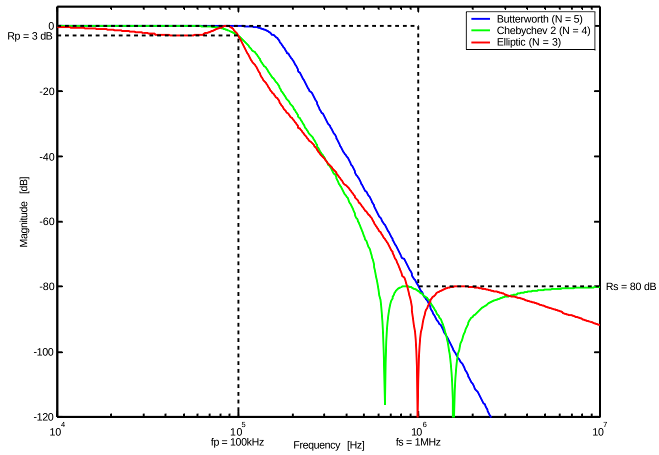
Filter Synthesis

- Analog filter synthesis
  - IIR filters
  - LP, HP, BP, BS
  - Magnitude response templates
  - Filter prototypes
  - Synthesis with biquads
- Phase response
  - Group delay
  - Step response
  - All-pass filters

Filter Approximation

- Objective: Magnitude specification → H(s)
- Classic IIR Filters
  - **Chebychev 2**
    Uses zeros for good attenuation but has no passband ripples. A good all-round combination.
  - **Butterworth**
    Poles only and no passband ripple. Less ringing in step response than Chebychev 2.
  - **Elliptic**
    Passband and stopband ripples. Use this when only magnitude response counts. Large overshoot in step response. High Q poles result in high sensitivity.
  - **Bessel**
    Great step response at the cost of poor attenuation.

Design Example



Matlab Code

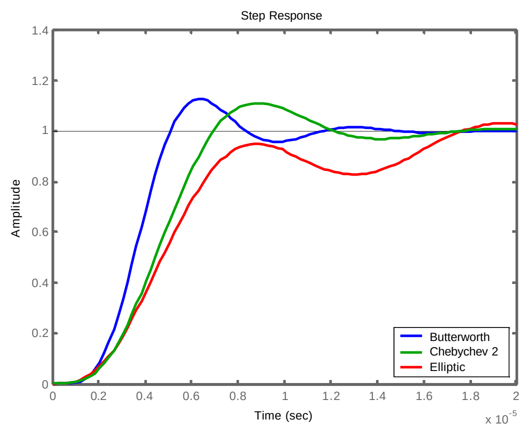
```
fp = 1.0e5;       wp = 2*pi*fp;
fs = 1.0e6;       ws = 2*pi*fs;
rp =  3;
rs = 80;

% filter prototype
[Nb, Wb] = buttord(wp, ws, rp, rs, 's');
[b, a] = butter(Nb, Wb, 's');
Hb = tf(b, a);

[Nc, Wc] = cheb2ord(wp, ws, rp, rs, 's');
[b, a] = cheby2(Nc, rs, Wc, 's');
Hc = tf(b, a);

[Ne, We] = ellipord(wp, ws, rp, rs, 's');
[b, a] = ellip(Ne, rp, rs, We, 's');
He = tf(b, a);
```

