146 04 319

① <u>Given:</u> $N=4$, $\begin{cases} f_p = 500\,\text{kHz} & \text{Passband Edge} \\ R_p = 0.3\,\text{dB} & \text{Passband Atten.} \end{cases}$

<u>Needed for Synthesis:</u> N, ω_{-3dB}

<u>Butterworth:</u>

$$|H(\omega)|^2 = \frac{1}{1+\left(\omega/\omega_{-3dB}\right)^{2N}} = \frac{1}{1+\epsilon^2\left(\omega/\omega_p\right)^{2N}}$$

with $\frac{1}{1+\epsilon^2} = \left(10^{-R_p/20}\right)^2$ Passband Atten.

$$\left(\frac{\omega}{\omega_{-3dB}}\right)^{2N} = \epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}$$

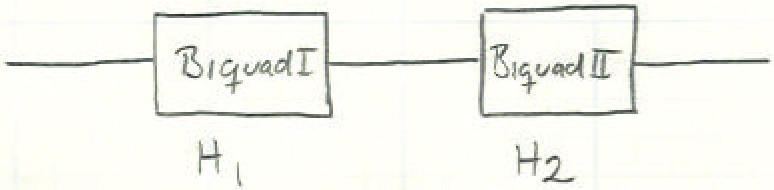
$$\Rightarrow \quad \frac{\omega_{-3dB}}{\omega_p} = \frac{1}{\epsilon^{1/N}}$$

$$\epsilon = \sqrt{10^{R_p/10} - 1} = 0.2674$$

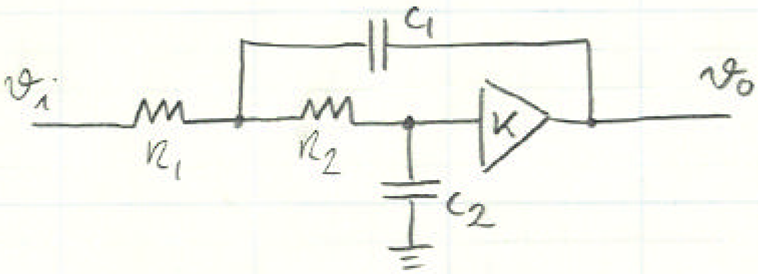
$$\Rightarrow \quad \omega_{-3dB} = 2\pi\, 500\,\text{kHz} \cdot \frac{1}{0.2674^{1/4}}$$

$$\underline{\underline{\omega_{-3dB} = 2\pi \cdot 695.3\ \text{kHz}}}$$

Implementation:



$$H(s) = \frac{G\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



$$\omega_0 = \frac{1}{\sqrt{R_1C_1R_2C_2}} \qquad G = K \qquad Q = \frac{\omega_0}{\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-K}{R_2C_2}}$$

from Matlab:

$\omega_{01} = 4.3687$ Mrad/s $\qquad \omega_{02} = 4.3687$ Mrad/s

$Q_1 = 0.5412 \qquad Q_2 = 1.3066$

$G = K = 1$, choose $R_1 = R_2 = R = R_{max} = 100k$ (NOM)

$\Rightarrow Q = \frac{\omega_0 R C_1}{2}$ (note that Q does not change with systematic RC variations!)

$$\boxed{C_1^+ = \frac{2Q}{\omega_0 R^+}}$$ (maximum component values)

$$\omega_0 = \frac{1}{R\sqrt{C_1 C_2}}$$

WORST CASE: $R = R^+ = 1.15\,R$

$C_1 = C_1^+ = 1.1\,C_1$

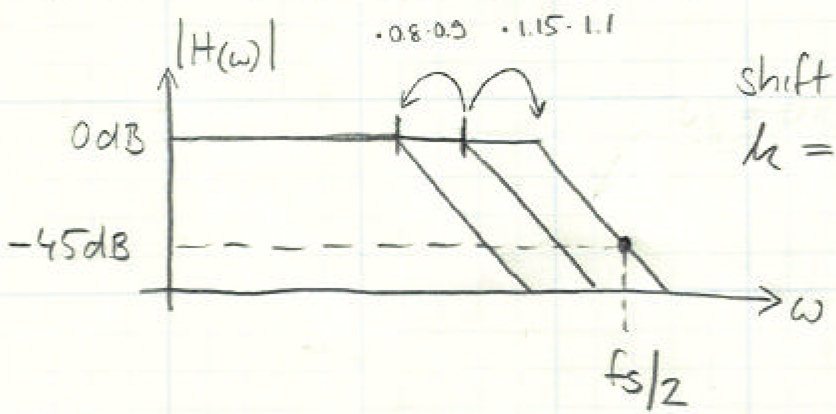
$C_2 = C_2^+ = 1.1\,C_2$

$$\omega_0 = \frac{1}{1.1 \cdot 1.15} \frac{1}{R\sqrt{C_1 C_2}}$$

$$\boxed{C_2^+ = \left(\frac{1}{\omega_0 R^+}\right)^2 \cdot \frac{1}{C_1^+}}$$ (maximum comp. values)

