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B. E. Boser **Homework 9 Solution** **EECS 247**
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The amplifier transfer function is

$$V_o = V_{\max} \tanh\left(A_v V_i\right)$$

$$\Rightarrow V_i = \frac{1}{A_v}\tanh^{-1}\left(\frac{V_o}{V_{\max}}\right) = \frac{1}{2A_v}\ln\frac{1+\frac{V_o}{V_{\max}}}{1-\frac{V_o}{V_{\max}}} \qquad (1)$$

Using the nomenclature of slide 3, lecture 18, (1) becomes

$$V_{1N} = \frac{1}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}}{V_{\max}}\right) = \frac{1}{2A_v}\ln\frac{1+\frac{V_{1OUT}}{V_{\max}}}{1-\frac{V_{1OUT}}{V_{\max}}} \qquad (1a)$$

We will now find the integrator transfer function using the charge conservation principle. We choose $V_{CM} = 0$ and $C_{R1} = C_{R2} = C_{IN} = C_{FB}/2$.

During φ_1:

$$Q_{IN}(k-1) = C_{IN}V_{IN}(k-1) \qquad (2)$$

$$Q_{R1}(k-1) = C_{R1}V_{REF} \qquad (3)$$

$$Q_{R2}(k-1) = 0 \qquad (4)$$

$$Q_{FB}(k-1) = \left[V_{1N}(k-1) - V_{1OUT}(k-1)\right]C_{FB}$$

$$\Rightarrow Q_{FB}(k-1) = \left[-\frac{1}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k-1)}{V_{\max}}\right) - V_{1OUT}(k-1)\right]C_{FB} \qquad (5)$$

During φ_2, two cases are distinguished:

If the comparator output $D = -1$,

$$Q_{FB}(k) = Q_{FB}(k-1) - Q_{IN}(k-1) - Q_{R1}(k-1) + Q_{IN}(k) + Q_{R1}(k)$$

$$\left[-\frac{1}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k)}{V_{\max}}\right) - V_{1OUT}(k)\right]C_{FB} - \left[-\frac{1}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k-1)}{V_{\max}}\right) - V_{1OUT}(k-1)\right]C_{FB} =$$

$$= -C_{IN}V_{IN}(k-1) - C_{R1}V_{REF} + \frac{C_{IN}}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k)}{V_{\max}}\right) + \frac{C_{R1}}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k)}{V_{\max}}\right) \qquad (6)$$

Equation (6) simplifies to

$$V_{1OUT}(D=-1)=\frac{0.5}{z-1}\left(V_{IN}+V_{REF}\right)+\frac{z}{z-1}\frac{1}{A_v}\left[\tanh^{-1}\left(\frac{V_{1OUT}z^{-1}}{V_{\max}}\right)-2\tanh^{-1}\left(\frac{V_{1OUT}}{V_{\max}}\right)\right] \quad (6a)$$

If the comparator output $D=1$,

$$Q_{FB}(k)=Q_{FB}(k-1)-Q_{IN}(k-1)-Q_{R2}(k-1)+Q_{IN}(k)+Q_{R2}(k)$$

$$\left[-\frac{1}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k)}{V_{\max}}\right)-V_{1OUT}(k)\right]C_{FB}-\left[-\frac{1}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k-1)}{V_{\max}}\right)-V_{1OUT}(k-1)\right]C_{FB}=$$

$$=-C_{IN}V_{IN}(k-1)+\frac{C_{IN}}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k)}{V_{\max}}\right)+C_{R2}\left[V_{REF}+\frac{1}{A_v}\tanh^{-1}\left(\frac{V_{1OUT}(k)}{V_{\max}}\right)\right] \quad (7)$$

Equation (7) simplifies to

$$V_{1OUT}(D=1)=\frac{0.5}{z-1}\left(V_{IN}-V_{REF}\right)+\frac{z}{z-1}\frac{1}{A_v}\left[\tanh^{-1}\left(\frac{V_{1OUT}z^{-1}}{V_{\max}}\right)-2\tanh^{-1}\left(\frac{V_{1OUT}}{V_{\max}}\right)\right] \quad (7a)$$

Combining (6a) and (7a) we get

$$V_{1OUT}=\underbrace{\frac{0.5}{z-1}\left(V_{IN}-DV_{REF}\right)}_{\text{ideal term}}+\underbrace{\frac{z}{z-1}\frac{1}{A_v}\left[\tanh^{-1}\left(\frac{V_{1OUT}z^{-1}}{V_{\max}}\right)-2\tanh^{-1}\left(\frac{V_{1OUT}}{V_{\max}}\right)\right]}_{\text{term due to nonlinearity}} \quad (8)$$

The corresponding Sigma Delta is modeled in Simulink as shown in Figure 1.

