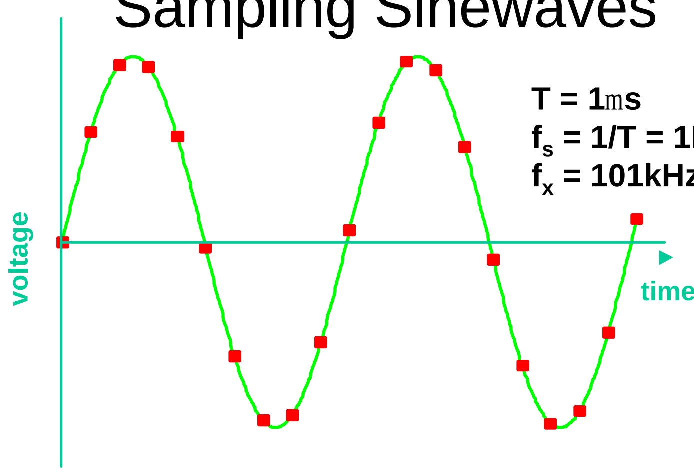
Sampling and Aliasing

- Any continuous time signal can be sampled and processed in the sampled-data domain
  - A/D converters "look" at signals at only discrete points in time
  - Computer models of continuous systems must operate at discrete time intervals
  - Even "analog" filters can use <u>continuous</u> *or* <u>sampled</u> time signal representations
- Multiple continuous time signals can yield exactly the same sampled data signal → aliasing
  - Let's look at samples taken at 1μsec intervals of several sinusoidal waveforms …

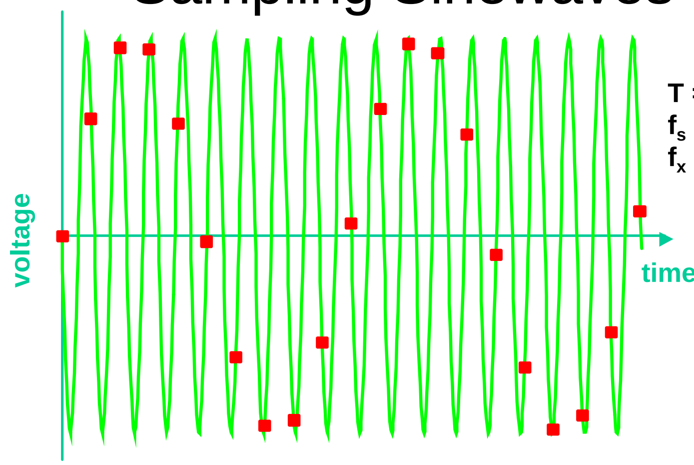
Sampling Sinewaves



T = 1μs
f_s = 1/T = 1MHz
f_x = 101kHz

$v(t) = \sin[2\pi(101000)t]$

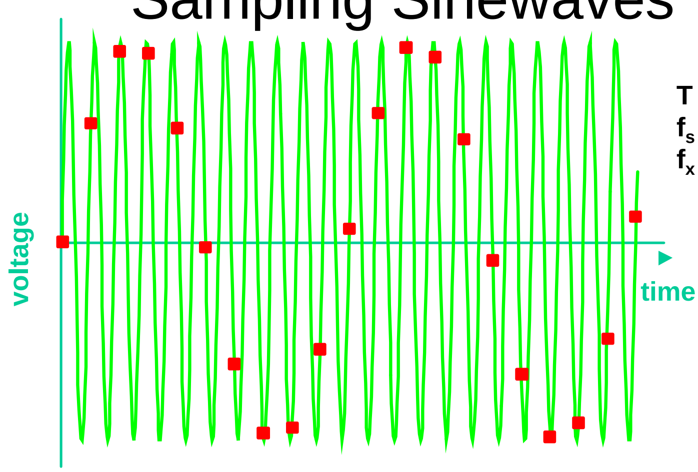
Sampling Sinewaves



T = 1ms
f_s = 1MHz
f_x = 899kHz

v(t) = - sin [2p(899000)t]

