

Higher Order Filter Options

- Cascade of Biquads
 - High-Q poles
 - High component sensitivity
- Ladders
 - Low sensitivity (Orchard)
 - Synthesize from LC prototypes
- Digital filters
 - Preferred solution when possible

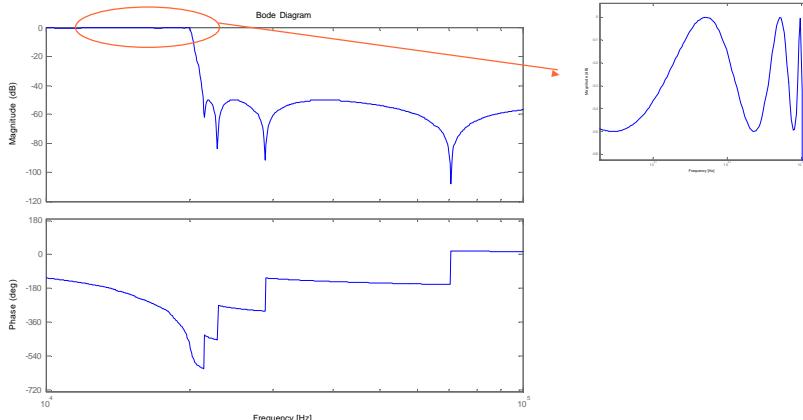


Cascade of Biquads

- LPF with
 - $f_{\text{pass}} = 20 \text{ kHz}$
 - $f_{\text{stop}} = 22.05 \text{ kHz}$
 - $r_{\text{pass}} = 0.5 \text{ dB}$
 - $r_{\text{stop}} = 50 \text{ dB}$
- 8th order Elliptic Filter
- Implementation with Biquads
Goal: maximize dynamic range
 - Pair poles and zeros
highest Q poles with closest zeros is a good starting point, but not necessarily optimum
 - Ordering
lowest Q poles is a good start



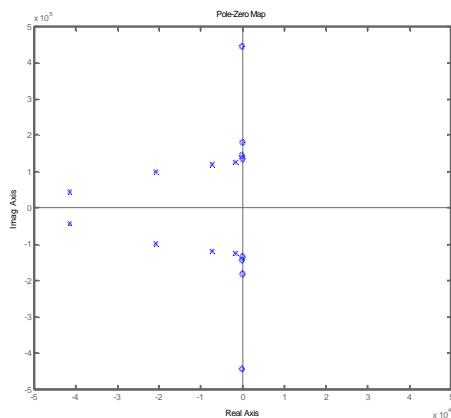
Filter Response



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Pole-Zero Map



Q_{pole}

38.4389
8.2903
2.4134
0.7130

f_{pole} [kHz]

20.0501
19.0959
16.0142
9.4282

f_{zero} [kHz]

