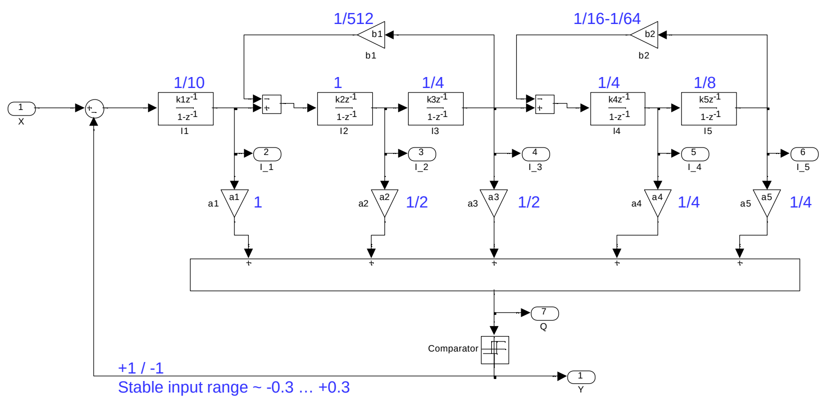
Tones

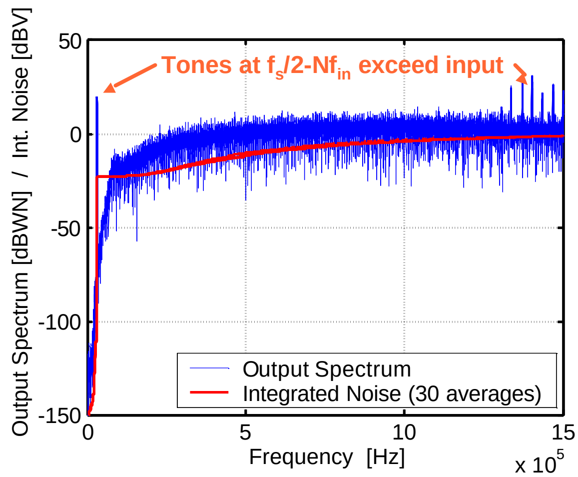
- 5^{th} order $\Sigma\Delta$ modulator
  - DC inputs
  - Tones
  - Dither
  - kT/C noise