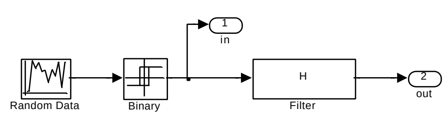
Step response

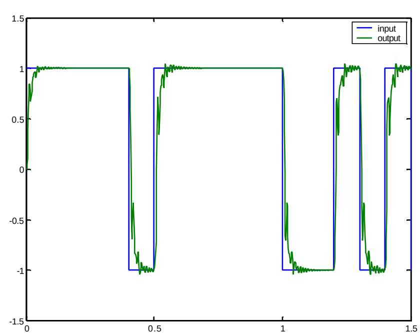


- Slow rise time
- Overshoot

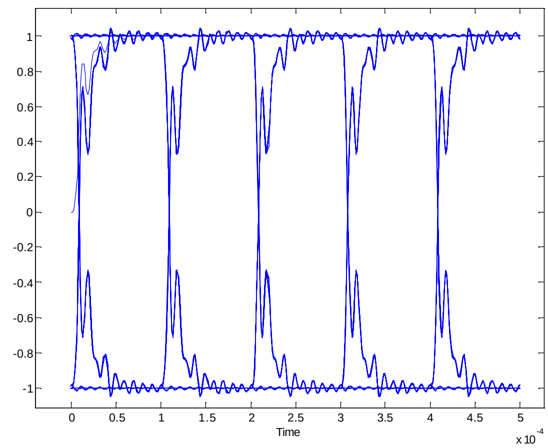
Acceptable in some applications, problematic in others.

Data Transmission

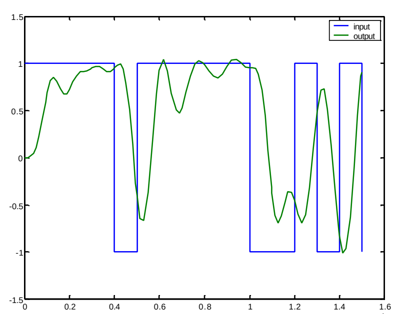


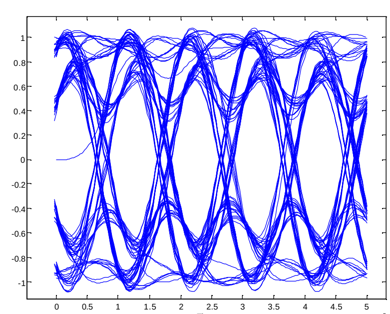


Eye Diagram



Increased Data Rate





Group Delay

- Nonuniform group delay is one source of eye closure
- Group delay?

Amplitude and Phase Distortion

- Consider a continuous time filter with s-domain transfer function G(s):

$$G(j\omega) \equiv |G(j\omega)| e^{j\theta(\omega)}$$

- The filter input is the sum of two sinewaves at slightly different frequencies ($\Delta\omega << \omega$):

$$v_{IN}(t) = A_1 \sin(\omega t) + A_2 \sin[(\omega + \Delta\omega) t]$$

Amplitude and Phase Distortion

- The filter output is:

$$v_{OUT}(t) = A_1 ½G(jw)½ \sin[wt+q(w)] + A_2 ½G[j(w+Dw)]½ \sin[(w+Dw)t+ q(w+Dw)]$$

$$= A_1 ½G(jw)½ \sin\left\{w\left[t+\frac{q(w)}{w}\right]\right\} +$$

$$+ A_2 ½G[j(w+Dw)]½ \sin\left\{(w+Dw)\left[t+\frac{q(w+Dw)}{w+Dw}\right]\right\}$$

Amplitude and Phase Distortion

$$v_{OUT}(t) = A_1 ½G(jw)½ \sin\left\{w\left[t+\frac{q(w)}{w}\right]\right\} +$$

$$+ A_2 ½G[j(w+Dw)]½ \sin\left\{(w+Dw)\left[t+\frac{q(w+Dw)}{w+Dw}\right]\right\}$$

$$\frac{q(w+Dw)}{w+Dw} @ \left[q(w)+\frac{dq(w)}{dw}Dw\right]\left[\frac{1}{w}\left(1-\frac{Dw}{w}\right)\right]$$

$$@ \frac{q(w)}{w} + \left(\frac{dq(w)}{dw} - \frac{q(w)}{w}\right)\frac{Dw}{w}$$

