N247 HW3

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Shown in figure 2 below is the normalized prototype for a fourth order butterworth ladder filter.

- Denormalize the component values so as to achieve 20MHz bandwidth using $R_s = R_t = 1\Omega$. Subsequently, rescale the capacitors to obtain total integrated output noise=1mV.

- Notice that the normalized impedances of capacitors and inductors respectively have the form $Z_C = \frac{1}{\omega_0 R_N C_0}$; $Z_L = \frac{\omega_0 L_0}{R_N}$, with $\omega_0 = 1rad/s; R_N = 1\Omega$. To obtain the values corresponding to $R_S = 1$, $\omega_p = 2\pi 20MHz$, we need to use the values $C^1 = \frac{\omega_0 R_N}{\omega_p R_S} C_0$, $L^1 = \frac{\omega_0 R_N}{\omega_p R_S} L_0$. This gives the following values for the capacitors and the inductors: Now we need to perform noise scaling. The intuition behind noise scaling is that to increase(respectively decrease) the noise level in the circuit, we need to increase(decrease) the impedance level of the circuit, mantaining the same frequency responce. This is easy to see in a first order RC filter example, where the total integrated noise is KT/C, and the corner frequency is set by $\frac{1}{RC}$. In this case, a higher noise(smaller C) requires an increase in R to avoid broadening the bandwidth. In this case, the simulated noise of the filter is $.78\mu V$. To increase the noise preserving the filter shape, first we decrease all the capacitor sizes by a factor of 1.65e6. Subsequently, we increase all the inductor values by the same amount(this keeps all the LC products unchanged with respect to the original filter); finally, we increase all the resistors by the same values (this keeps all the L/R ratios and the RC products of the new filter the same as the old ones). The resulting filter components are listed in the table above and the resulting filter shape are shown in figure 2.

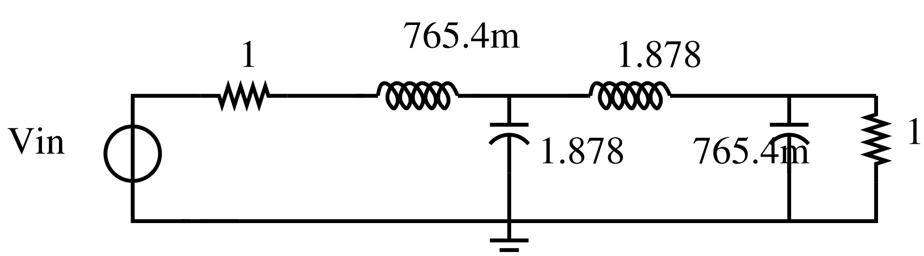

Fig. 1: LC Prototype for Fourth Order Butterworth Filter

$R_s 1$	1	$1.65M\Omega$
$L_1 765.4mH$	$6nH$	$10.055mH$
$C_1 1.878F$	$14.95nF$	$9fF$
$L_2 1.878$	$14.95nH$	$24mH$
$C_2 765.4m$	$6nF$	$3.63fF$
$R_L 1$	1	$1.65M\Omega$