Sampling Sinewaves



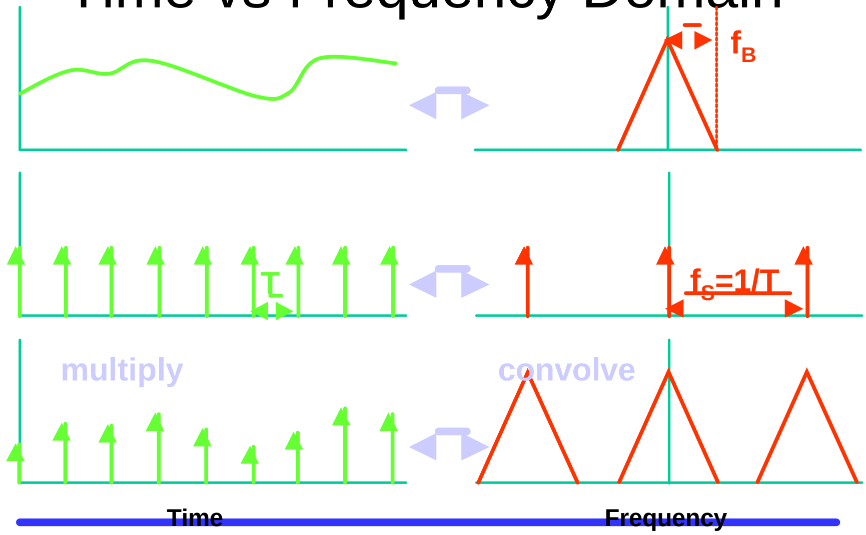
T = 1ms
f_s = 1MHz
f_x = 1101kHz

v(t) = sin [2p(1101000)t]

Aliasing

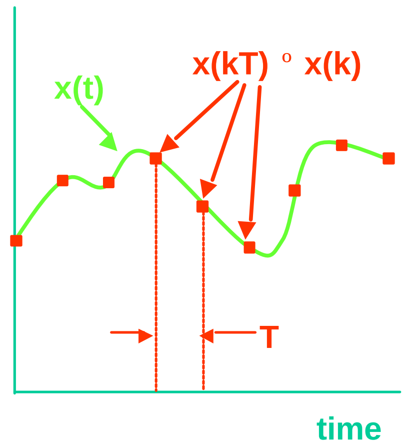
- Multiple continuous time signals can produce identical series of sampled voltages
- The translation of signals from $Nf_S \pm f_{IN}$ down to f_{IN} is called <u>aliasing</u>
  - Sampling theorem: $f_s > 2f_B$
- If aliasing occurs, no signal processing operation downstream of the sampling process can recover the original continuous time signal
  - If you don't like it, sample faster!

Time vs Frequency Domain



Nomenclature

Continuous time signal	x(t)
Sampling interval	T
Sampling frequency	$f_s = 1/T$
Sampled signal	x(kT) = x(k)



Sampled Signal Properties

- It's obvious from the preceding slides that the frequency content of a continuous signal can be grossly distorted by sampling
- What <u>do</u> we know about the relationships between x(t) and x(k)?
  - We'll assume that x(t) is stationary; that is, its mean is constant and its autocorrelation $R(t_1,t_2)$ depends only on the time difference t_1-t_2

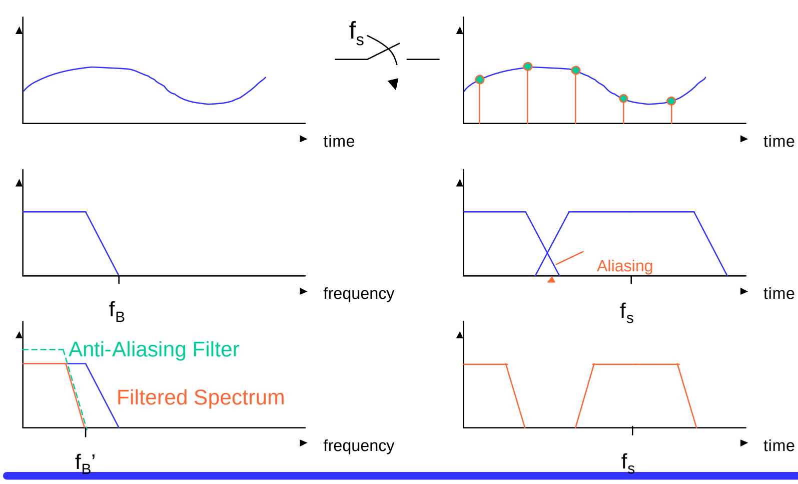
Sampled Signal Properties

- If x(t) is stationary, then
  (Athanasios Papoulis, *Signal Analysis*, 1977, Section 9.4.):
  - x(k) is also stationary
  - Sampling doesn't change the mean: E{x(k)}=E{x(t)}
    - E{} is the expectation operator
  - Sampling doesn't change energy:

$$E\{[x(k)]^2\}= E\{[x(t)]^2\}$$

- Continuous time energy will show up someplace in the frequency domain after sampling
  - Aliasing may move continuous signal frequency components to “wrong” frequencies