$\Longrightarrow$ COMPONENT VALUES SEE MATLAB PRINTOUT

MINIMUM SAMPLING FREQUENCY:



shift by

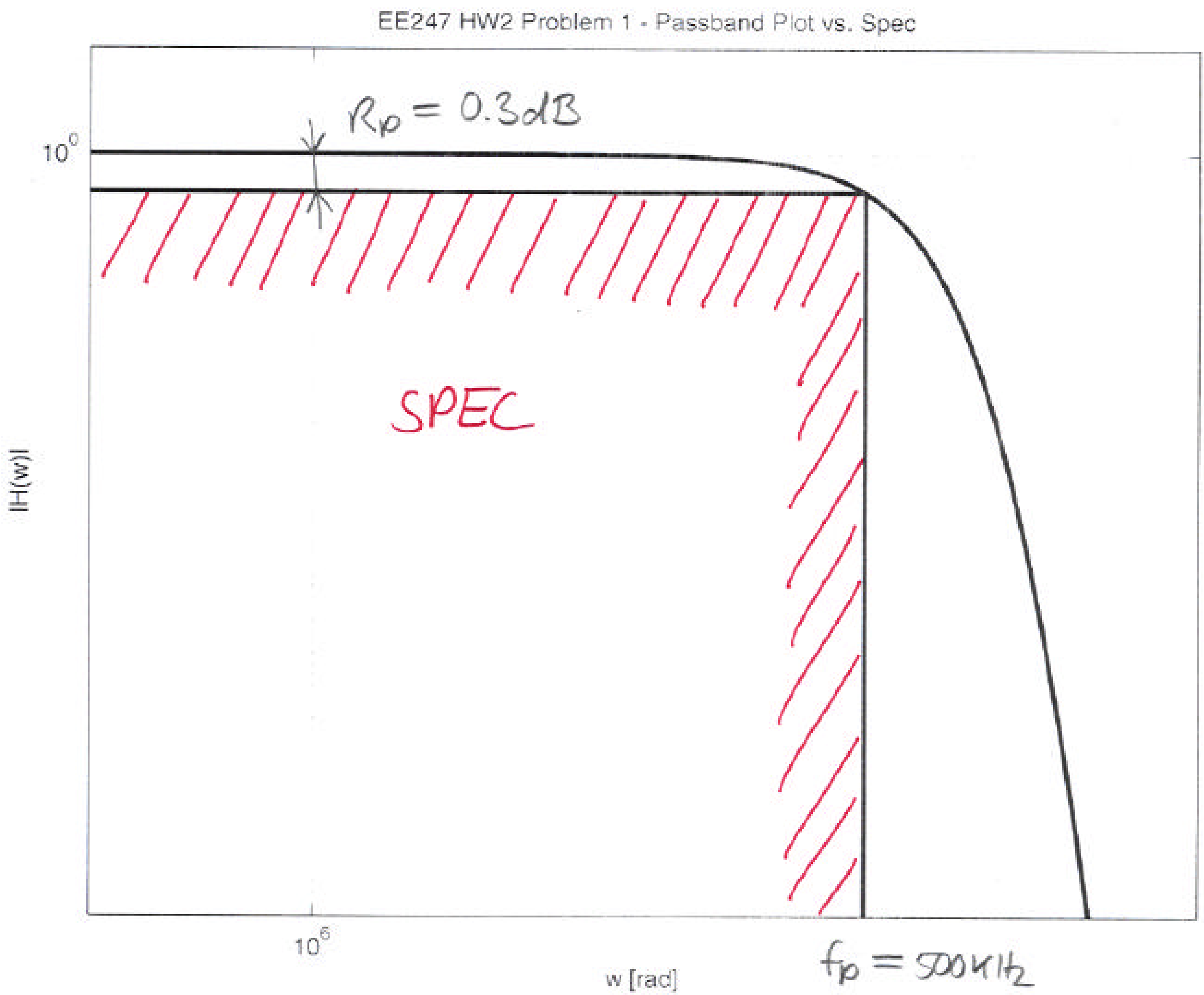
$$k = \frac{1}{0.8 \cdot 0.9} \cdot 1.15 \cdot 1.1 = 1.757$$

$$\left|H(f_s/2)\right|^2 = \frac{1}{1 + \left(\frac{f_s}{2f^{+}_{-3dB}}\right)^{2N}} \cong \left(\frac{2f^{+}_{-3dB}}{f_s}\right)^{2N}$$

$$f_s \geq \frac{2f^{+}_{-3dB}}{\left|H(f_s/2)\right|^{1/N}} = \frac{2 \cdot 1.757 \cdot 695.3\text{ kHz}}{10^{-45/(20 \cdot 4)}}$$