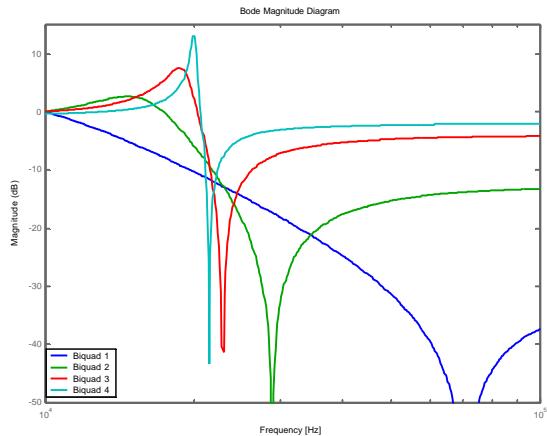
70.6923
28.7992
22.8585
21.4663



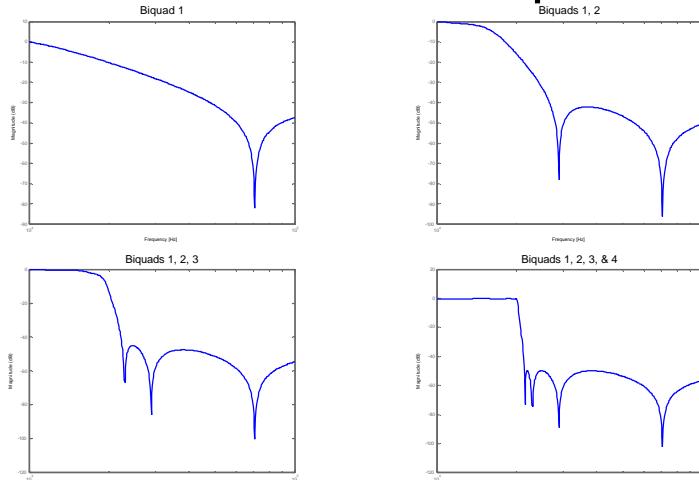
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Biquad Response



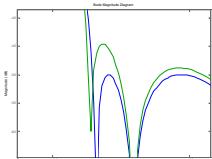
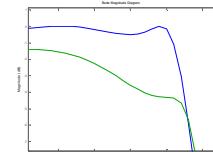
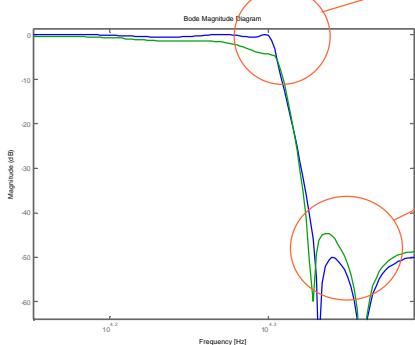
Intermediate Outputs



Sensitivity

Component variation in Biquad 4:

- Increase ω_p by 1%
- Decrease ω_z by 1%



High Q poles \rightarrow High sensitivity
in Biquad realizations



Ladder Filters

- Ladder example
 - Table
 - De-normalization
 - State variable synthesis
 - Transmission zeros
- Implementation
 - Tuning
 - Gm-C



LC Ladder Synthesis

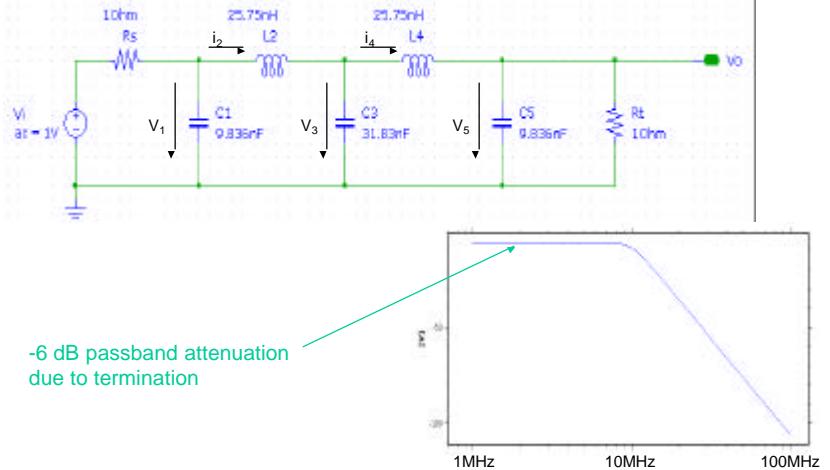
- CAD tool
- Filter table
 - A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.
 - R. Saal, *Handbook of filter synthesis*, AEG-Telefunken, 1979.
 - A. B. Williams and F. J. Taylor, *Electronic filter design*, 3rd edition, McGraw-Hill, 1995.
- Example:
 - $f_{\text{corner}} = 10\text{MHz}$, $f_{\text{stop}} = 20\text{MHz}$, $R_p = 2\text{dB}$, $R_s = 25\text{dB}$
 - 5th order Butterworth (from Matlab)