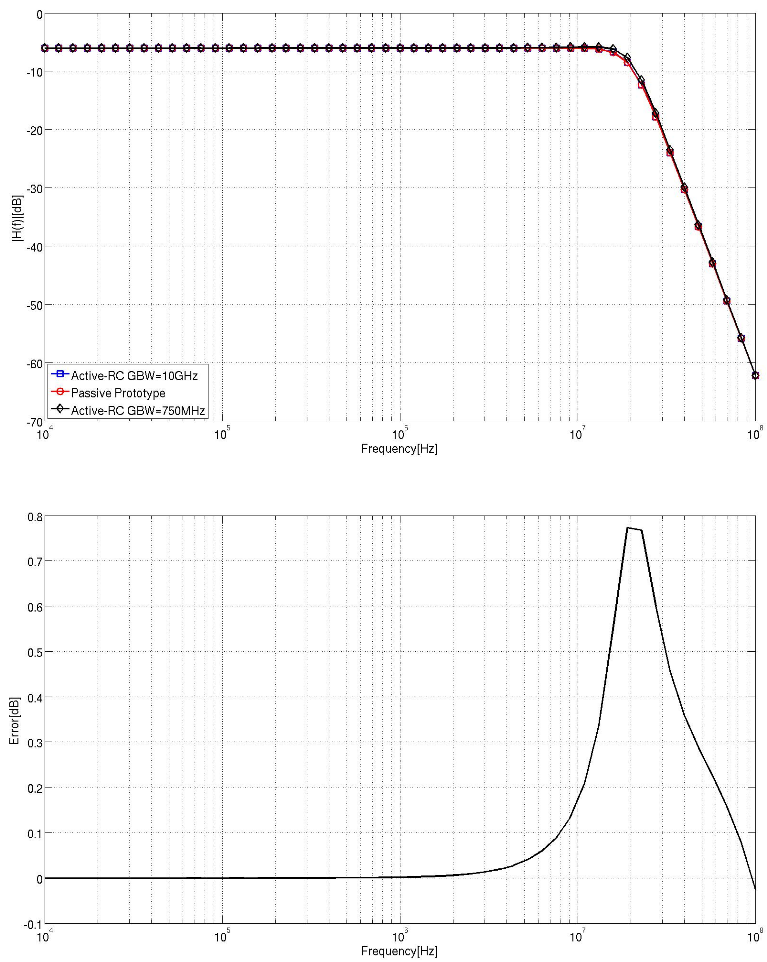

Fig. 2: Transfer funtion of discrete prototype and active-RC implementation.Error of the implementation with GBW=750MHz is also shown

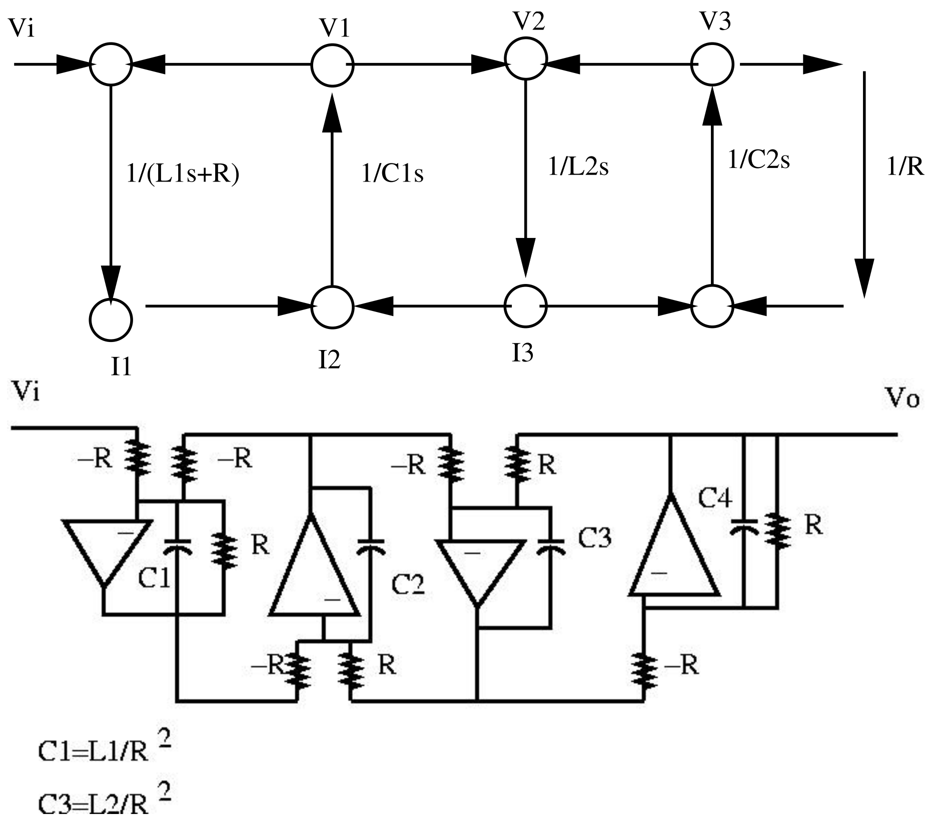

Fig. 3: Signal Flow Graph and leapfrog implementation

- Using the state-space approach described in lecture 6, realize an active-RC implementation of the filter using noiseless operational amplifiers with open loop gain=80dB and GBW=10GHz.
- The state variables of the circuit are the voltages across the capacitors and the currents in the inductors. The signal-flow graph of the filter is shown in figure 3. The leapfrog implementation is shown in figure 3
- Reduce the GBW until when you see a 1dB passaband error. What is the resulting GBW?
- Through simulation, operating on the noise-scaled filter, I found GBW=750MHz. (See Figure 2)
- Tranform the filter into the switched-capacitor counterpart using LDI tranformation for the integrators. For the switched capacitor circuit, choose $F_s = 160MHz$. Compare your periodic tranfer function(simulated using PSS+PAC) with the original continuous-time prototype.