$$\boxed{f_s \geq = 8.36\text{ MHz}}$$

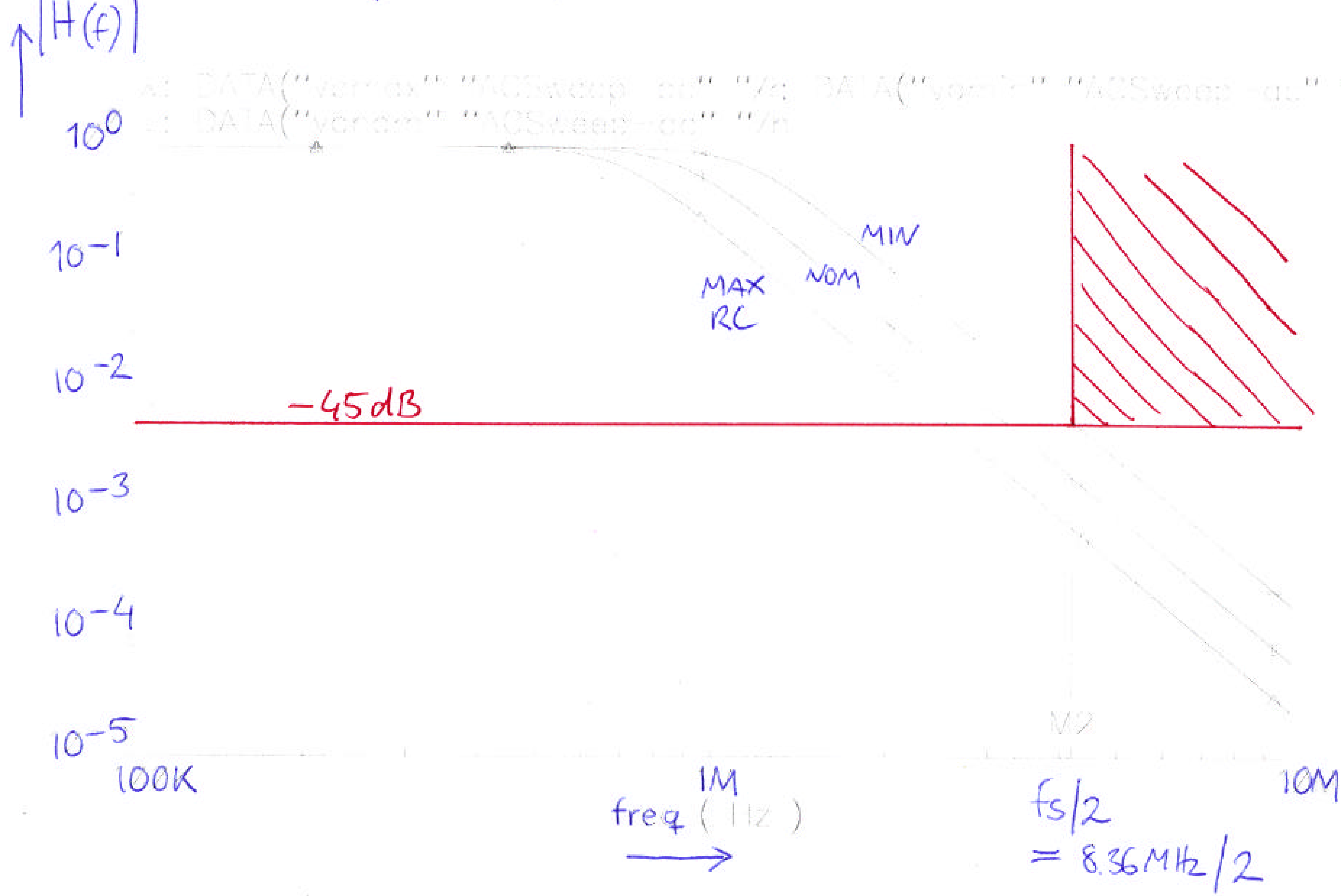
```
MATLAB OUTPUT:
--------------

w01 =
  4.3687e+006

w02 =
  4.3687e+006


Q1 =
    0.5412

Q2 =
    1.3066


COMPONENT VALUES (MAX, NOM, MIN)

R =
  1.0e+005 *
    1.1500    1.0000    0.8500

C11 =
  1.0e-011 *
    0.2154    0.1959    0.1763

C12 =
  1.0e-011 *
    0.5201    0.4728    0.4256

C21 =
  1.0e-011 *
    0.1839    0.1672    0.1505

C22 =
  1.0e-012 *
    0.7617    0.6925    0.6232
```


EE247 HW2 Problem 1 - Passband Plot vs. Spec
$R_p = 0.3dB$
10^0
SPEC
|H(w)|
10^6
w [rad]
$f_p = 500kHz$