5^{th} Order Modulator



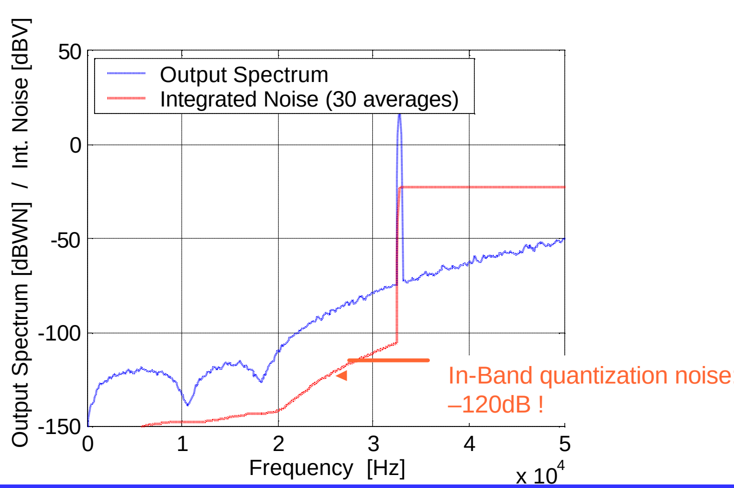
see L20_L5_sim.mdl and L20_L5.m

5^{th} Order Noise Shaping

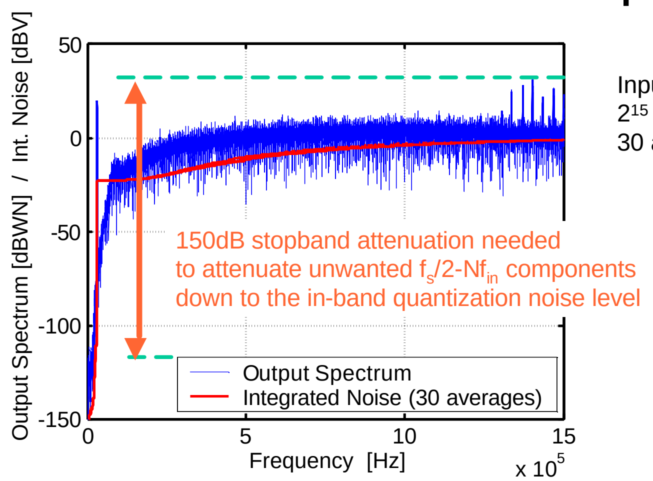


Input: 0.1V, sinusoid
2^{15} point DFT
30 averages

In-Band Noise



5^{th} Order Noise Shaping

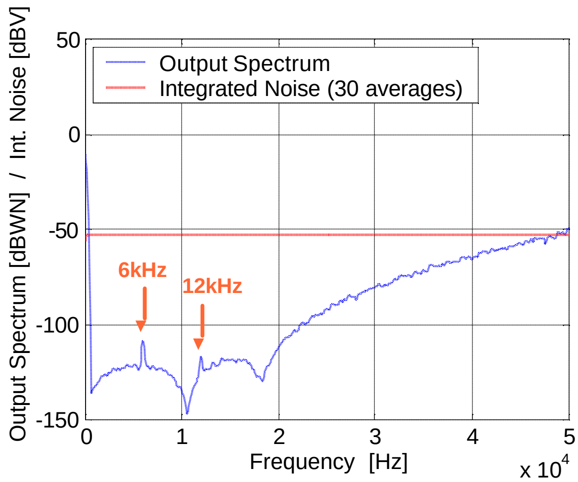


Input: 0.1V, sinusoid
2^{15} point DFT
30 averages

Out-of-Band vs In-Band Signals

- A digital (low-pass) filter with suitable coefficient precision can eliminate out-of-band quantization noise
- No filter can attenuate unwanted in-band components without attenuating the signal
- We'll spend some time making sure the components at $f_s/2\text{-}Nf_{in}$ will not "mix" down to the signal band
- But first, let's look at the modulator response to small DC inputs (or offset) …

ΣΔ Tones



2mV DC input
(1V full-scale)

Simulation technique:
A random 1st input randomizes the noise and enables averaging. Without the small tones are not visible.

Limit Cycles

- Representing a DC term with a –1/+1 pattern … e.g.

$$\tfrac{1}{11} \rightarrow \left\{ \underbrace{\underbrace{\underbrace{-1\ +1}_{1}\ \underbrace{-1\ +1}_{2}\ \underbrace{-1\ +1}_{3}\ \underbrace{-1\ +1}_{4}\ \underbrace{-1\ +1}_{5}}_{\langle 0\rangle}\ +1}_{\langle \frac{1}{11}\rangle} \right\}$$

- Spectrum

$$\frac{f_s}{11} \quad 2\frac{f_s}{11} \quad 3\frac{f_s}{11} \quad \ldots$$

Limit Cycles

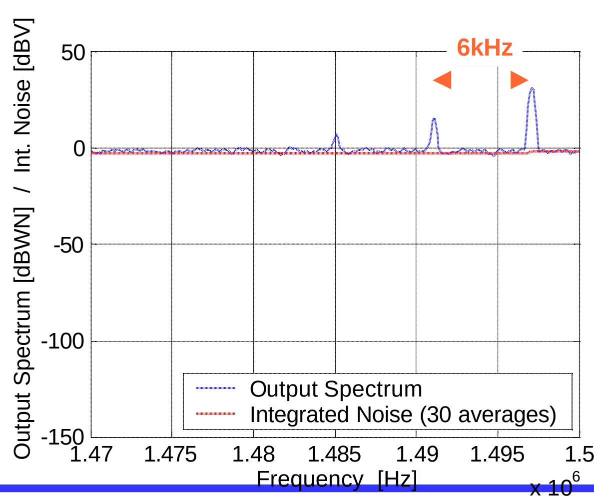
- Fundamental

$$f_d = f_s \frac{V_{DC}}{V_{DAC}}$$
$$= 3\text{MHz}\frac{2\text{mV}}{1\text{V}}$$
$$= \underline{6\text{kHz}}$$