Filter Table

NORMALIZED FILTER DESIGN TABLES								11.2
TABLE H-2 Butterworth LC Filter Design Values (Continued)								
#	R_p	C_1	L_2	C_2	L_3	C_4	L_4	C_5
5	1.0000	0.4196	1.199	0.9995	1.8198	0.6180		
	0.9999	0.4196	1.0995	0.9995	1.7562	0.6185		
	0.9998	0.4196	0.9999	0.9995	1.6443	1.7386		
	0.9997	0.4196	0.8993	0.9995	1.5326	1.7185		
	0.9996	0.4196	0.7987	0.9995	1.4215	2.5894		
	0.9995	0.4196	0.6981	0.9995	1.3104	2.5991		
	0.9994	0.4196	0.5975	0.9997	1.1993	3.3734	5.9948	
	0.9993	0.4196	0.4969	0.9997	1.0882	0.7357	0.5267	5.3673
	0.9992	0.4196	0.3963	0.9997	0.9771	0.3833	0.2567	5.2673
	0.9991	0.4196	0.2957	0.9997	0.8660	0.1318	0.0918	5.1665
	0.9990	0.4196	0.1951	0.9997	0.7549	0.0512	0.0312	5.0655
	0.9989	0.4196	0.0945	0.9997	0.6438	0.0172	0.0112	4.9645
	0.9988	0.4196	0.0039	0.9997	0.5327	0.0052	0.0032	4.8635
	0.9987	0.4196	-0.0945	0.9997	0.4216	-0.0172	-0.0112	4.7625
	0.9986	0.4196	-0.1951	0.9997	0.3105	-0.0512	-0.0312	4.6615
	0.9985	0.4196	-0.2957	0.9997	0.2094	-0.1318	-0.0918	4.5605
	0.9984	0.4196	-0.3963	0.9997	0.1083	-0.3833	-0.2567	4.4595
	0.9983	0.4196	-0.4969	0.9997	0.0972	-0.7357	-0.5267	4.3585
	0.9982	0.4196	-0.5975	0.9997	0.0861	-0.1318	-0.0918	4.2575
	0.9981	0.4196	-0.6981	0.9997	0.0750	-0.0512	-0.0312	4.1565
	0.9980	0.4196	-0.7987	0.9997	0.0639	0.0172	0.0112	4.0555
	0.9979	0.4196	-0.8981	0.9997	0.0528	0.0052	0.0032	3.9545
	0.9978	0.4196	-0.9975	0.9997	0.0417	-0.0052	-0.0032	3.8535
	0.9977	0.4196	-1.0969	0.9997	0.0306	0.0012	0.0002	3.7525
	0.9976	0.4196	-1.1963	0.9997	0.0195	-0.0012	-0.0002	3.6515
	0.9975	0.4196	-1.2957	0.9997	0.0084	0.0002	0.0000	3.5505
	0.9974	0.4196	-1.3951	0.9997	0.0073	0.0000	0.0000	3.4495
	0.9973	0.4196	-1.4945	0.9997	0.0062	0.0000	0.0000	3.3485
	0.9972	0.4196	-1.5939	0.9997	0.0051	0.0000	0.0000	3.2475
	0.9971	0.4196	-1.6933	0.9997	0.0040	0.0000	0.0000	3.1465
	0.9970	0.4196	-1.7927	0.9997	0.0029	0.0000	0.0000	3.0455
	0.9969	0.4196	-1.8921	0.9997	0.0018	0.0000	0.0000	2.9445
	0.9968	0.4196	-1.9915	0.9997	0.0007	0.0000	0.0000	2.8435
	0.9967	0.4196	-2.0909	0.9997	0.0000	0.0000	0.0000	2.7425
	0.9966	0.4196	-2.1903	0.9997	0.0000	0.0000	0.0000	2.6415
	0.9965	0.4196	-2.2897	0.9997	0.0000	0.0000	0.0000	2.5405
	0.9964	0.4196	-2.3891	0.9997	0.0000	0.0000	0.0000	2.4395
	0.9963	0.4196	-2.4885	0.9997	0.0000	0.0000	0.0000	2.3385
	0.9962	0.4196	-2.5879	0.9997	0.0000	0.0000	0.0000	2.2375
	0.9961	0.4196	-2.6873	0.9997	0.0000	0.0000	0.0000	2.1365
	0.9960	0.4196	-2.7867	0.9997	0.0000	0.0000	0.0000	2.0355
	0.9959	0.4196	-2.8861	0.9997	0.0000	0.0000	0.0000	1.9345
	0.9958	0.4196	-2.9855	0.9997	0.0000	0.0000	0.0000	1.8335
	0.9957	0.4196	-3.0849	0.9997	0.0000	0.0000	0.0000	1.7325
	0.9956	0.4196	-3.1843	0.9997	0.0000	0.0000	0.0000	1.6315
	0.9955	0.4196	-3.2837	0.9997	0.0000	0.0000	0.0000	1.5305
	0.9954	0.4196	-3.3831	0.9997	0.0000	0.0000	0.0000	1.4295
	0.9953	0.4196	-3.4825	0.9997	0.0000	0.0000	0.0000	1.3285
	0.9952	0.4196	-3.5819	0.9997	0.0000	0.0000	0.0000	1.2275
	0.9951	0.4196	-3.6813	0.9997	0.0000	0.0000	0.0000	1.1265
	0.9950	0.4196	-3.7807	0.9997	0.0000	0.0000	0.0000	1.0255
	0.9949	0.4196	-3.8701	0.9997	0.0000	0.0000	0.0000	0.9245
	0.9948	0.4196	-3.9695	0.9997	0.0000	0.0000	0.0000	0.8235
	0.9947	0.4196	-4.0689	0.9997	0.0000	0.0000	0.0000	0.7225
	0.9946	0.4196	-4.1683	0.9997	0.0000	0.0000	0.0000	0.6215
	0.9945	0.4196	-4.2677	0.9997	0.0000	0.0000	0.0000	0.5205
	0.9944	0.4196	-4.3671	0.9997	0.0000	0.0000	0.0000	0.4195
	0.9943	0.4196	-4.4665	0.9997	0.0000	0.0000	0.0000	0.3185
	0.9942	0.4196	-4.5659	0.9997	0.0000	0.0000	0.0000	0.2175
	0.9941	0.4196	-4.6653	0.9997	0.0000	0.0000	0.0000	0.1165
	0.9940	0.4196	-4.7647	0.9997	0.0000	0.0000	0.0000	0.0155
	0.9939	0.4196	-4.8641	0.9997	0.0000	0.0000	0.0000	0.0095
	0.9938	0.4196	-4.9635	0.9997	0.0000	0.0000	0.0000	0.0035
	0.9937	0.4196	-5.0629	0.9997	0.0000	0.0000	0.0000	0.0005
