UNIVERSITY OF CALIFORNIA

College of Engineering

NTU 776CA (EECS 247)

Midterm (120 minutes)

October 15-19, 2001

Exam is open-book, open-notes. Clearly mark results with box around. No credit for ambiguous solutions. Show derivations. Return this cover page. Good luck!

Name: Solution

PROBLEM	SCORE
1	
2	
Total	

(Total # of pages = 7 including this one)

1. [40] Consider the following Z-domain filter transfer function.

$$H(z) = \frac{1 - z^{-1}}{2 + z^{-1}}$$

a) Find the poles and zeros of *H(z)*. [10]

b) Sketch the frequency response of the filter between $-\pi$ and π. Is it highpass, lowpass, or bandpass? [10]

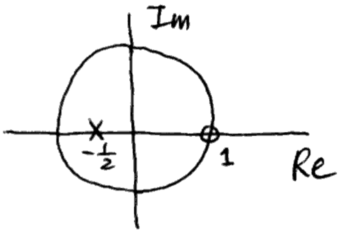
c) Draw a fully differential circuit that realizes this transfer function using LDI integrators and switched capacitors. Assume you have non-overlapping two-phase clocks available. Your answer will be graded on the number of components used in your circuit. [20]

Hint: you may realize negative numbers with fully differential circuits.

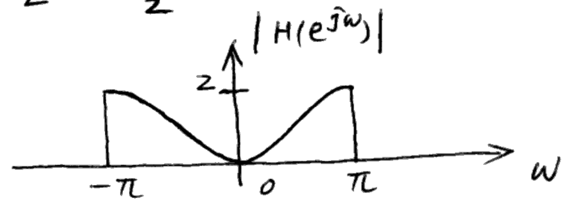
a) zero : $1 - z^{-1} = 0 \Rightarrow z = 1$

pole : $2 + z^{-1} = 0 \Rightarrow z = -\frac{1}{2}$

b)



$\Rightarrow$



"high-pass"

c) $$\frac{Y(z)}{X(z)} = H(z) = \frac{1 - z^{-1}}{2 + z^{-1}}$$

$$\Rightarrow \quad Y(z)(2 + z^{-1}) = X(z)(1 - z^{-1})$$

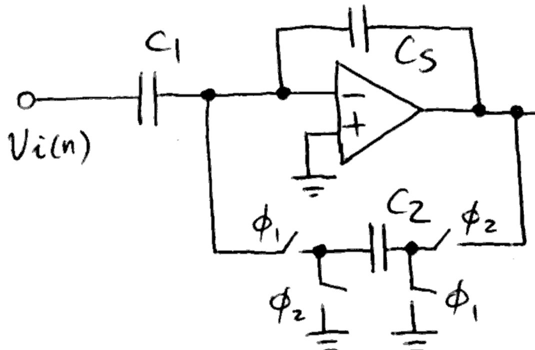
$$2y(n) + y(n-1) = x(n) - x(n-1)$$

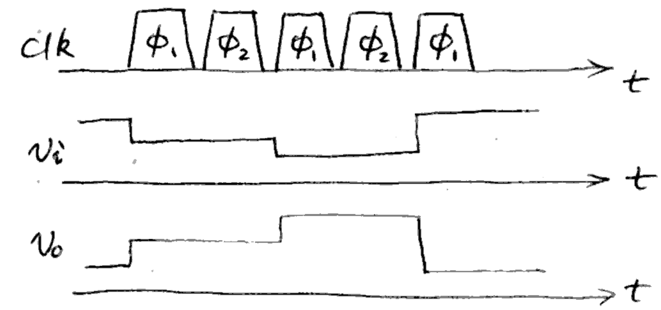
$$y(n) = -\frac{1}{2}y(n-1) + \frac{1}{2}[x(n) - x(n-1)] \qquad (*)$$

#1. c) contd

we need 1 LDI, 2 caps (one switched).

single-ended prototype





Derive TF using charge conservation principle.