- Tone velocity

$$\frac{df_d}{dV_{DC}} = \frac{f_s}{V_{DAC}}$$
$$= \underline{3\text{kHz/V}}$$

ΣΔ Tones

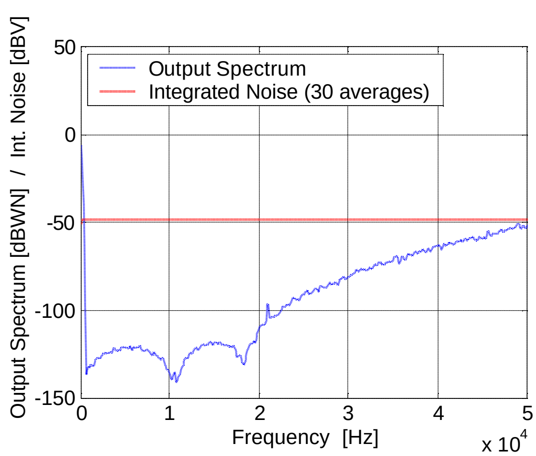


ΣΔ Tones

- Tones follow the noise shape
- The fundamental of a tone that falls into a “quantization noise null” disappears …

$$V_{DC} = V_{FB} \frac{f_d}{f_s}$$
$$= 1\text{V} \frac{10.5\text{kHz}}{3\text{MHz}}$$
$$= \underline{3.5\text{mV}}$$

ΣΔ Tones



3.5mV DC input

$\Sigma\Delta$ Tones

- In-band tones look like signals
- Can be a big problems in some applications
  - E.g. audio → even tones with power below the quantization noise floor can be audible
- Tones near $f_s/2$ can be aliased down into the signal band
  - Since they are often strong, even a small alias can be a big problem
  - We will look at mechanisms that alias tones in the next lecture
- First let's look at dither as a means to reduce or eliminate in-band tones

Dither

- DC inputs can of course be represented by many possible bit patterns
- Including some that are random but still average to the DC input
- The spectrum of such a sequence has no tones
- How can we get a SD modulator to produce such "randomized" sequences?

Dither

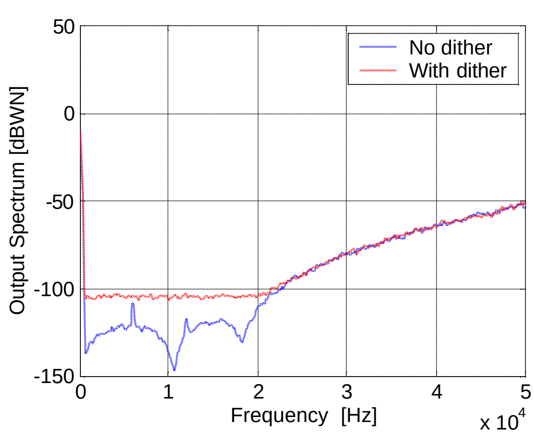
- The target DR for our audio SD is 16 Bits, or 98dB
- Let's choose the sampling capacitor such that it limits the dynamic range:

$$DR = \frac{\frac{1}{2}(V_{FS})^2}{k_B T / C}$$

$$C = DR \frac{k_B T}{\frac{1}{2}(V_{FS})^2}$$

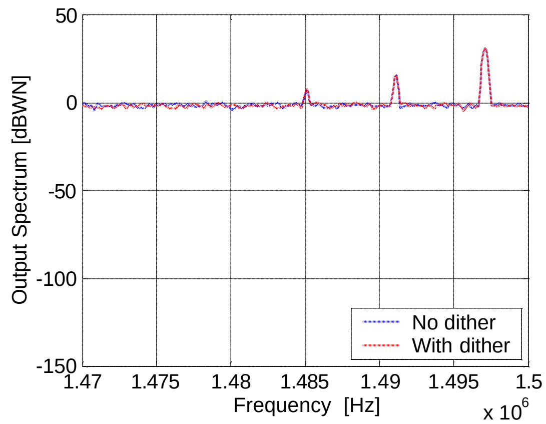
$$= 10^{9.8} \frac{k_B T}{\frac{1}{2}(1\text{V})^2} = \underline{50.5\text{pF}} \quad \rightarrow \quad \sqrt{\overline{v_n^2}} = \sqrt{\frac{k_B T}{C}} = \underline{9\mu\text{V}}$$

