UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering and Computer Sciences
NTU IC-776CA

'For Fun' Homework Set Solution

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University of California
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a Let $X(z)$ be the input, $Y(z)$ the output and $Q(z)$ the quantization error. First, we calculate the signal transfer function *STF*:

$$\left[\left(X(z)-Y(z)\right)\frac{0.5}{z-1}-Y(z)\right]\frac{2}{z-1}=Y(z)$$

$$\Rightarrow STF=\frac{Y(z)}{X(z)}=z^{-2} \qquad (1)$$

Next, we calculate the noise transfer function *NTF*:

$$\left(-Y(z)\frac{0.5}{z-1}-Y(z)\right)\frac{2}{z-1}+Q(z)=Y(z)$$

$$\Rightarrow NTF=\frac{Y(z)}{Q(z)}=\left(1-z^{-1}\right)^2 \qquad (2)$$

$$\left|NTF(z)\right|^2=NTF(z)\times NTF(z^{-1})=\left(1-z^{-1}-z+1\right)^2=\left(2\sin\pi fT\right)^4\approx\left(2\pi fT\right)^4 \qquad (3)$$

The peak noise power is

$$\overline{S_Y}=\int_{-B}^{B}S_Q(f)\left|NTF(z)\right|^2_{z=e^{j2\pi fT}}df=\int_{-f_s/2M}^{f_s/2M}\frac{1}{f_s}\frac{\Delta^2}{12}\left(2\pi fT\right)^4 df=\frac{\Delta^2}{12}\frac{\pi^4}{5}\frac{1}{M^5} \qquad (4)$$

Hence,

$$SQNR=\frac{\overline{S_X}}{\overline{S_Y}}=\frac{\frac{1}{2}\left(\frac{\Delta}{2}\right)^2}{32\frac{\Delta^2}{12}\frac{\pi^4}{5}\frac{1}{M^5}}=\frac{15}{2\pi^4}M^5 \qquad (5)$$

We want

$$SQNR=100dB\Rightarrow 10^{10}=\frac{15}{64\pi^4}M^5\Rightarrow M=167 \qquad (6)$$

We choose the next higher power of 2:

$$M=256 \qquad (7)$$

Hence, the required sampling frequency is

$$f_s = Mf_N = M \times 2B = 256 \times 2 \times 20kHz = 10.24MHz \quad (8)$$

b The output spectrum and integrated noise plot is shown in Figure 1.