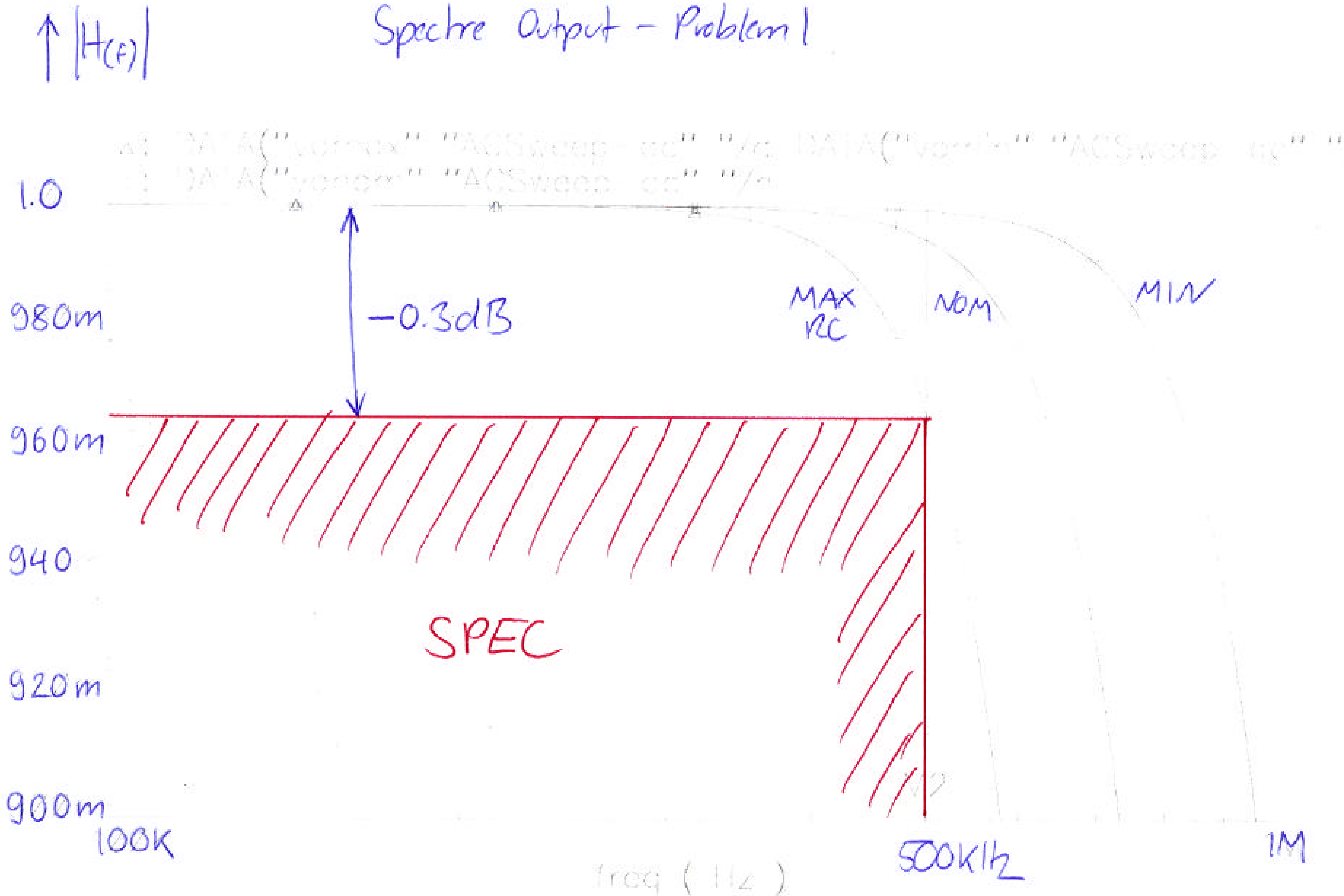
```
% EE247 Homework#2 Problem 1
% Boris Murmann

% filter specification and synthesis
N  = 4;
Wn = 2*pi*695.3e3;        % -3dB frequnency [rad] (hand calculated)
[Z, P, K] = butter(N, Wn, 's');

% Plot frequency response and check vs. spec
w1 = 2*pi*100e3;
w2 = 2*pi*1000e3;
wp = 2*pi*500e3;
Ap= 10^(-0.3/20);
Amin = 0.5;

[NUM, DEN]= zp2tf(Z, P, K);
[H,W]= freqs(NUM, DEN);
loglog( W, abs(H), 'LineWidth',2  );
axis([w1 w2 Amin 1.1]);
line( [w1 wp], [Ap Ap], 'LineWidth',2, 'Color','red');
line( [wp wp], [Ap Amin], 'LineWidth',2, 'Color','red');
title('EE247 HW2 Problem 1 - Passband Plot vs. Spec');
xlabel('w [rad]');
ylabel('|H(w)|');
grid;

% Break up into biquads
[NUM1, DEN1] = zp2tf( [], P(1:2), 1)
[NUM2, DEN2] = zp2tf( [], P(3:4), 1)
w01 = sqrt(DEN1(3))
w02 = sqrt(DEN2(3))
Q1 = w01/DEN1(2)
Q2 = w02/DEN2(2)

% Calculate componenents
Rnom=100e3;
MR=1.15;
MC=1.1;
mR=0.85;
mC=0.9;

% Maximum component values
R(1) = Rnom*MR;
C11(1) = 2*Q1/(w01*R(1));
C12(1) = 2*Q2/(w02*R(1));
C21(1) = (1/C11(1)) * (1/(w01*R(1)))^2;
C22(1) = (1/C12(1)) * (1/(w02*R(1)))^2;

% Nominal component values
R(2) = R(1)/MR;
C11(2) = C11(1)/MC;
C12(2) = C12(1)/MC;
C21(2) = C21(1)/MC;
C22(2) = C22(1)/MC;

% Minimum component values
R(3) = R(2)*mR;
C11(3) = C11(2)*mC;
C12(3) = C12(2)*mC;
C21(3) = C21(2)*mC;
C22(3) = C22(2)*mC;

R
C11
C12
C21
C22
```

```
* EE247 Homework#2 - Problem 1
* Spectre Input
* Boris Murmann

********** Circuit Desription *************

simulator lang=spectre

vin      (vi  0)      vsource mag=1

b1nom    (vi vo1nom)     skbp res=100k cap1=1.959p cap2=1.672p  k=1
b2nom    (vo1nom vonom)  skbp res=100k cap1=4.728p cap2=0.6925p k=1

b1min    (vi vo1min)     skbp res=85k  cap1=1.763p cap2=1.505p k=1
b2min    (vo1min vomin)  skbp res=85k  cap1=4.256p cap2=0.623p k=1

b1max    (vi vo1max)     skbp res=115k cap1=2.154p cap2=1.839p k=1
b2max    (vo1max vomax)  skbp res=115k cap1=5.201p cap2=0.762p k=1

subckt skbp (vi vo)
  parameters cap res cap1 cap2 k
  r1 (vi 1) resistor r=res
  r2 (2  1) resistor r=res
  c1 (1 vo) capacitor c=cap1
  c2 (2  0) capacitor c=cap2
  k1 (vo 0 2 0) vcvs gain=k
ends skbp

********** Control Statements *************

SimOptions options
+        rawfmt=   psfbin
+        gmin=     1E-12
+        reltol=   1E-03
+        vabstol=  1E-06
+        iabstol=  1E-12
+        temp=     27

ACSweep    ac   start=100k stop=10M dec=100
```