Dither



2mV DC input

Dither

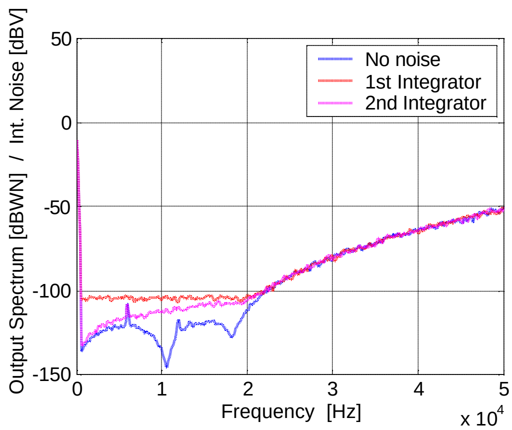


Dither at an amplitude which buries the in-band tones has virtually no effect on tones near $f_s/2$

kT/C Noise

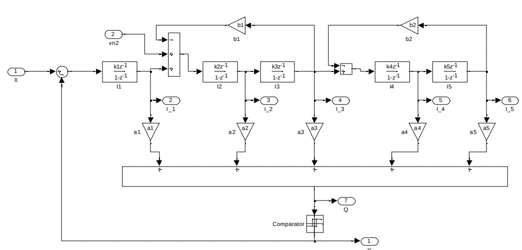
- So far we've looked at noise added to the input of the SD modulator, which is also the input of the first integrator
- Now let's add noise also to the input of the second integrator
- Let's assume a 4pF sampling capacitor
  - This gives 1.4 x 32μV rms noise (two uncorrelated 32μV samples per clock)

kT/C Noise



- 2mV DC input
- Noise from 2nd integrator
  - smaller than 1st integrator noise
  - shaped
- Why?

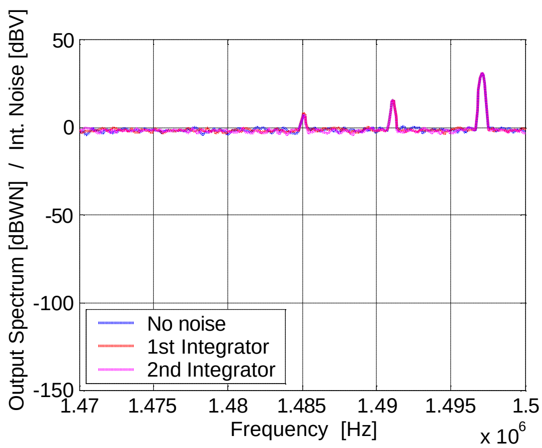
kT/C Noise



- Noise from 1st integrator is added directly to the input
- Noise from 2nd integrator is first-order noise shaped
- Noise from subsequent integrators is attenuated even further

→ Especially for high oversampling ratios, only the first 1 or 2 integrators add significant thermal noise. This is true also for other imperfections.

Dither

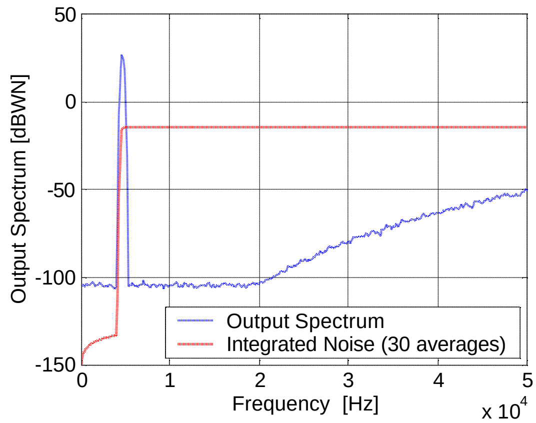


No practical amount of dither eliminates the tones near $f_s/2$

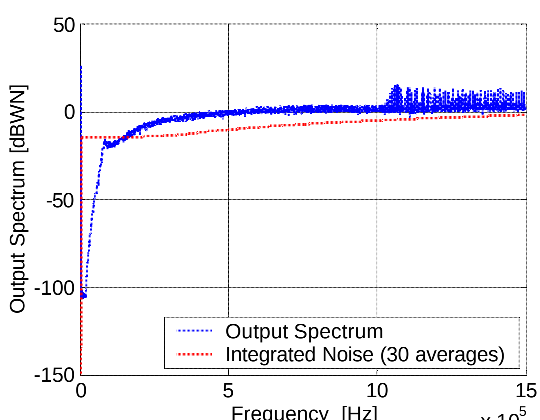
Full-Scale Inputs

- With practical levels of thermal noise added, let's try a 5kHz sinusoidal input near full-scale (0.3V)
- No distortion is visible in the spectrum
  - 1-Bit modulators are intrinsically linear
  - But tones exist at high frequencies
    → to the oversampled modulator, a sinusoidal input looks like two "slowly" alternating DCs … hence giving rise to limit cycles