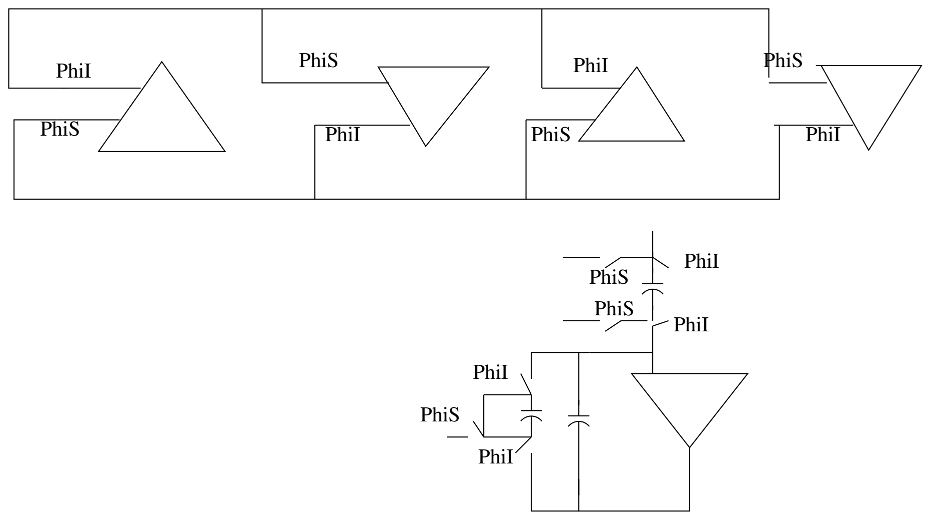

Fig. 4: Conceptual Representation of SC filter highlighting integrator phase relationships and complex-conjugate termination(shown is a non-delaying termination)

- To transform the filter into a switched capacitor implementation(See reference by Brodersen and Choi), replace all resistors by switched capacitors of value $C_R = \frac{1}{Rf_S}$. This will transform continuous time integrators into LDI discrete-time counterparts. Now, care has to be taken so that on average, every two integrator loop in the circuit is composed of a delaying integrator and a non-delaying integrator. This can be done by simply alternating the sampling and integrating phases of such integrators (See figure 4). For the termination resistors, this cannot be done and so we resort to the complex conjugate terminations described in Choi and Brodersen-i.e. one of the termination resistors is connected across the corresponding integrating capacitor during the integrating phase (non-delayed feedback), while the other one is connected during the sampling phase (delayed-feedback). This results into an effective averging of the phase errors that improves matching of the tranfer function to the original template. The transfer function is shown in figure 5 together with that of the continuous-time prototype.
- Now reduce the G_m until when your tranfer function shows a passband error of 1dB WHEN COMPARING TO THE RESULT OF YOUR SAMPLED DATA IMPLEMENTATION WITH $G_m = 100mS$. What is the value of G_m obtained?
- From simulation of the noise-scaled filter, I found $G_m \geq 50\mu S$.

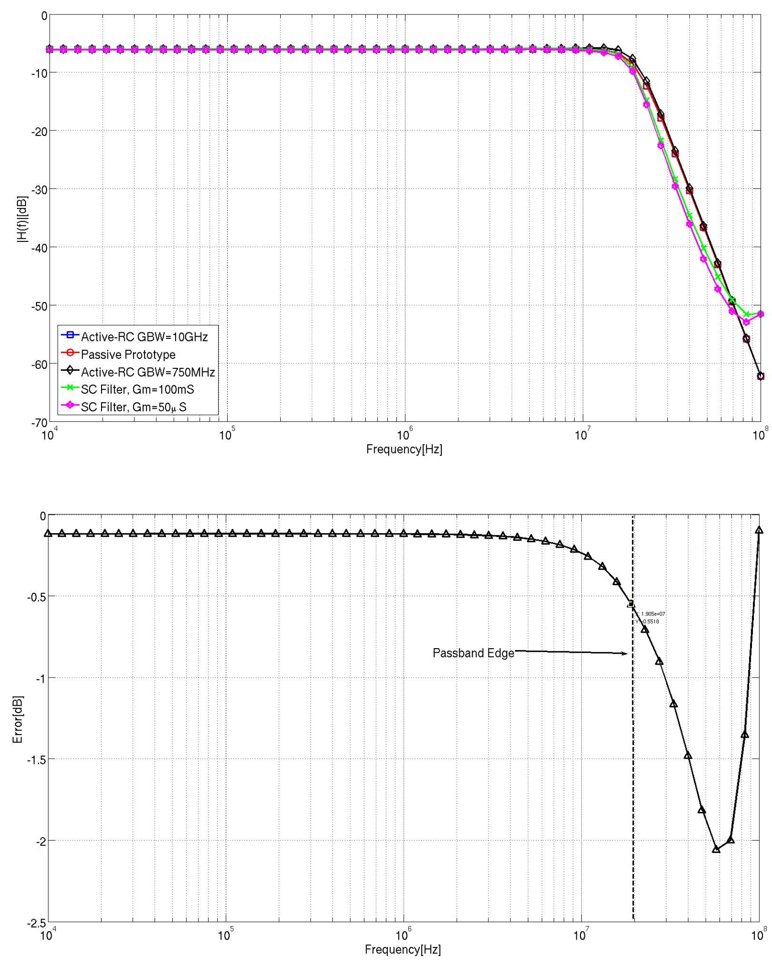

Fig. 5: Sc Filter tranfer function of the CT filter compared to SC implementation(above) and error between Response at Gm=100mS and at $Gm = 50\mu S$ for the noise-scaled filter.