Spectre Output - Problem 1



Spectre Output - Problem 1



(2) $$H_1(s) = \frac{-K_{oa}}{s+\omega_{oa}}$$

$$H_2(s) = \frac{K_{2b}s^2 + K_{ob}}{s^2 + \frac{\omega_{ob}}{Q} + \omega_{ob}^2}$$ $K_{1b}=0$ (complex conj. zeros)

From Matlab:

$$K \cdot \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)\;(s-p_3)}$$

$\downarrow$ $H_2(s)$ $\qquad$ $\downarrow$ $H_1(s)$

- $H_1(s=0) = -\frac{K_{oa}}{\omega_{oa}} \overset{!}{=} -1$ (want unity gain)

$$\omega_{oa} = \frac{-1}{p_3} \;;\; K_{oa} = \omega_{oa}$$

- use $zp2tf\left([z_1, z_2], [p_1, p_2], K/K_{oa}\right)$

$\Rightarrow \omega_{ob}, Q, K_{ob}, K_{2b}$

$\rightarrow$ $\omega_{oa} = 3.29\ Mrad/s = K_{oa}$] $\underline{\underline{H_1(s)}}$

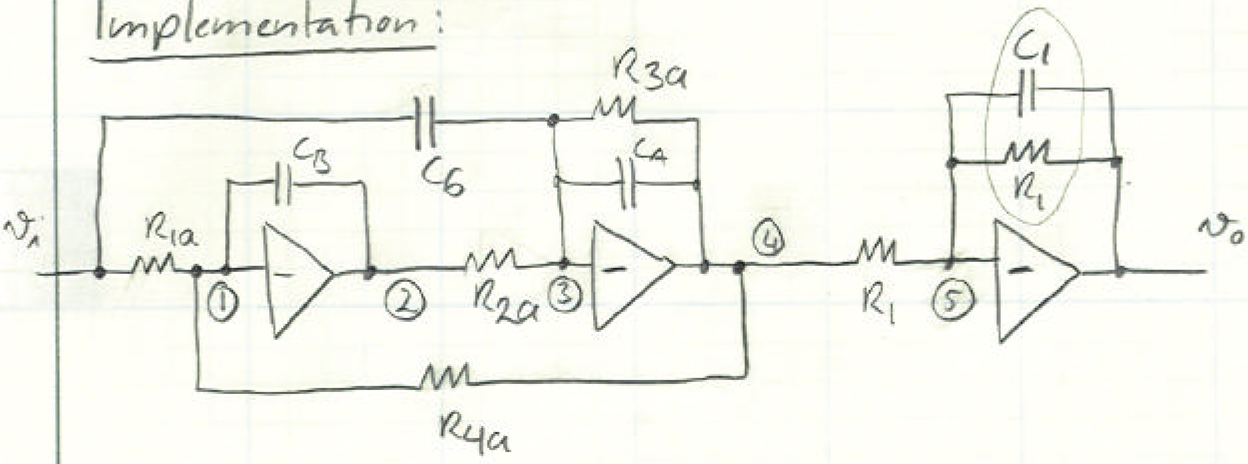
$\rightarrow$ $\omega_{ob} = 6.3\ Mrad/s$

$Q = 2.21$

$K_{ob} = 3.97 \cdot 10^{15}$

$K_{2b} = 0.13\ \frac{1}{rad^2}$] $\underline{\underline{H_2(s)}}$

Implementation:



- $C_1 = \dfrac{1}{\omega_{0a} R_1} = 303.9\,fF$ with $R_1 = R_{max} = 100K$
- → iteravively scale CA, CB until all $R < R_{max}$

using Matlab:

$R_{1a} = 79.37K$

$R_{2a} = -39.68K$

$R_{3a} = 87.54K$

$R_{4a} = 79.37K$

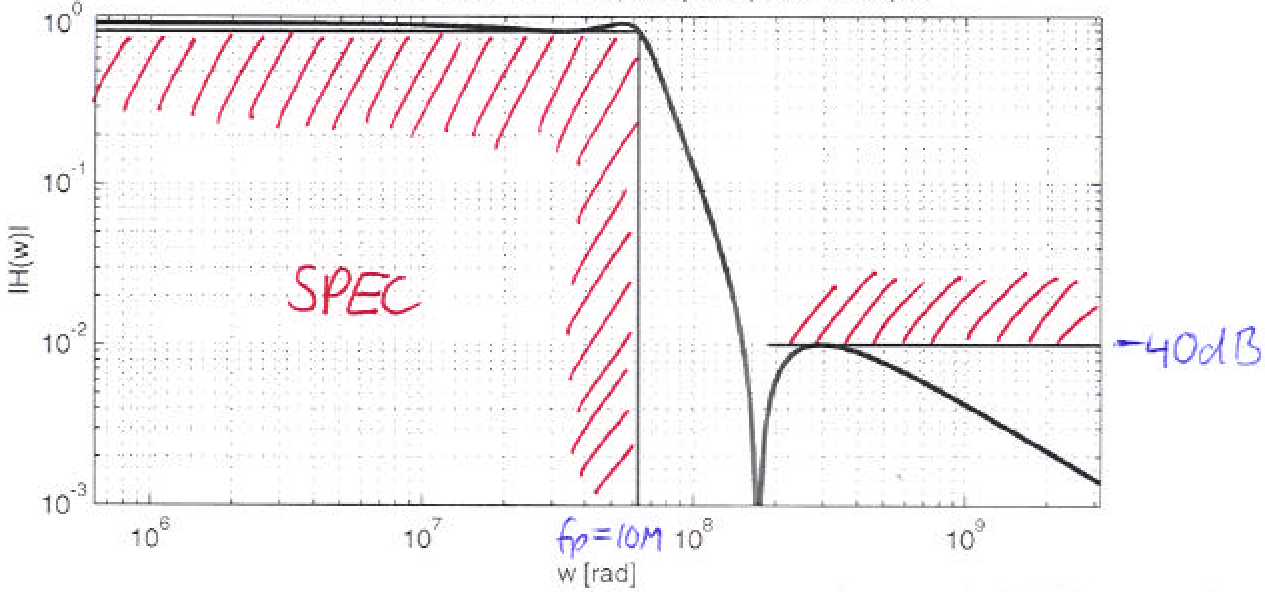
$C_A = 400fF$

$C_B = 200fF$

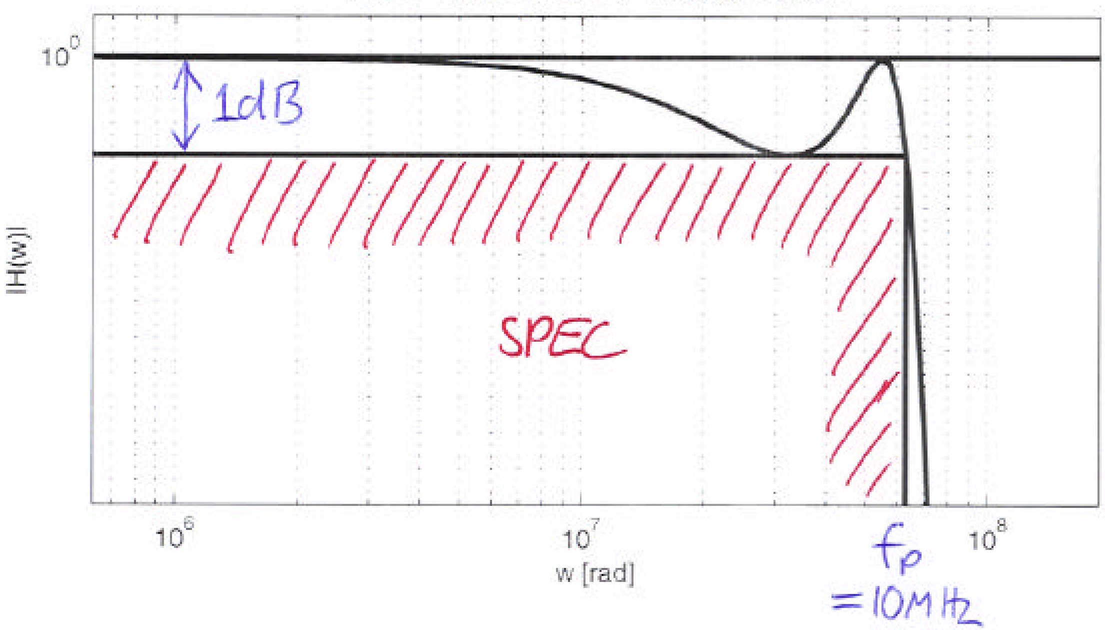
$C_6 = 52.8\,fF \overset{\checkmark}{>} 50fF$