Amplitude and Phase Distortion

$$\frac{\theta(\omega+\Delta\omega)}{\omega+\Delta\omega} \cong \frac{\theta(\omega)}{\omega} + \left(\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega}\right)\frac{\Delta\omega}{\omega}$$

- If the second term in this equation is non-zero, then the filter's output at frequency $\omega+\Delta\omega$ is time-shifted differently than the filter's output at frequency ω
  → "Phase distortion"

Amplitude and Phase Distortion

$$\frac{\theta(\omega+\Delta\omega)}{\omega+\Delta\omega} \cong \frac{\theta(\omega)}{\omega} + \left(\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega}\right)\frac{\Delta\omega}{\omega}$$

- If the second term in this equation is zero, then the filter's output at frequency $\omega+\Delta\omega$ and the output at frequency ω are each delayed in time by $-\theta(\omega)/\omega$
- $\tau_{PD} \equiv -\theta(\omega)/\omega$ is called the "phase delay" and has units of time

Amplitude and Phase Distortion

- Since $\Delta\omega\neq 0$, phase distortion is avoided only if:

$$\frac{dq(w)}{dw} - \frac{q(w)}{w} = 0$$

- Clearly, if $\theta(\omega)=k\omega$, k a constant, we avoid phase distortion
- This type of filter phase response is called “linear phase”
  - Phase shift varies linearly with frequency

Amplitude and Phase Distortion

- $\tau_{GR} \equiv -d\theta(\omega)/d\omega = k$ is called the “group delay” and also has units of time
- $\tau_{GR} = \tau_{PD}$ implies linear phase
- Note: Filters with $\theta(\omega)=k\omega+c$ are also called linear phase filters, but they’re not free of phase distortion

Amplitude and Phase Distortion

- If $\tau_{GR} = \tau_{PD}$,

$$v_{OUT}(t) = A_1 ½G(jw)½ \sin\left[w\left(t - t_{GR}\right)\right] +$$

$$+ A_2 ½G[j(w+Dw)]½ \sin\left[(w+Dw)\left(t - t_{GR}\right)\right]$$

- If $|G(j\omega)| = |G[j(\omega+\Delta\omega)]|$ for all inputs within the signal-band, v_{OUT} is a scaled, time-shifted replica of the input, with no “amplitude distortion”

Amplitude and Phase Distortion

- No amplitude distortion:

$$|G(j\omega)| = |G[j(\omega+\Delta\omega)]|$$

- No phase distortion:

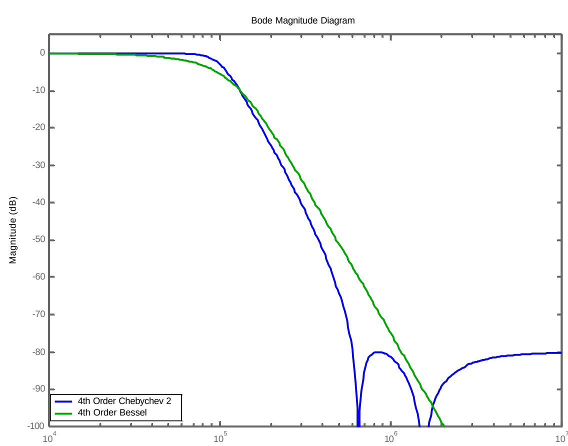
$$\tau_{GR} = \tau_{PD}$$

- Neither of these conditions are realizable exactly in continuous time filters
  - Real passbands can’t have flat amplitude responses
  - Derivatives of sums of arctangents aren’t constant