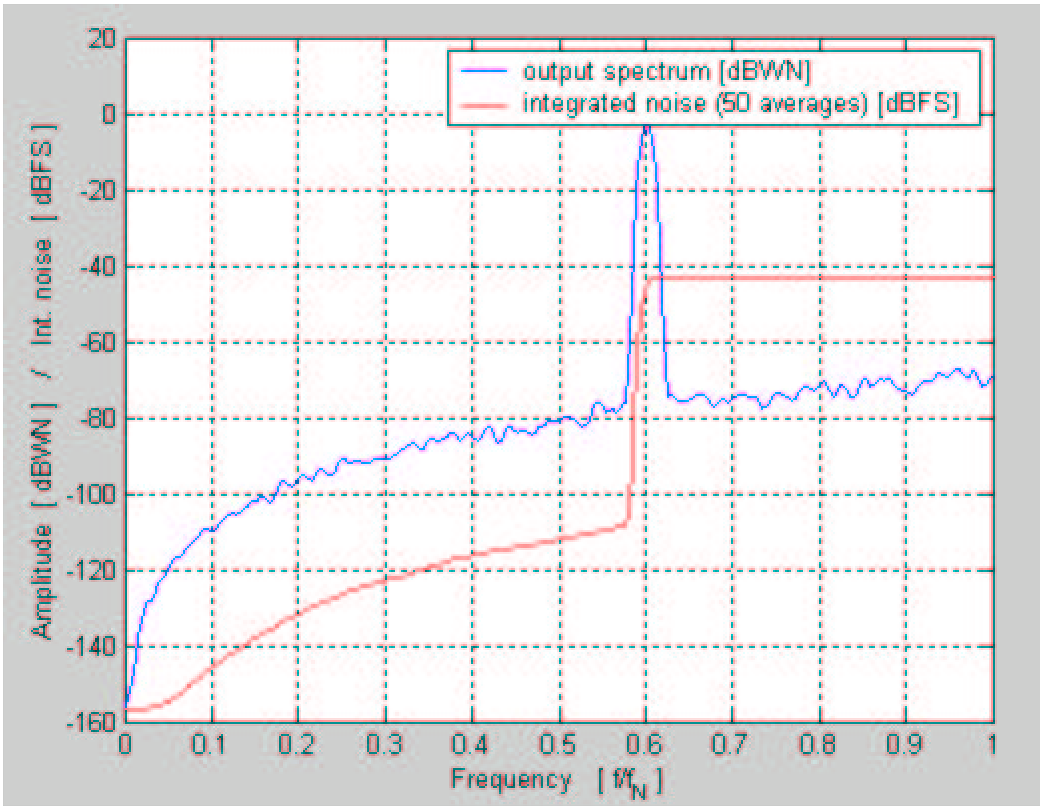

Figure 1. Output spectrum and integrated noise plot.

The dynamic range is approximately 111dB. Note that it is larger than the target value (100dB), since we overdesigned M. Substituting $M = 256$ in equation (6) yields
$SQNR = 109dB$
which is close to the observed value.

c The root locus is shown in Figure 2. The modulator is stable for $G_{eff} < 1.33$.

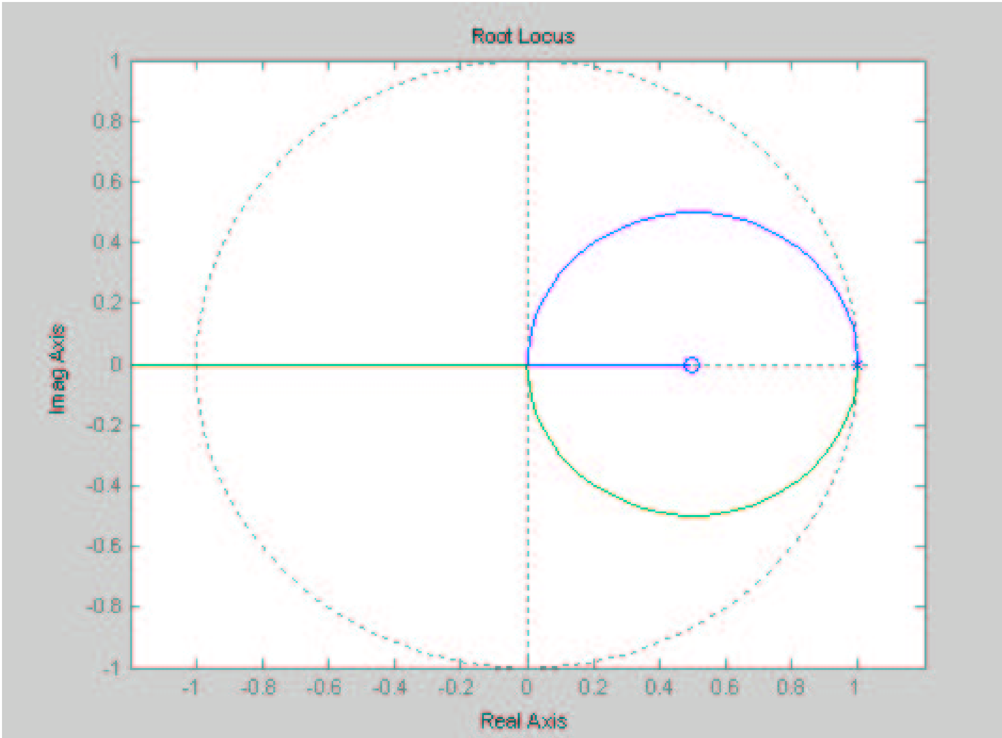

Figure 2. Root locus.

d In Figure 3, the effective gain is plotted as a function of input power. As it is shown, the modulator is stable for all the inputs.