→ Simulation Results see attached

EE247 HW2 Problem 2 - Frequency Response vs. Spec

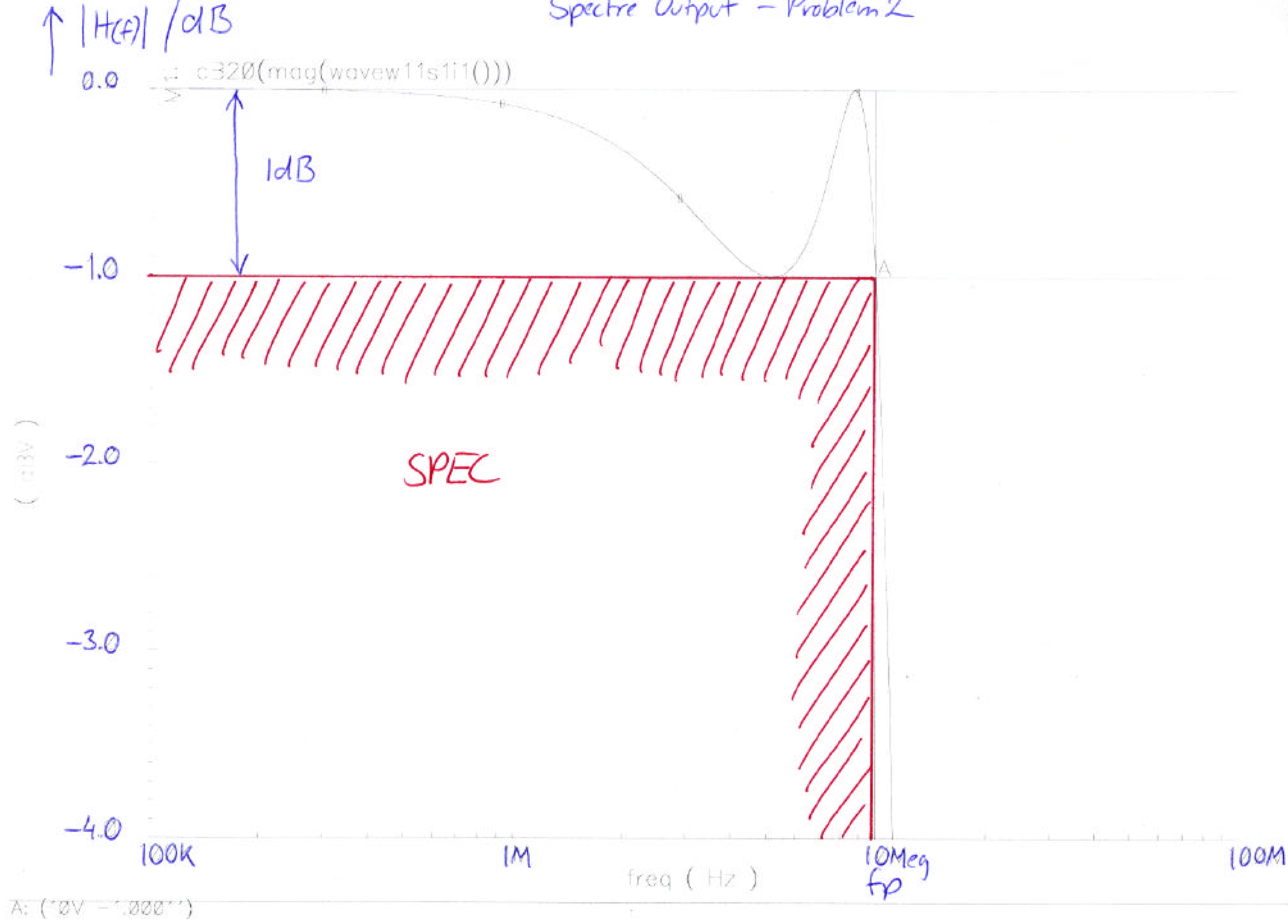


EE247 HW2 Problem 2 - Passband Zoom

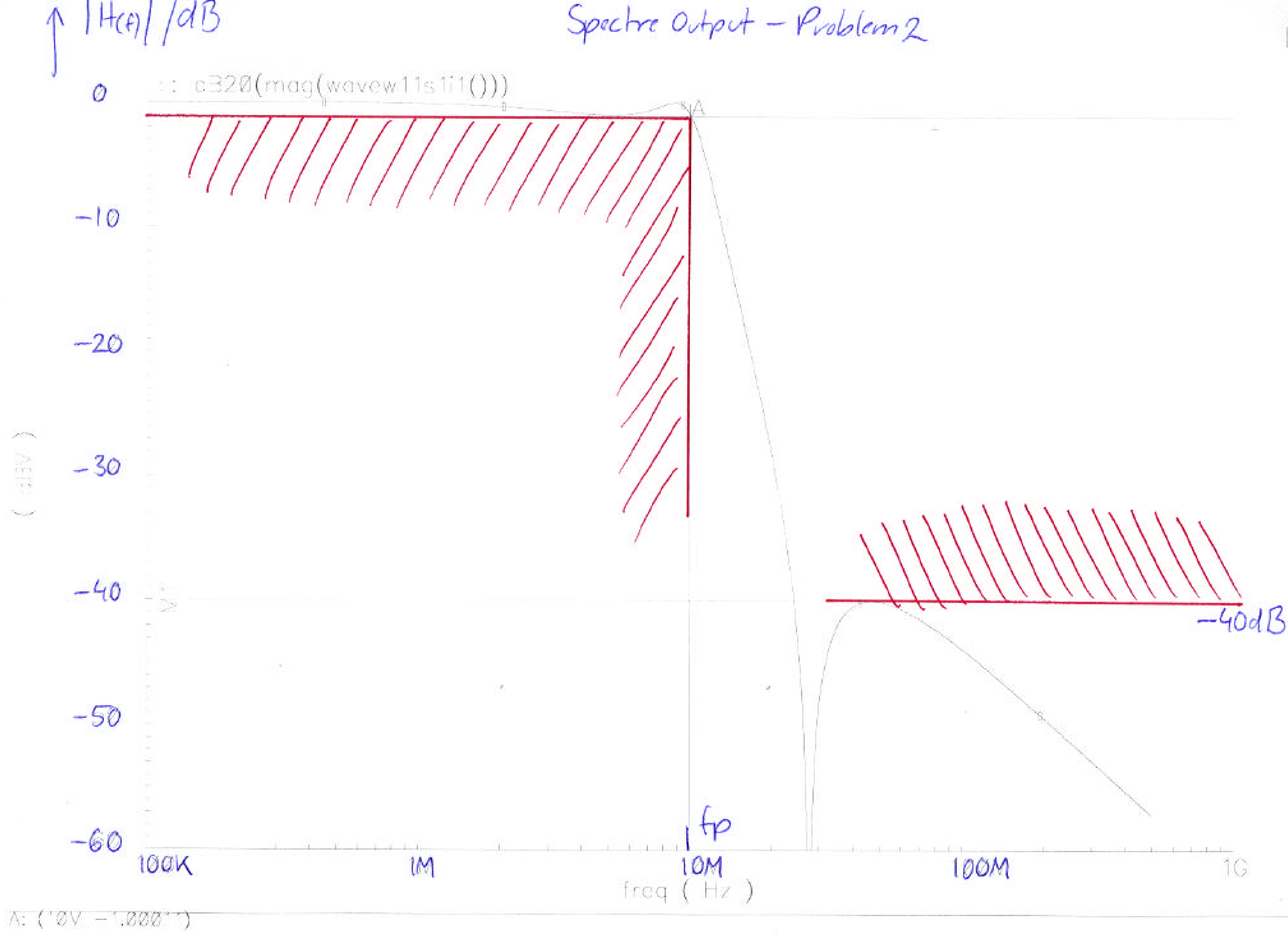


```
% EE247 Homework#2 Problem 2
% Boris Murmann

% filter specification and synthesis
clear;
N  = 3;
Rp = 1;
Rs = 40;
Wn = 2*pi*10e6;       % cutoff frequency [rad]
[Z, P, K] = ellip(N, Rp, Rs, Wn, 's');

% Plot frequency response and check vs. spec
% Plot range
w1 = 2*pi*100e3;
w2 = 2*pi*500e6;
wp = Wn;
ws = 2*pi*30e6;
Ap= 10^(-Rp/20);
As= 10^(-Rs/20);
Amin = 10^((-Rs-20)/20);
[NUM, DEN]= zp2tf(Z, P, K);
[H,W]= freqs(NUM, DEN, w1:(w2-w1)/1000:w2) ;

subplot(2,1,1)
loglog( W, abs(H), 'LineWidth',2  );
axis([w1 w2 Amin 1.1]);
% horizontal passband line
line( [w1 wp], [Ap Ap], 'LineWidth',1, 'Color','red');
%line( [w1 ws], [1  1 ], 'LineWidth',1, 'Color','red');
% horizontal stopband line
line( [ws w2], [As As], 'LineWidth',1, 'Color','red');
% vertical passband line
line( [wp wp], [Ap Amin], 'LineWidth',1, 'Color','red');
% vertical stopband line
%line( [ws ws], [1 As], 'LineWidth',1, 'Color','red');
title('EE247 HW2 Problem 2 - Frequency Response vs. Spec');
xlabel('w [rad]');
ylabel('|H(w)|');
grid;

subplot(2,1,2)
loglog( W, abs(H), 'LineWidth',2  );
axis([w1 ws Ap-0.3 1.05]);
% horizontal passband line
line( [w1 wp], [Ap Ap], 'LineWidth',2, 'Color','red');
line( [w1 ws], [1  1 ], 'LineWidth',2, 'Color','red');
% horizontal stopband line
line( [ws w2], [As As], 'LineWidth',2, 'Color','red');