	0.9936	0.4196	-5.1623	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9935	0.4196	-5.2617	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9934	0.4196	-5.3611	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9933	0.4196	-5.4605	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9932	0.4196	-5.5599	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9931	0.4196	-5.6593	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9930	0.4196	-5.7587	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9929	0.4196	-5.8581	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9928	0.4196	-5.9575	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9927	0.4196	-6.0569	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9926	0.4196	-6.1563	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9925	0.4196	-6.2557	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9924	0.4196	-6.3551	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9923	0.4196	-6.4545	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9922	0.4196	-6.5539	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9921	0.4196	-6.6533	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9920	0.4196	-6.7527	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9919	0.4196	-6.8521	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9918	0.4196	-6.9515	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9917	0.4196	-7.0509	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9916	0.4196	-7.1503	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9915	0.4196	-7.2597	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9914	0.4196	-7.3591	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9913	0.4196	-7.4585	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9912	0.4196	-7.5579	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9911	0.4196	-7.6573	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9910	0.4196	-7.7567	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9909	0.4196	-7.8561	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9908	0.4196	-7.9555	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9907	0.4196	-8.0549	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9906	0.4196	-8.1543	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9905	0.4196	-8.2537	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9904	0.4196	-8.3531	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9903	0.4196	-8.4525	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9902	0.4196	-8.5519	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9901	0.4196	-8.6513	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9900	0.4196	-8.7507	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9899	0.4196	-8.8501	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9898	0.4196	-8.9495	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9897	0.4196	-9.0489	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9896	0.4196	-9.1483	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9895	0.4196	-9.2477	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9894	0.4196	-9.3471	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9893	0.4196	-9.4465	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9892	0.4196	-9.5459	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9891	0.4196	-9.6453	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9890	0.4196	-9.7447	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9889	0.4196	-9.8441	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9888	0.4196	-9.9435	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9887	0.4196	-10.0429	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9886	0.4196	-10.1423	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9885	0.4196	-10.2417	0.9997	0.0000	0.0000	0.0000	0.0000
	0.9884	0.41						