@ ϕ_2: $\quad v_i(n-1)\,C_1 + v_o(n-1)\,C_2 + v_o(n-1)\,C_s = \Sigma Q$

@ ϕ_1: $\quad v_i(n)\,C_1 + \emptyset + v_o(n)\,C_s = \Sigma Q'$

but $\Sigma Q = \Sigma Q'$, so

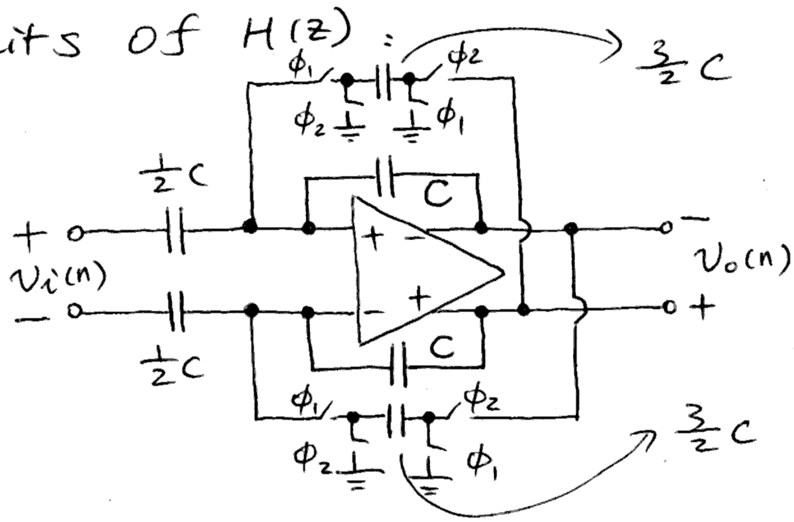
$$v_i(n-1)\,C_1 + v_o(n-1)\,C_2 + v_o(n-1)\,C_s = v_i(n)\,C_1 + v_o(n)\,C_s$$

$$\Rightarrow \quad v_o(n) = \left(\frac{C_2 + C_s}{C_s}\right) v_o(n-1) + \frac{C_1}{C_s}\left[\,v_i(n-1) - v_i(n)\,\right]$$

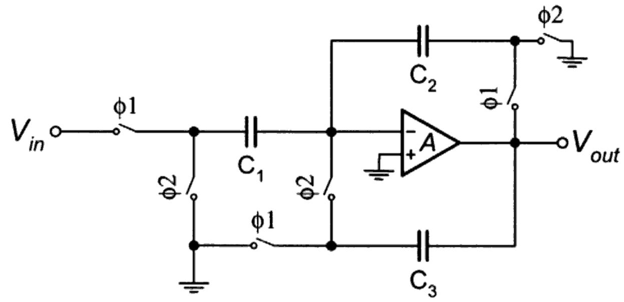
compare this with (*), we have

$$C_1 = -\tfrac{1}{2}C_s, \quad C_2 = -\tfrac{3}{2}C_s$$

This leads to the final implementation using differential circuits of $H(z)$:



2. [60] Performance of switched-capacitor circuits often suffers from nonidealities of op amps, i.e., finite bandwidth, finite gain, offset etc. The following predictive SC amplifier is claimed to be able to compensate the finite-gain effect of the op amp. Assume the op amp gain is A (frequency independent). $\phi 1$ and $\phi 2$ are non-overlapping two-phase clocks. V_{in} and V_{out} updates when $\phi 1=1$.



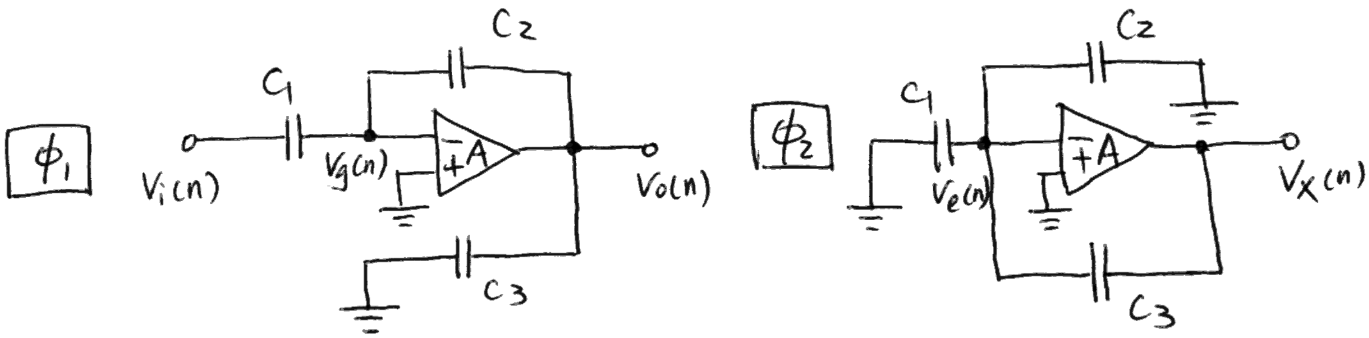
a) Derive the Z-domain voltage transfer function $V_{out}(z)/V_{in}(z)$ with finite A. [20]

b) Explain why the predictive SC amplifier works. What is the limitation of this compensation scheme? You may explain intuitively and justify your answer with the result you obtain in a). [20]

Hint: think about when the frequency of V_{in} approaches f_{clk}, the clock frequency.

c) Someone also claims this compensation scheme can remove the input offset of the op amp. Can you prove that? [20]

a) Redraw the ckt in ϕ_1 & ϕ_2 respectively.



We will use the charge conservation law to derive the VTF. We need to take A into account in the process.

#2. a) cont'd

Step 1 start in ϕ_2, time $(n-1)$. Assume C_1 & C_2 are fully discharged. Then in time n, we balance the charge.