% vertical passband line
line( [wp wp], [Ap Amin], 'LineWidth',2, 'Color','red');
% vertical stopband line
%line( [ws ws], [1 As], 'LineWidth',2, 'Color','red');
title('EE247 HW2 Problem 2 - Passband Zoom');
xlabel('w [rad]');
ylabel('|H(w)|');
grid;

% Calculate Coefficients

% First order section
% Desired DC Gain is G2=1
G1=1;
w0a=-P(3)
K0a=G1*w0a
% Second order section
% DC Gain is also one, by using K/K0b as the K parameter in zp2tf
[NUM1, DEN1] = zp2tf( [Z(1:2)], P(1:2), K/K0a);
w0b=sqrt(DEN1(3))
```

Spectre Output – Problem 2



Spectre Output – Problem 2



```
* EE247 Homework#2 - Problem 2
* Spectre Input
* Boris Murmann


********** Circuit Desription *************

* component values from matlab
parameters
+R1  = 100k
+C1  = 3.0389e-013
+CA  = 4.0000e-013
+CB  = 2.0000e-013
+R1a = 7.9366e+004
+R2a =-3.9683e+004
+R3a = 8.7539e+004
+R4a = 7.9366e+004
+C6  = 5.2853e-014

vin (vi 0) vsource  mag=1

r1x (vi 1) resistor r=R1a
r2x (2  3) resistor r=R2a
r3x (3  4) resistor r=R3a
r4x (1  4) resistor r=R4a
cax (3  4) capacitor c=CA
cbx (1  2) capacitor c=CB
c6x (vi 3) capacitor c=C6
amp1(2 0 1 0) vcvs gain=-1e6
amp2(4 0 3 0) vcvs gain=-1e6

r11 (4  5) resistor  r=R1
r12 (5 v0) resistor  r=R1
c1x (5 v0) capacitor c=C1
amp3(v0 0 5 0) vcvs gain=-1e6


********** Control Statements *************

SimOptions options
+       rawfmt=   psfbin
+       gmin=     1E-12
+       reltol=   1E-03
+       vabstol=  1E-06
+       iabstol=  1E-12
+       temp=     27

ACSweep    ac  start=100k stop=500Meg dec=100
```