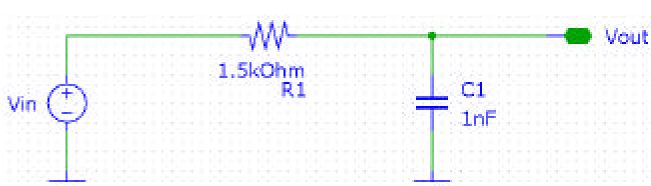
Introduction to Filters

- Filtering = frequency-selective signal processing
  - It's the most common type of signal processing
  - Examples:
    - Extract desired signal from many (radio)
    - Separating signal and noise
    - Amplifier bandwidth limitations

- Where to start
  - Perfectionist: ideal (low-pass) filter
  - Engineer: continuous time, first-order low-pass filter

First-Order RC Filter (LPF1)



Steady-state frequency response:

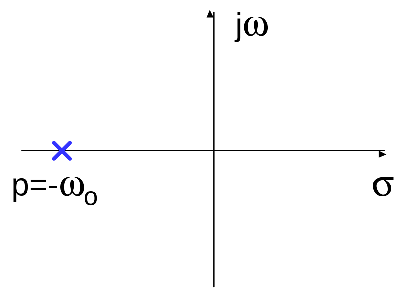
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + \frac{s}{w_o}}$$

with $$w_o = \frac{1}{RC} = 2p \times \underline{100kHz}$$

Poles and Zeros

$$H(s) = \frac{1}{1 + \frac{s}{w_o}}$$

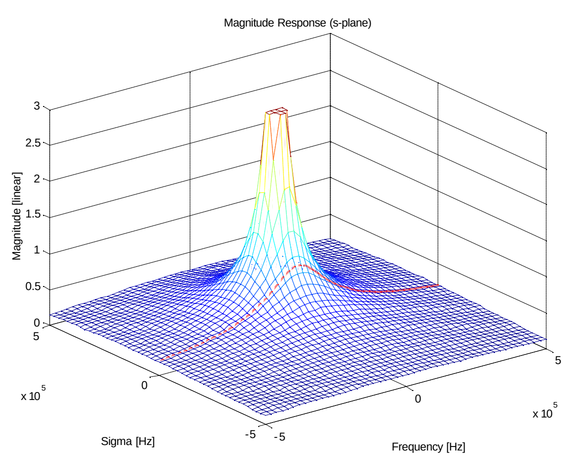
s-plane (pzmap):



$$\text{Pole}: \quad p = -w_o$$

$$\text{Zero}: \quad z \to \infty$$

Magnitude Response



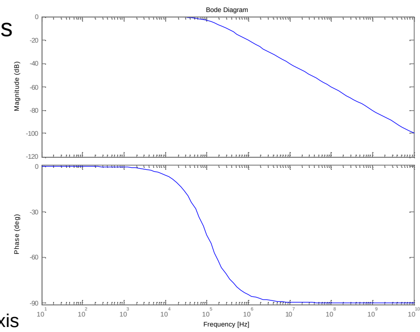
L02_bode3_lpf1.m

Frequency Response

Asymptotes:
- 20 dB/dec rolloff
- 90 degrees phase shift per 2 decades

$$\left|H(s=jw)\right|_{w=0}=1$$

$$\left|H(s=jw)\right|_{w\to\infty}=0$$



Matlab code (L02_bode_lpf1.m):

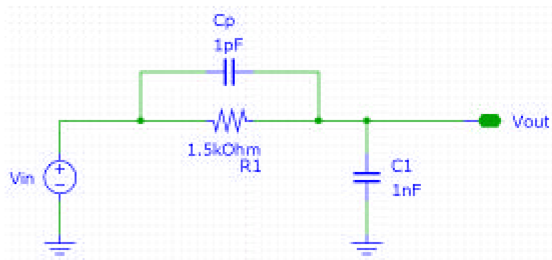
```
wo = 2*pi*100e3;
s = tf('s');
h = 1 / (1+s/wo);
bodehz(h, logspace(1, 10, 100));
```

Note: `bodehz` is same as `bode`, but frequency axis is in Hz, rather than rad/s.