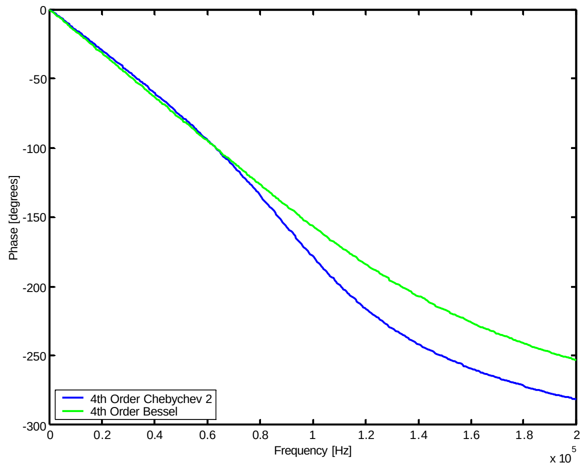
Group Delay Comparison

- 100kHz corner frequency
- Chebychev II versus Bessel

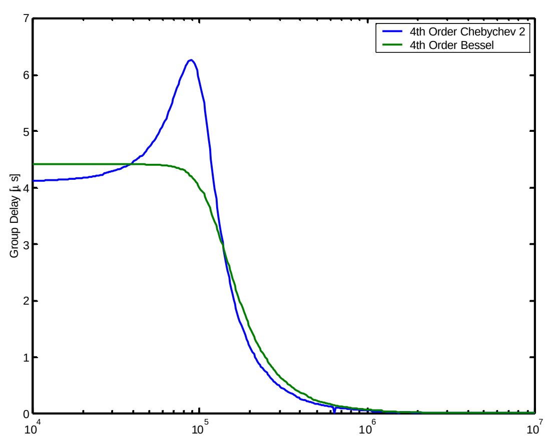
Magnitude Response



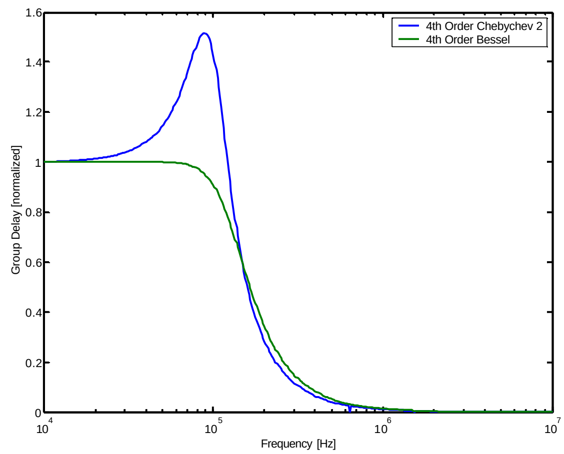
Phase Response



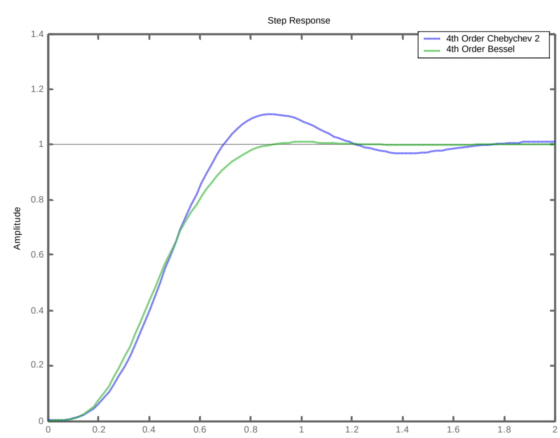
Group Delay



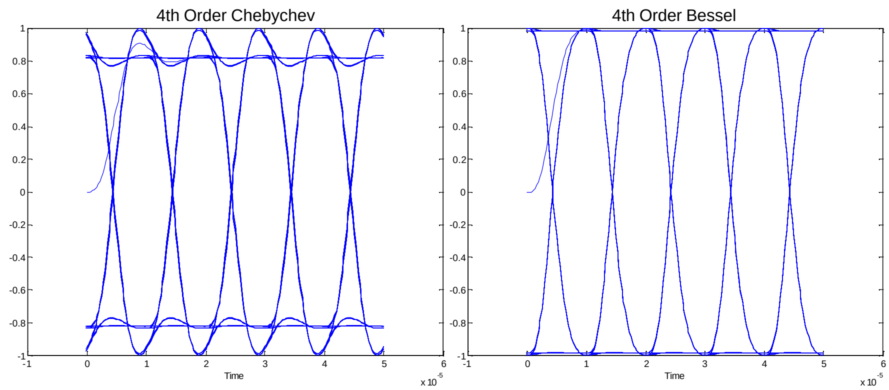
Normalized Group Delay



Step Response



Eye Diagrams

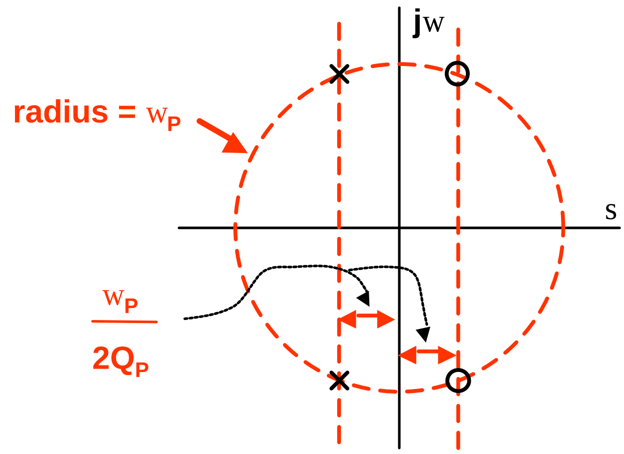


Note: the “Chebychev” output should be scaled to the same peak amplitude as the output from the Bessel. The Bessel has a clear advantage.

Allpass Filters

- Delay equalization
  - Unity magnitude response
  - “arbitrary” delay
- Can compensate arbitrary phase distortion (e.g. from a “given” channel)

Second-Order All Pass Filter



Second-Order All Pass Filter

- From graphical considerations alone, the second-order all pass filter has unity magnitude response at all frequencies
  - Its phase shift is twice that of its pole