SPICE Verification

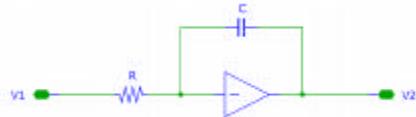


State Space Description

$$V_1 = \frac{1}{sC_1} \left[\frac{V_i - V_1}{R_s} - i_2 \right]$$

RC Integrator:

$$i_2 = \frac{1}{sL_2} [V_1 - V_3]$$



$$V_3 = \frac{1}{sC_3} [i_2 - i_4]$$

$$\frac{V_2}{V_1} = -\frac{1}{sRC}$$

$$i_4 = \frac{1}{sL_4} [V_3 - V_5]$$

$$V_5 = \frac{1}{sC_5} \left[i_4 - \frac{V_5}{R_L} \right]$$



Normalize

$$V_1 = \frac{1}{sC_1} \left[\frac{V_i - V_1}{R_s} - i_2 \right]$$

$$i_2 = \frac{1}{sL_2} [V_1 - V_3]$$

$$V_3 = \frac{1}{sC_3} [i_2 - i_4]$$

$$i_4 = \frac{1}{sL_4} [V_3 - V_5]$$

$$V_5 = \frac{1}{sC_5} \left[i_4 - \frac{V_5}{R_t} \right]$$

$$\begin{aligned} V_2 &= i_2 R^* \\ V_4 &= i_4 R^* \end{aligned}$$

$$V_1 = \frac{1}{sC_1} \left[\frac{V_i - V_1}{R_s} - \frac{V_2}{R^*} \right]$$

$$V_2 = \frac{R^*}{sL_2} [V_1 - V_3] = \frac{1}{sC_2 R^*} [V_1 - V_3]$$

$$V_3 = \frac{1}{sC_3 R^*} [V_2 - V_4]$$

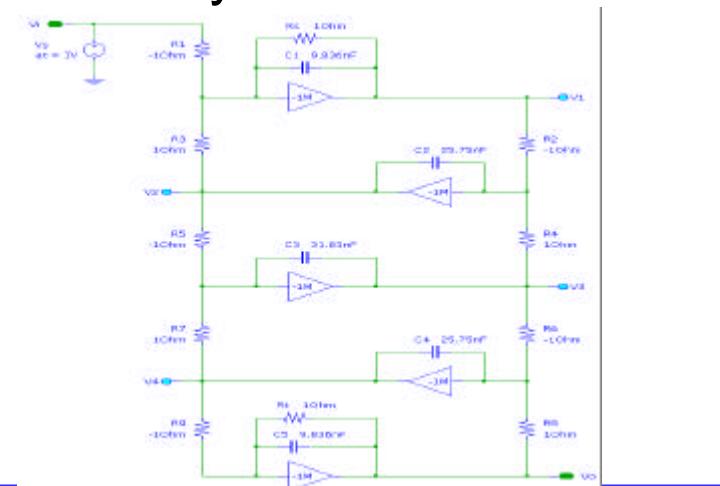
$$V_4 = \frac{R^*}{sL_4} [V_3 - V_5] = \frac{1}{sC_4 R^*} [V_3 - V_5]$$

$$V_5 = \frac{1}{sC_5} \left[\frac{V_4}{R^*} - \frac{V_5}{R_t} \right]$$

with $C_2 = \frac{L_2}{(R^*)^2}$ and $C_4 = \frac{L_4}{(R^*)^2}$

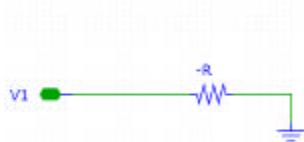


Synthesize

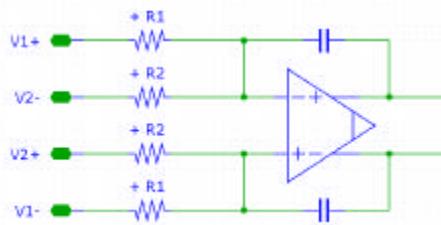
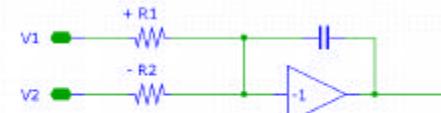