Parasitics

Can we really get 100dB attenuation at 10GHz?
- Probably not
- Parasitics limit the performance of analog components
- E.g.
  - Shunt capacitance
  - Feed-through capacitance
  - Finite inductor, capacitor Q

LPF2



$$H(s) = \frac{1 + sRC_P}{1 + sR\left(C + C_P\right)}$$

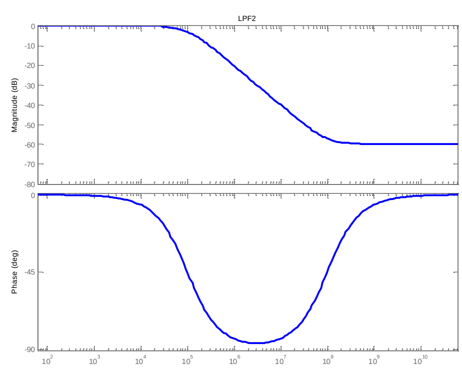
Pole : $p = -\frac{1}{R\left(C + C_P\right)} \approx -\frac{1}{RC}$

Zero : $z = -\frac{1}{RC_P}$

Frequency Response

$$\left|H(jw)\right|_{w=0} = 1$$

$$\left|H(jw)\right|_{w\to\infty} = \frac{C_P}{C + C_P} \approx \frac{C_P}{C} = 10^{-3} = -60dB$$



- Why not just make C larger?
- Beware of other parasitics not included in this model …

Continuous Time Analog

- Analog passive components aren't ideal
  - Extra real poles/zeroes result from parasitics
  - Parasitic effects begin to appear "50dB beyond" desired component characteristics
  - Common sense helps you anticipate them

- Digital filters do not suffer from these effects

Second-Order LPF

- Improved attenuation (compared to 1st order)
- Complex poles (rather than multiple real ones)
  - Why?
  - Visualize 3D s-plane plot!
- Biquadratic (2nd order) transfer function:

$$H(s) = \frac{1}{1 + \frac{s}{w_P Q_P} + \frac{s^2}{w_P^2}}$$

$$\left|H(jw)\right|_{w=0} = 1$$

$$\left|H(jw)\right|_{w\to\infty} = 0$$

$$\left|H(jw)\right|_{w=w_P} = Q_P$$

Biquad Poles

$$H(s) = \frac{1}{1 + \frac{s}{w_P Q_P} + \frac{s^2}{w_P^2}}$$

has poles at $$s = -\frac{w_P}{2Q_P}\left(1 \pm \sqrt{1 - 4Q_P^2}\right)$$

for $Q_P \leq \frac{1}{2}$ poles are real, complex otherwise

Complex Poles

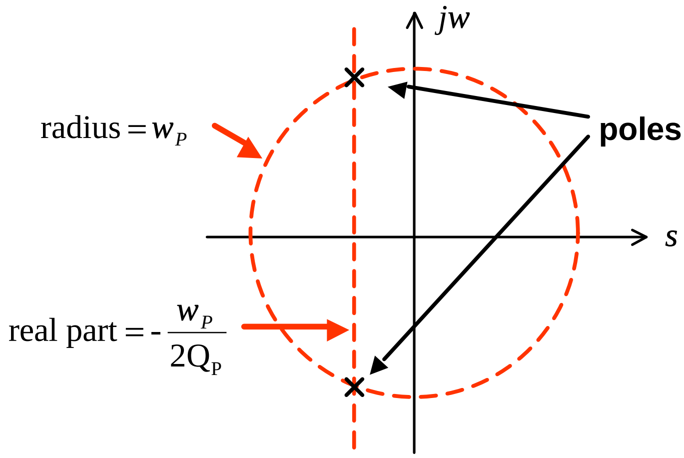
$$Q_P > \tfrac{1}{2}$$

$$s = -\frac{w_P}{2Q_P}\left(1 \pm j\sqrt{4Q_P^2 - 1}\right)$$

Distance from origin in s-plane:

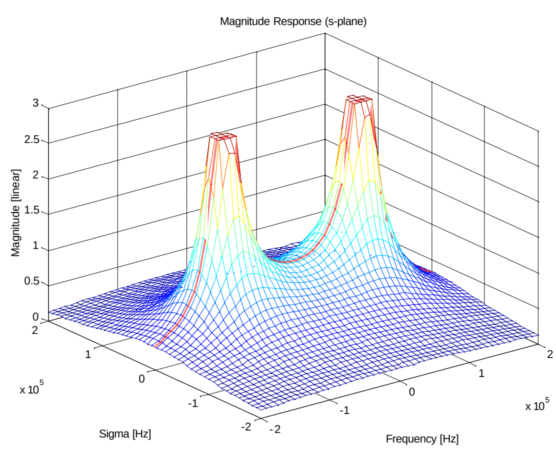
$$\begin{aligned} d^2 &= \left(\frac{w_P}{2Q_P}\right)^2 \left(1 + 4Q_P^2 - 1\right) \\ &= w_P^2 \end{aligned}$$

s-Plane

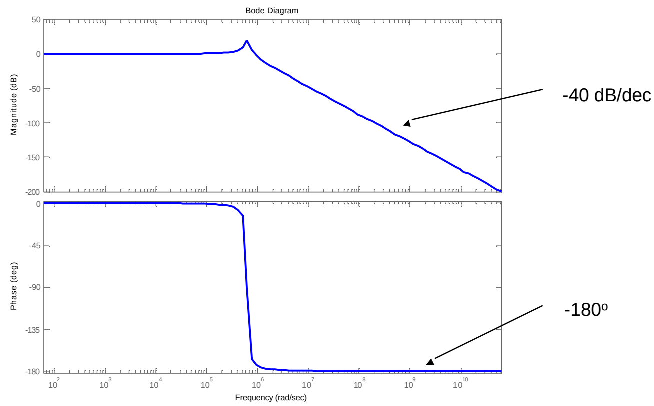


LPF3

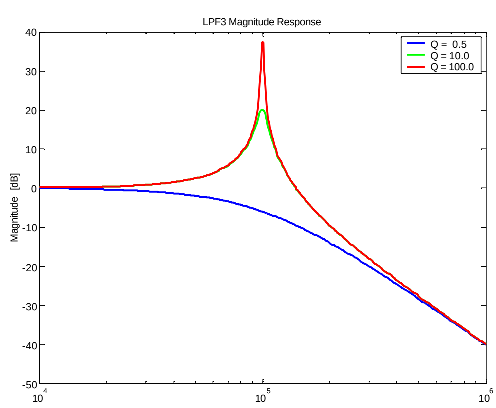
$$\omega_P = 2\pi \times 100kHz$$
$$Q_P = 10$$



Frequency Response



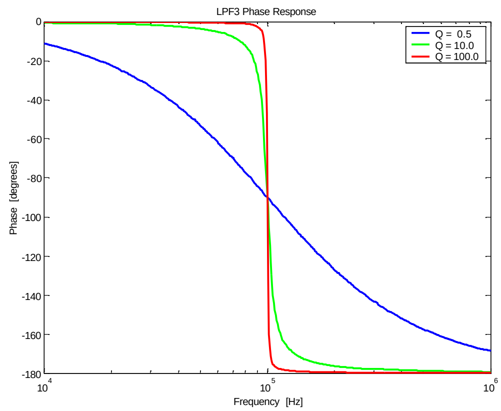
Varying Q … Magnitude



Gain at ω_p:

20 log Q [dB]

Phase



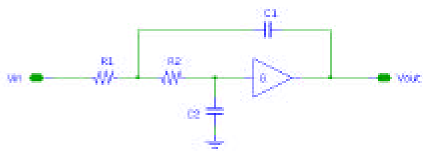
Slope at ω_p :

-45 Q deg/decade

Implementation of Biquads

- Passive RC: only real poles
- Terminated LC
  - "lowest power" (well … it's passive!)
  - No noise (except load and source)
- Active Biquad
  - Filter texts give you dozens of topologies. Who needs or wants that many choices?
  - Single-opamp biquad: Sallen-Key
  - Two-opamp biquad: Tow-Thomas

Sallen-Key LPF



$$H(s) = \frac{G}{1 + \frac{s}{w_P Q_P} + \frac{s^2}{w_P^2}}$$

$$w_P = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

$$Q_P = \frac{w_P}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}}$$

- Single gain element
- “Parasitic sensitive”
- Versions for LPF, HPF, BP, …

Ref: K. L. Su, “Analog Filters,” Chapman & Hall, 1996, pp. 215.