- All pass sections can't cancel out the delay of other filter sections,
  - But they can add strategic delay to improve the phase response of a companion filter

All Pass Filters

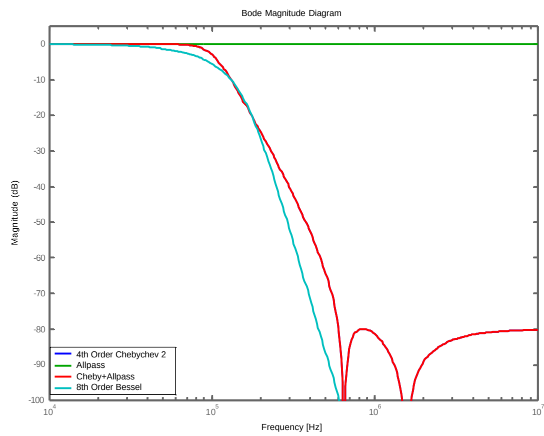
- An all pass filter used to make a companion filter's group delay more constant in the filter passband is called a "phase equalizer" or "delay equalizer"

- Group-delay-critical applications frequently devote as many poles to phase response "reshaping" as they devote to magnitude response shaping

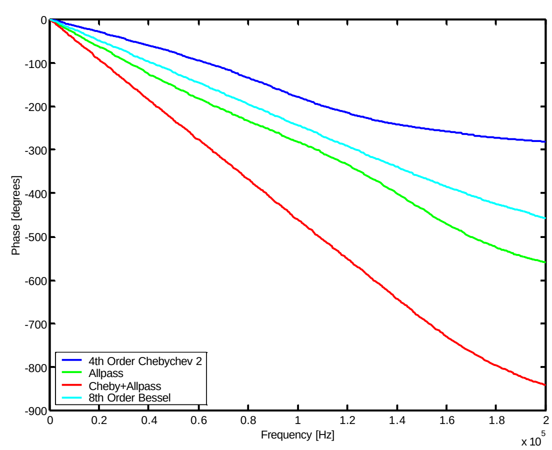
All Pass Filters

- Phase equalizers are commonly used in digital data receivers
- If you're not sure whether or not you need a phase equalizer, build one
  - Start with just as many poles as your magnitude shaper
  - CAD tools like MATLAB's iirgrpdelay help synthesize optimal phase equalizers
  - Unfortunately iirgrpdelay is for sampled data filters only → use the bilinear transform (see later)