Negative Resistors

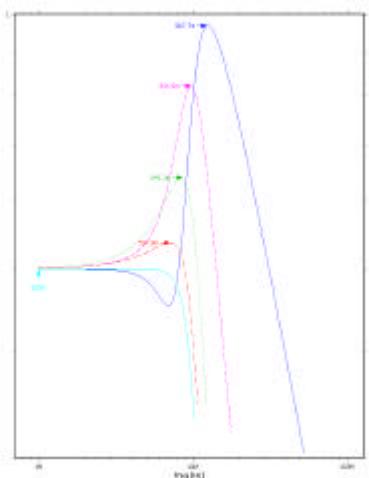
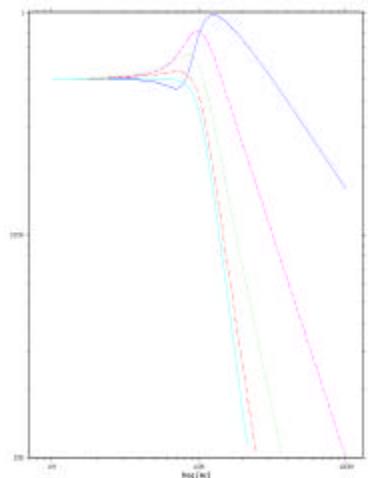
Single ended



Differential

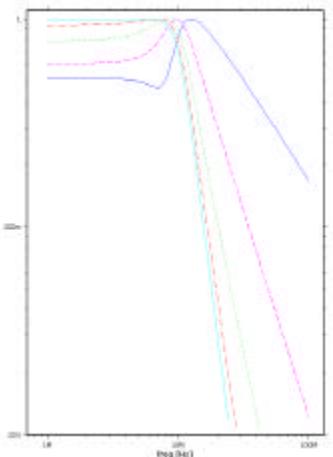
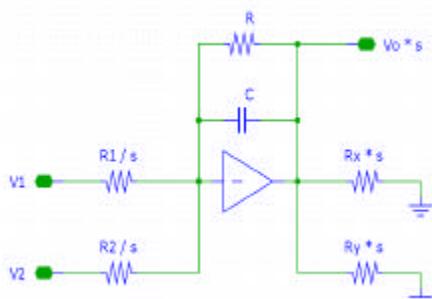


Frequency Response

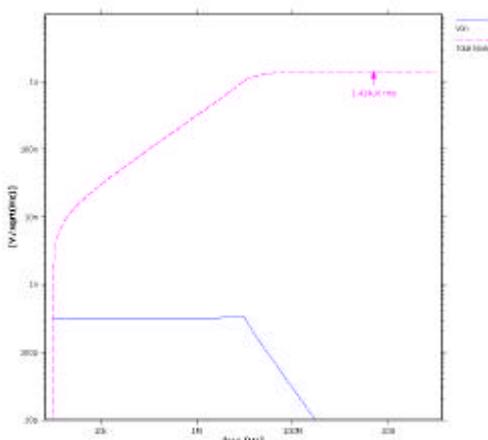


Scale Node Voltages

Scale V_o by factor "s"



Noise



Total noise: $1.4 \mu\text{V rms}$
(noiseless opamps)

