \sqrt{\frac{k_b T}{C}} = 9\mu V
\]
Dither

Dither at an amplitude which buries the in-band tones has virtually no effect on tones near $f_s/2$

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kT/C Noise

- So far we’ve looked at noise added to the input of the SD modulator, which is also the input of the first integrator

- Now let’s add noise also to the input of the second integrator

- Let’s assume a 4pF sampling capacitor
  - This gives $1.4 \times 32 \mu V$ rms noise (two uncorrelated $32 \mu V$ samples per clock)
**kT/C Noise**

- 2mV DC input
- Noise from 2nd integrator
  - smaller than 1st integrator noise
  - shaped
- Why?

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- Noise from 1st integrator is added directly to the input
- Noise from 2nd integrator is first-order noise shaped
- Noise from subsequent integrators is attenuated even further

→ Especially for high oversampling ratios, only the first 1 or 2 integrators add significant thermal noise. This is true also for other imperfections.
A/D DSP

Dither

No practical amount of dither eliminates the tones near $f_s/2$

Full-Scale Inputs

- With practical levels of thermal noise added, let's try a 5kHz sinusoidal input near full-scale (0.3V)
- No distortion is visible in the spectrum
  - 1-Bit modulators are intrinsically linear
  - But tones exist at high frequencies
    - to the oversampled modulator, a sinusoidal input looks like two “slowly” alternating DCs ...
    - hence giving rise to limit cycles
Full-Scale Inputs

Output Spectrum [dBWN]

Integrated Noise (30 averages)

Output Spectrum

Frequency [Hz] x 10^5

-150 -100 -50 0 50

-150 -100 -50 0 50

Frequency [Hz] x 10^5

-150 -100 -50 0 50
**V_{ref} Interference**

- Dither successfully removes in-band tones that would corrupt the signal
- The high-frequency tones in the quantization noise spectrum will be removed by the digital filter following the modulator
- What if some of these strong tones are demodulated to the base-band before digital filtering?
- Why would this happen?

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**AM Modulation**

\[ x_1(t) = X_1 \cos(\omega_1 t) \]
\[ x_2(t) = X_2 \cos(\omega_2 t) \]
\[ x_1(t) \times x_2(t) = \frac{X_1 X_2}{2} \left[ \cos(\omega_1 t + \omega_2 t) + \cos(\omega_1 t - \omega_2 t) \right] \]
AM Modulation in DAC

\[ y(t) = D_{out} = \pm 1 \]
\[ V_{ref} = 1V + 1mV \ t_s/2 \] square wave
\[ v(t) = y(t) \times V_{ref} = \text{fundamental} \]
\[ + \ 0.05\% \ of \ spectrum \ at \ t_s/2 \]
\[ V_{\text{ref}} \text{ Interference} \]

- Simulation are for specified amounts of \( \frac{f_s}{2} \) interference in the DAC reference.
- As predicted interference demodulates the high-frequency tones.
- Since the high frequency tones are strong, a small amount (1\( \mu \)V) of interference suffices to create huge base-band tones.
- Stronger interference (1mV) rises the noise floor also.
- Amplitude of demodulated tones is proportional to interference.
Symmetry of the spectra at $f_s/2$ and DC confirm that this is AM modulation.

$V_{ref}$ Tone Velocity

$V_m = 6mV / 12mV$ DC

$V_{ref} = 1V$ DC

$+ 1mV f_s/2$ square wave

$V_{in} = 0.5kHz/mV$
$V_{\text{ref}}$ Tone Velocity

- The velocity of AM demodulated tones is half that of the native tone.
- Such differences help debugging of real silicon.
- How clean does the reference have to be?

$V_{\text{ref}}$ Interference

- Tone dominated noise floor
- w/o thermal noise
### V_{ref} Interference

- 120dB of clock-to-V_{ref} isolation is not sufficient for digital audio applications
- Achieving this level of performance requires careful engineering
- Getting an accurate requirement is the first (and an essential) step

### Summary

- Our stage 2 model can drive almost all capacitor sizing decisions
  - Gain scaling
  - kT/C noise
  - Dither
- Dither removes effectively in-band tones
  - Actual tonality determined by demodulation of limit cycles near f_s/2
- Next we will add relevant component imperfections, e.g.
  - Real capacitors aren’t perfect
  - Real opamps aren’t ideal
- **We’ll model nonlinearities in the \( \Sigma \Delta \) system next time ...**