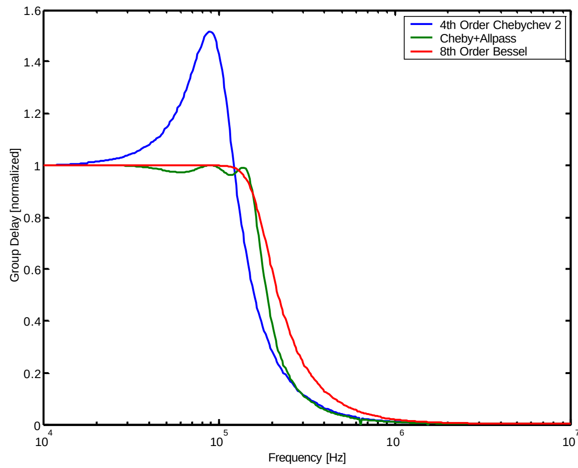
Magnitude Response



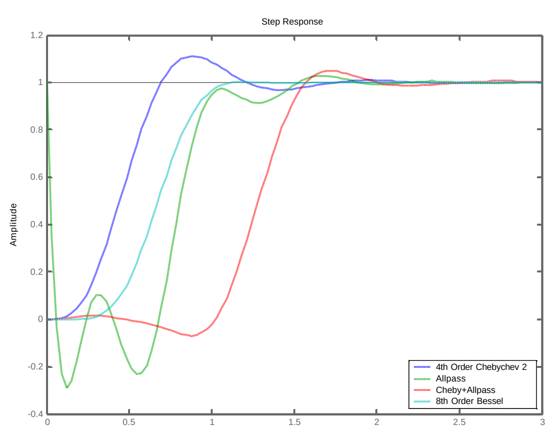
Phase Response



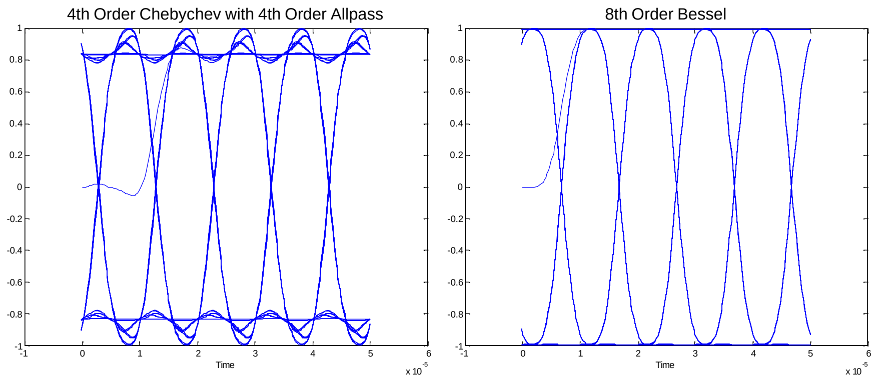
Normalized Group Delay



Step Response



Eye Diagrams



FIR Filter Phase Response

- A significant advantage of FIR over IIR filters is that FIR filters *can* be realized with linear phase response
- But (with few exceptions) FIR filters are only practical in the digital domain

Summary

Filter design:

- Magnitude response
  - Template
  - Approximation (matlab) → H(s)
  - Realization with
    - Cascades of biquads
      Sensitivity limits practical designs to <3 sections
    - Ladder filters → next lecture
- Phase / group delay
  - Phase equalization (allpass)