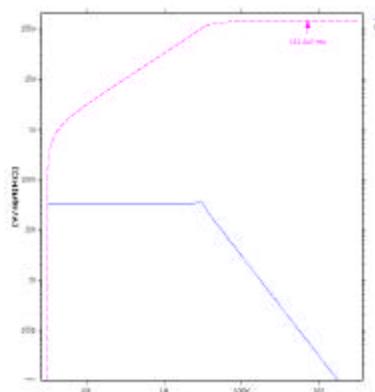
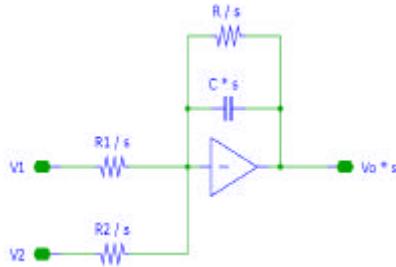
That's excellent, but the capacitors are very large (and the resistors small).

Suppose our application calls for $140 \mu\text{V rms}$...



Scale to Meet Noise Target

Scale capacitors and resistors
to meet noise objective

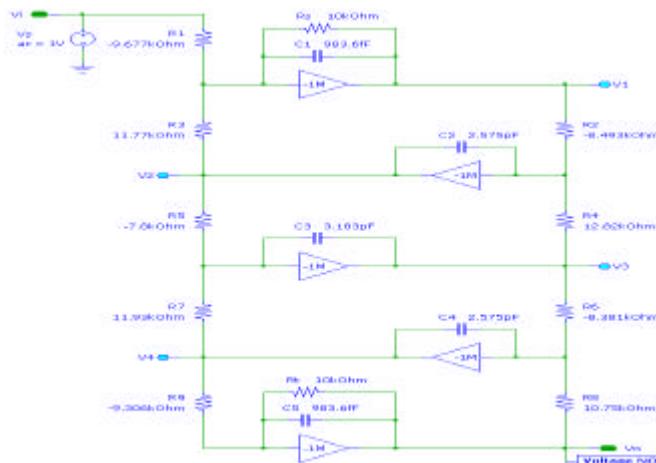


$$s = 10^{-4}$$

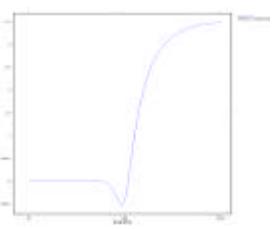
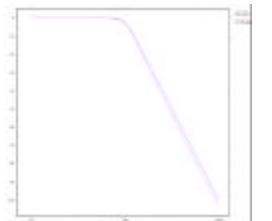
Noise: 141 μV rms (noiseless opamps)



Completed Design



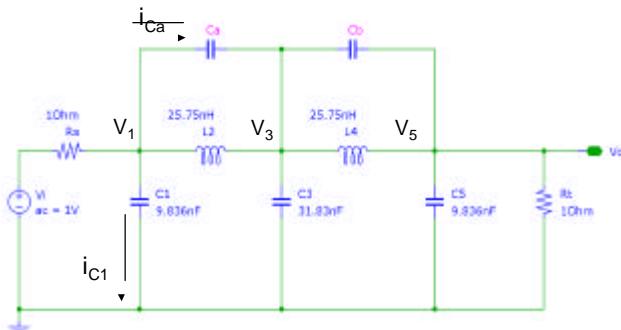
Sensitivity



- C_1 made (arbitrarily) 50% (!) larger than its nominal value
- 0.5 dB error at band edge
- 3.5 dB error in stopband
- Looks like very low sensitivity
- More analysis needed (Monte Carlo?)



Transmission Zeros



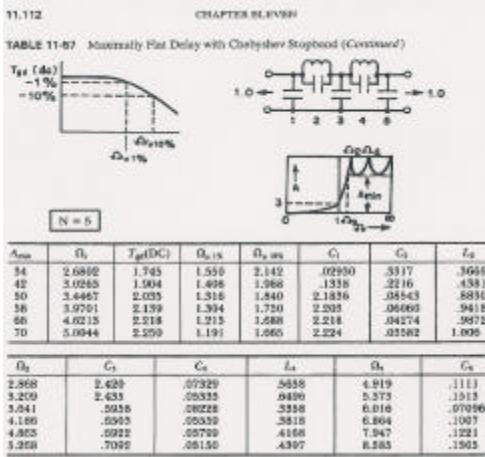
$$i_a = sC_a(V_1 - V_3)$$

$$i_{C1} = sC_1V_1$$

$$\rightarrow i_a + i_{C1} = s(C_a + C_1) \left[V_1 - V_3 \frac{C_a}{C_a + C_1} \right]$$