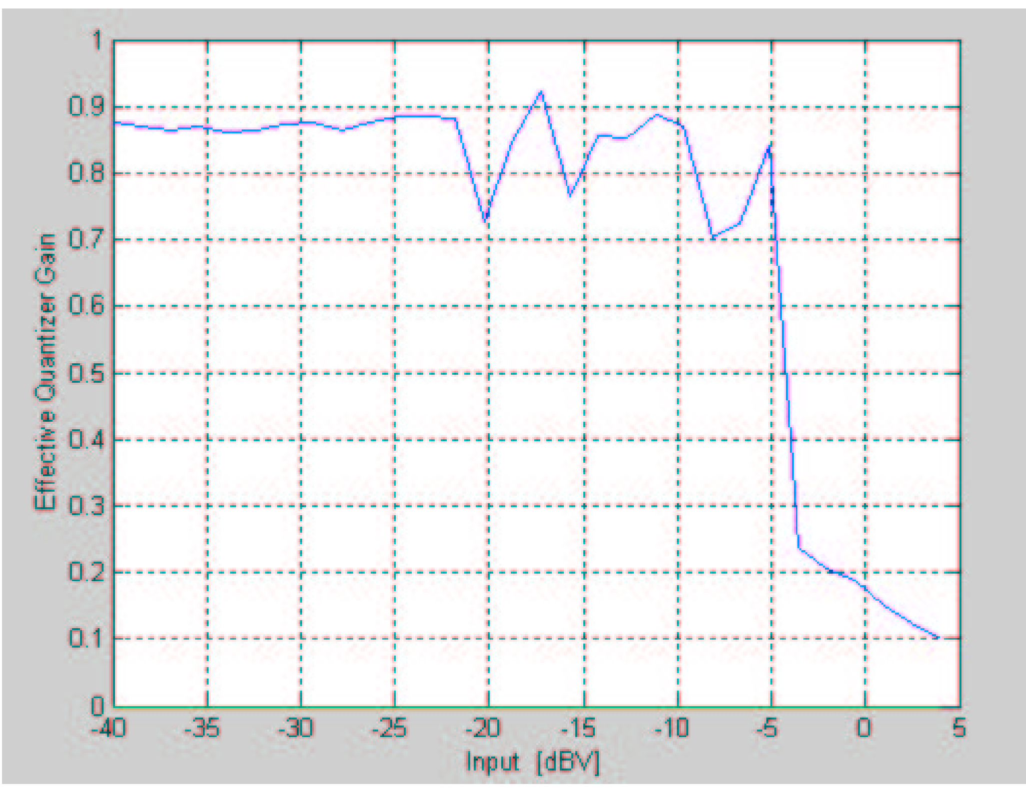

Figure 3. Effective gain vs input power.

e The modulator with the gain elements is shown in Figure 4.

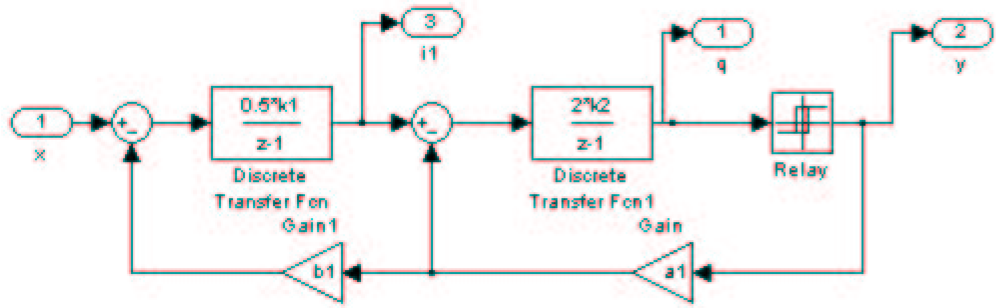

Figure 4. Modulator schematic.

The peak integrator outputs before scaling are shown in Figure 5.