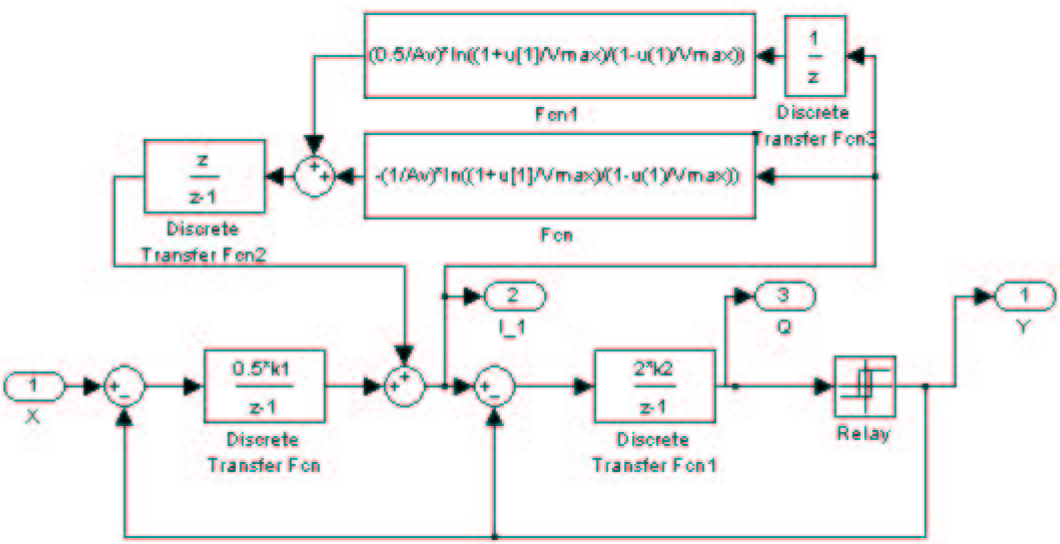

Figure 1. Sigma Delta Simulink model.

The in-band quantization noise using an ideal integrator is $-111dB$, as shown in Homework 8. Since only $3dB$ increase is accepted, the maximum acceptable value is $-108dB$, which corresponds to $A_v=100$ according to the simulation. This value however leads to $SFDR<95dB$. To meet this specification the amplifier gain is further increased to $A_v=600$. The corresponding graphs are shown in Fig. 2 and 3.