$\boxed{\phi_2}$ $(n-1)$ $\qquad\qquad$ $\boxed{\phi_1}$ (n)

$$\Sigma Q = \phi \equiv \Sigma Q = [V_i(n) - V_g(n)]C_1 + [V_o(n) - V_g(n)]C_2$$

$$\Rightarrow \boxed{V_o(n) = -\left\{\frac{C_1}{C_2 + \frac{C_1+C_2}{A}}\right\} V_i(n)} \qquad (*)$$

$$\text{or } \frac{V_o(n)}{V_i(n)} = -\frac{C_1}{C_2 + \frac{C_1+C_2}{A}}$$

"finite-gain effect"

Step 2 start in ϕ_2, time (n), work into ϕ_1, time $(n+1)$.

$\boxed{\phi_2}$ (n) $\qquad\qquad$ $\boxed{\phi_1}$ $(n+1)$

$$-C_1 V_e(n) - C_2 V_e(n) + [V_x(n) - V_e(n)]C_3 = V_o(n) C_3$$

$$\Downarrow$$

$$V_e(n) = -V_o(n)\frac{C_3}{C_1+C_2+C_3+AC_3}$$

$$\text{or } V_e(n) = -\frac{V_o(n)}{A + \frac{C_1+C_2+C_3}{C_3}}$$

$$[V_i(n+1) - V_g(n+1)]C_1 + [V_o(n+1) - V_g(n+1)]C_2 = -C_1 V_e(n) - C_2 V_e(n)$$

$$\Downarrow$$

$$V_i(n+1)C_1 + V_o(n+1)\left(C_2 + \frac{C_1+C_2}{A}\right) = -V_e(n)(C_1+C_2)$$

$$\Downarrow$$

$$V_i(n+1)C_1 + V_o(n+1)\left(C_2 + \frac{C_1+C_2}{A}\right) = V_o(n)\frac{C_1+C_2}{A + \frac{C_1+C_2+C_3}{C_3}}$$

#2

$$\Rightarrow \boxed{\frac{V_o}{V_i}(z) = \frac{-C_1}{C_2 + \frac{C_1+C_2}{A} - \frac{z^{-1}(C_1+C_2)}{A + \frac{C_1+C_2+C_3}{C_3}}}} \qquad (*)$$

if $\frac{C_1+C_2+C_3}{C_3} \ll A$ holds, we may write:

$$\frac{1}{A + \frac{C_1+C_2+C_3}{C_3}} \approx \frac{1}{A} - \underbrace{\left(\frac{C_1+C_2+C_3}{C_3} \Big/ A^2\right)}_{\text{ignore for } A \gg 1}$$

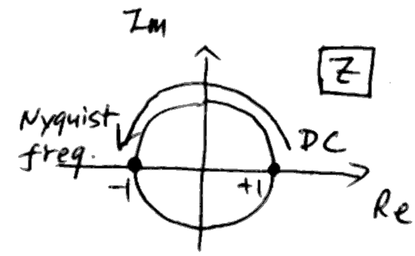
This leads to,

$$\Rightarrow \boxed{\frac{V_o}{V_i}(z) \approx \frac{-C_1}{C_2 + \frac{C_1+C_2}{A}(1-z^{-1})} = H(z)}$$

b) if $f_{in} \ll f_{clk}$, then $z \approx 1$ (low-freq input)

$$\Rightarrow \frac{V_o}{V_i}(z) \approx -\frac{C_1}{C_2}$$

The gain dependent term $\frac{C_1+C_2}{A}(1-z^{-1})$ vanishes for low-freq input. This explains why this SC amplifier is called "Predictive" since signals that are slow are easy to be predicted from sample to sample. So, ϕ_1 is <u>amplifying</u> phase, while ϕ_2 is <u>predicting</u> phase.



<u>Caveat</u> as f_{in} approaches f_{clk}, we see in the right plot that z approaches -1.

$$\Rightarrow \frac{V_o}{V_i}(z) \approx \frac{-C_1}{C_2 + \left(\frac{C_1+C_2}{A}\right)\cdot \underline{\underline{2}}}$$

Now, we actually enhance the error due to finite gain.

This is the limitation of this scheme.

c) This is simple. Assume A is large.

$\boxed{\phi_2}$ $(n-1)$ $\qquad\qquad$ $\boxed{\phi_1}$ (n)

$$\Sigma Q = -V_{os}(C_1+C_2) \equiv \Sigma Q = [V_i(n) - V_g(n)]C_1 + [V_o(n) - V_g(n)]C_2$$

$$\Rightarrow -V_{os}(C_1+C_2) = V_i(n)C_1 + V_o(n)C_2 - V_g(n)(C_1+C_2)$$

but $V_g(n) = V_{os}$, so the LHS and the last term in RHS cancel, we have,

$$\phi = V_i(n)C_1 + V_o(n)C_2$$

or

$$\frac{V_o(n)}{V_i(n)} = -\frac{C_1}{C_2}$$

no V_{os} effect.