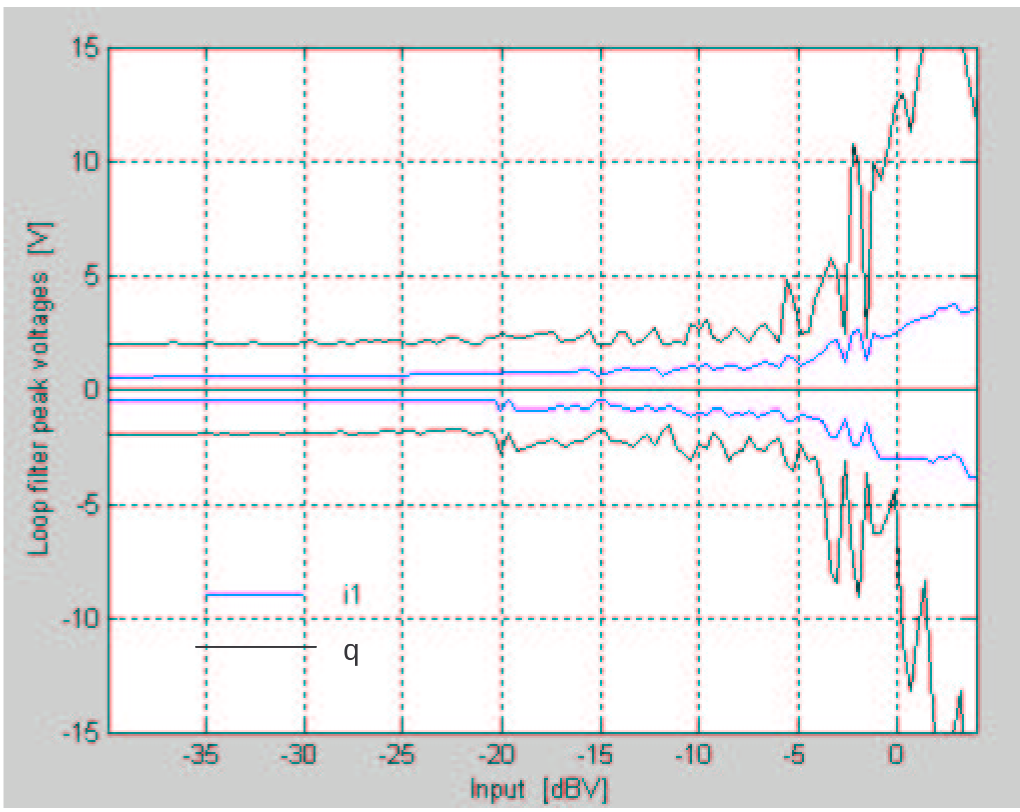

Figure 5. Integrator outputs before scaling.

From the plot, we determine $|i1|_{\max} = 2.5$ and $|q|_{\max} = 11$.

To limit them within the DAC outputs (± 1) for inputs up to $-2dB$, we scale the following factors:

$k_1 = 1/2.5 = 0.4$

$k_2 = 2.5/11 = 0.23$

$a_1 = 1/2.5 = 0.4$

$b_1 = 2.5$

The peak integrator outputs after scaling are shown in Figure 6.

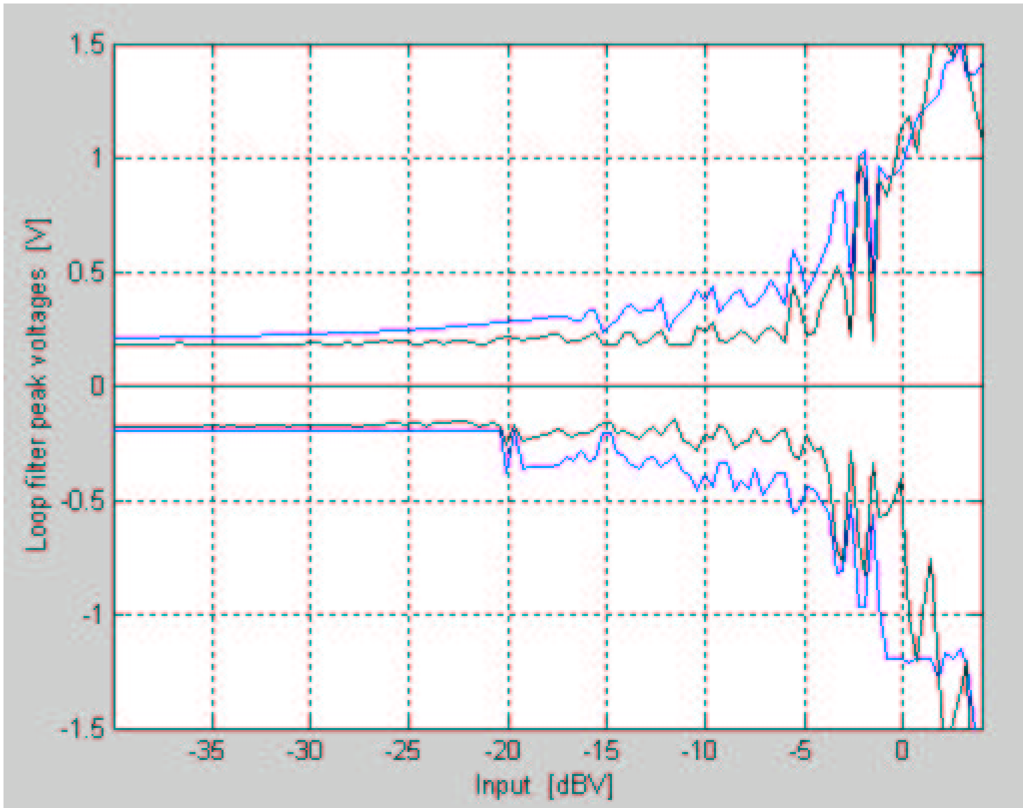

Figure 6. Integrator outputs after scaling.