Component Sizing Choice 1

4 unknowns: R_1, R_2, C_1, C_2
2 knowns: ω_P, Q_P
→ problem is underdetermined

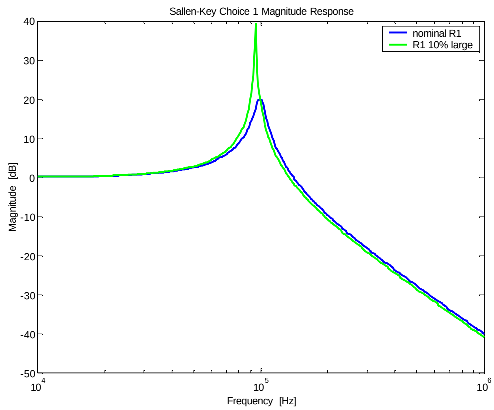
Choice 1: minimum component spread

$$C_1 = C_2 = 1nF$$

$$R_1 = R_2 = \frac{1}{w_P C_1} = 1.6k\Omega$$

$$G = 3 - \frac{1}{Q_P} = 2.9$$

SK Magnitude Response 1



10% increase of R_1 more than doubles Q_P!

→ The circuit is very sensitive to component variations.

Component Sizing Choice 2

Choice 2: minimum sensitivity

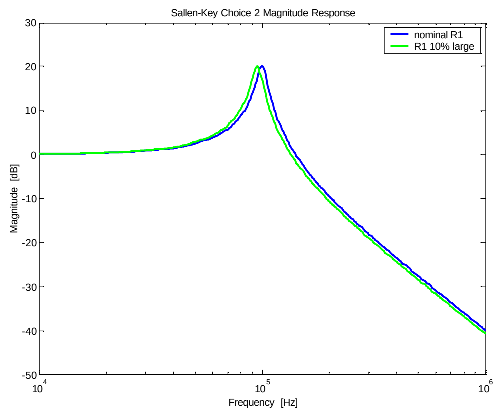
$$G = 1$$
$$R_1 = R_2 = 2k\Omega$$
$$C_1 = \frac{2Q_P}{w_P R_1} = 16nF$$
$$C_2 = \frac{1}{2Q_P w_P R_1} = 40pF$$

Note also:

$$\frac{C_1}{C_2} = 4Q_P^2 = 400$$

Huge element spread →
This topology is suitable only for low-Q filter implementations.

SK Magnitude Response 2



10% increase of R_1 has only small effect on response!

→ The circuit is NOT very sensitive to component variations.

Sensitivity

Definition $$\frac{\Delta y}{y} = S_x^y \frac{\Delta x}{x}$$

with $$S_x^y = \frac{x}{y}\frac{dy}{dx}$$

Example $$\frac{\Delta Q_P}{Q_P} = S_{R_1}^{Q_P} \frac{\Delta R_1}{R_1}$$

Choice 1 $$S_{R_1}^{Q_P} = Q_P - 0.5 = 9.5$$

$$\frac{\Delta Q_P}{Q_P} \approx 9.5 \frac{\Delta R_1}{R_1} = 95\%$$

Choice 2 $$S_{R_1}^{Q_P} = 0$$

- Implementation and component sizing have huge impact on sensitivity
- High-sensitivity circuits are problems in practice
- No theory for finding a low-sensitivity architecture
- Ladder filters are usually low sensitivity
- Use proven circuits & check!

Common sense: Sensitivity is a first order approximation, correct only for infinitesimally small errors

Summary

- Frequency Response
  - Poles and zeros are like tent poles and pegs
  - Frequency response is evaluated on $j\omega$ axis
  - Poles and zeros close to $j\omega$ axis dominate resonse
- Practical Implementation Constraints
  - Components are not ideal
  - Avoid solutions requiring large element spread
  - Beware of high-sensitivity architectures