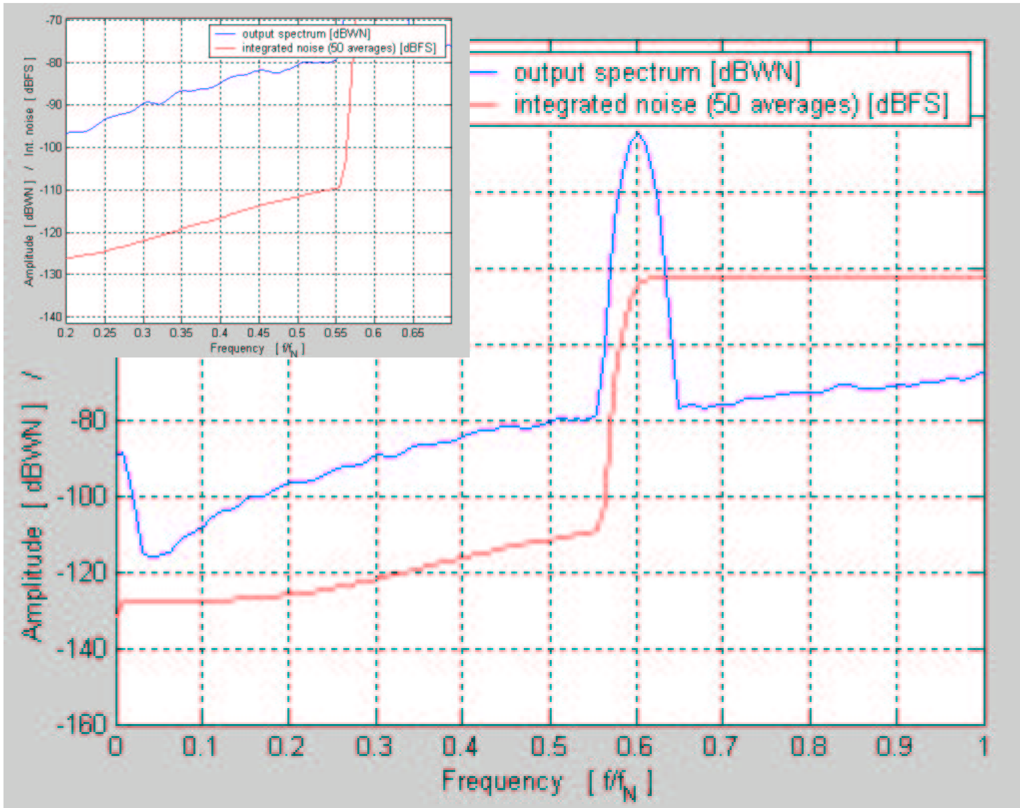

Figure 2. Quantization noise plot for gain = 600.

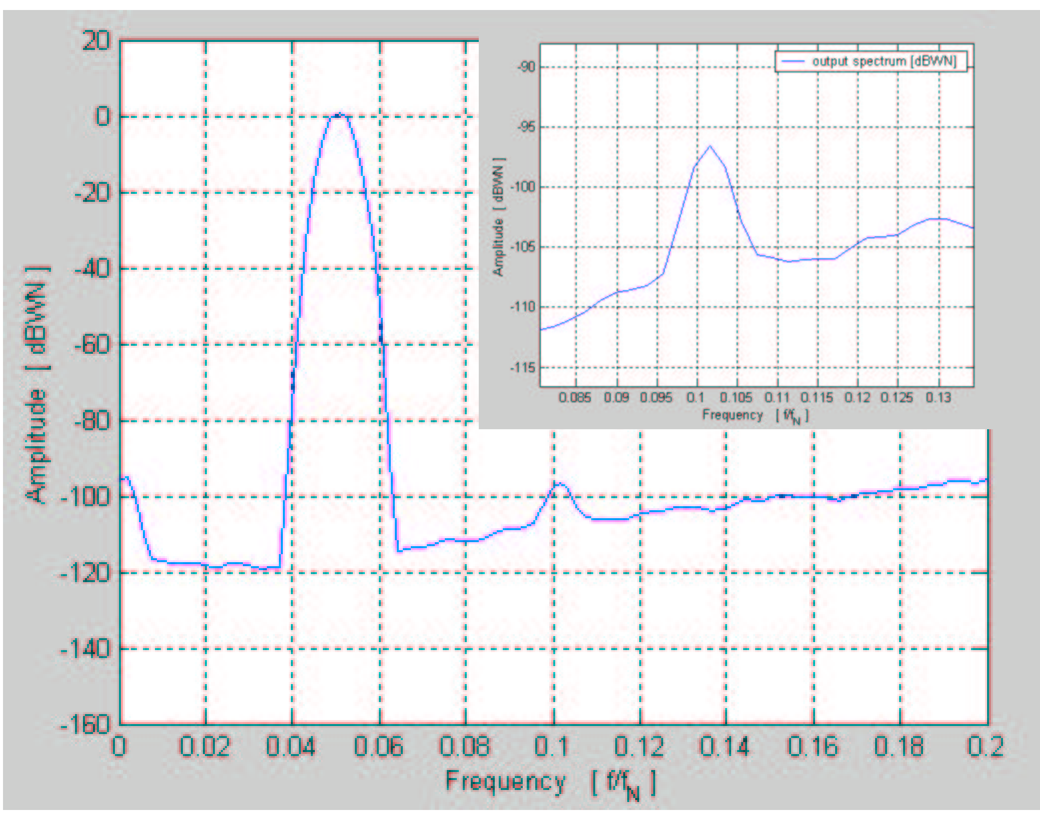

Figure 3. Harmonics plot for gain = 600.