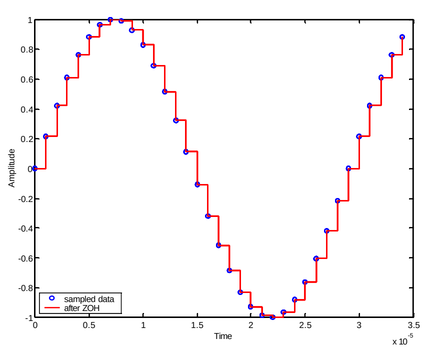
Anti-Aliasing Filter



Sampled Signals

- Sampled data signals are valid only at sampling instances
- In an actual circuit, the signal transitions to the new value between sampling instances
- The value between sampling instances is insignificant and often erroneous (e.g. amplifier slewing distortion)

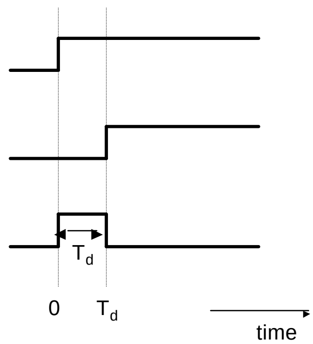
Zero-Order Hold



- Reconstructs CT signal from SD signal
- Frequency response?

Zero-Order Hold

Step response



Laplace transform

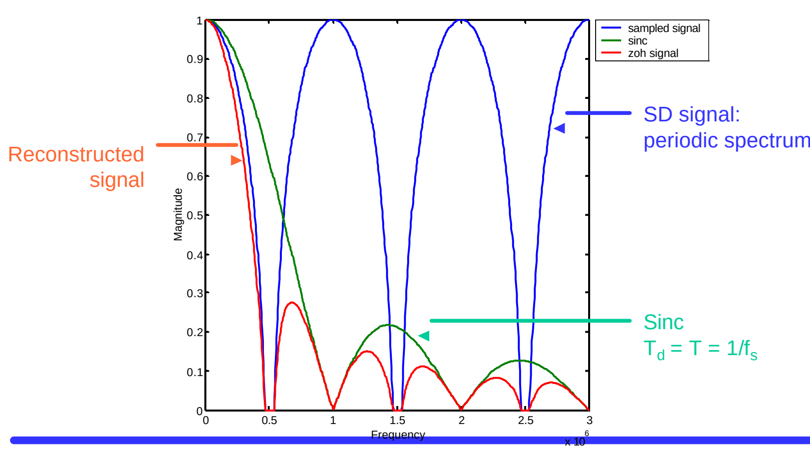
$$\frac{1}{sT_d}$$

$$-\frac{e^{-sT_d}}{sT_d}$$

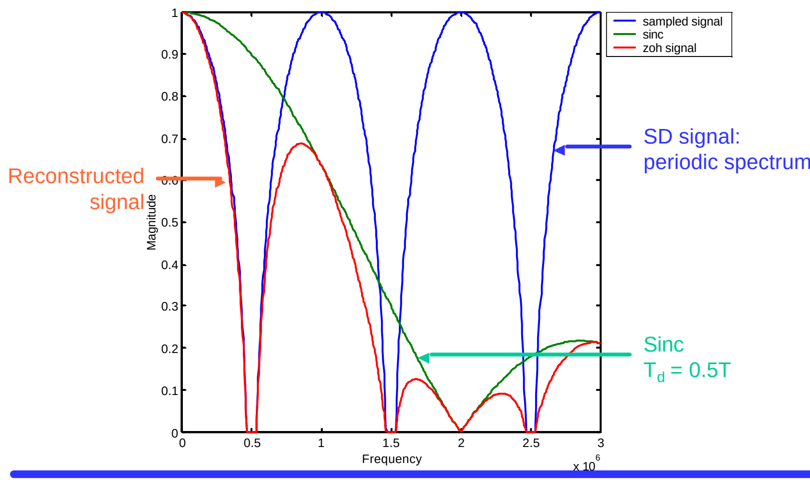
$$=\frac{1-e^{-sT_d}}{sT_d}$$

$$=\frac{\sin(2pfT_d)}{2pfT_d}e^{-jpfT_d}$$

Spectrum of Reconstructed Signal



Spectrum of Reconstructed Signal



Summary

- "Quantization in Time":
  Continuous time (CT) vs Sampled Data (SD)
- Sampling theorem: $f_s > 2f_B$
- IF sampling theorem is met, CT signal can be recovered from SD signal without loss of information
- ZOH reconstructs CT from SD signal