Filter Table



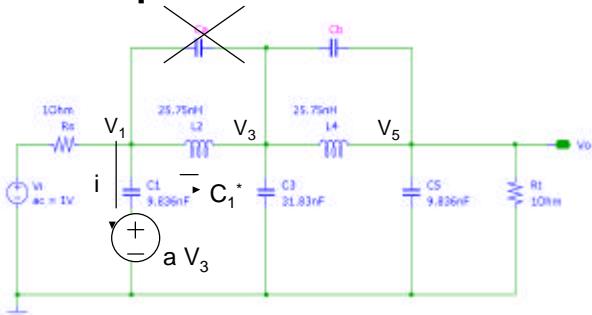
- 5th order Chebychev II
 - Williams & Taylor,
p. 11.112
 - 50dB stopband attenuation

A/D
DSP

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Equivalent Circuit



$$i = i_a + i_{C_1} = s(C_a + C_1) \left[V_1 - V_3 \frac{C_a}{C_a + C_1} \right] \\ \equiv sC_a^*(V_1 - aV_3)$$

$$\text{with : } C_1^* = C_a + C_1 \quad \text{and} \quad a = \frac{C_a}{C_a + C_1}$$

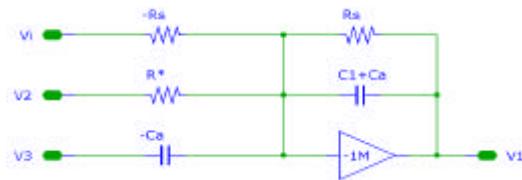
A/D
DSP

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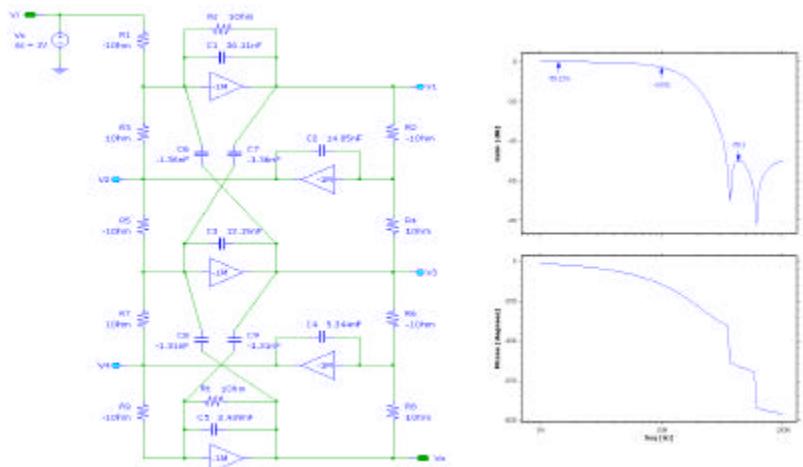
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Realization with Integrator

$$\begin{aligned}V_1 &= \frac{1}{s(C_a + C_1)} \left[\frac{V_i - V_1}{R_s} - i_{L2} \right] + aV_3 \\&= \frac{1}{s(C_a + C_1)} \left[\frac{V_i - V_1}{R_s} - i_{L2} \right] + \frac{C_a}{C_a + C_1} V_3 \\&= \frac{1}{s(C_a + C_1)} \left[\frac{V_i - V_1}{R_s} - \frac{V_2}{R^*} + sC_a V_3 \right]\end{aligned}$$



Active RC Simulation



Summary

Higher Order Filter Realization

- Cascade of Biquads
 - High sensitivity often problem for $N>4$
- Ladder Filters
 - Based on LC prototypes
 - Low sensitivity
 - Active RC simulation retains low sensitivity
 - Many implementation choices:
Active RC, Gm-C, MOSFET-C