Full-Scale Inputs



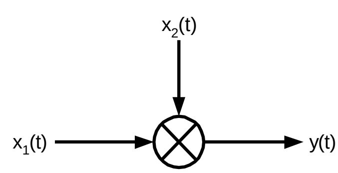
Full-Scale Inputs



V_{ref} Interference

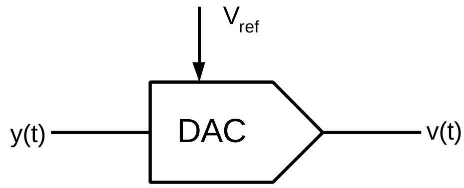
- Dither successfully removes in-band tones that would corrupt the signal
- The high-frequency tones in the quantization noise spectrum will be removed by the digital filter following the modulator
- What if some of these strong tones are demodulated to the base-band <u>before</u> digital filtering?
- Why would this happen?

AM Modulation



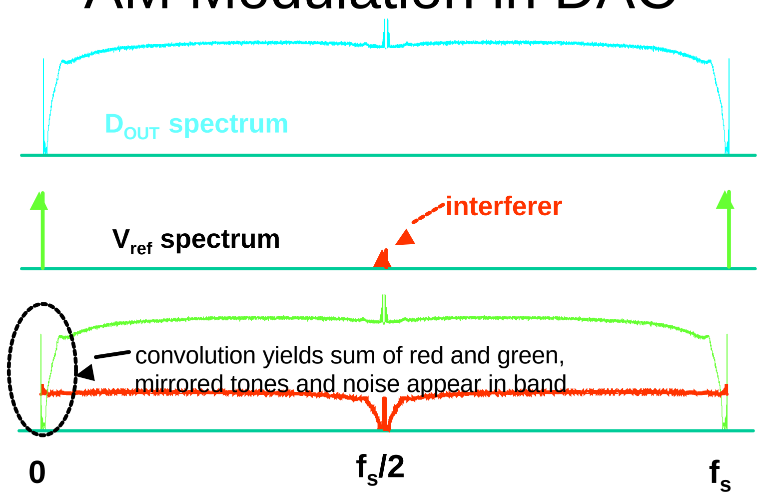
$$x_1(t) = X_1 \cos(w_1 t)$$
$$x_2(t) = X_2 \cos(w_2 t)$$
$$x_1(t) \times x_2(t) = \frac{X_1 X_2}{2} \left[\cos(w_1 t + w_2 t) + \cos(w_1 t - w_2 t)\right]$$

AM Modulation in DAC

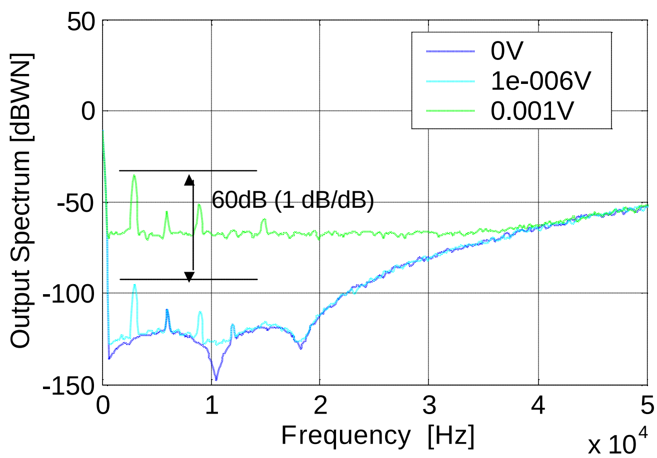


$$y(t) = D_{out} = \pm 1$$
$$V_{ref} = 1\text{V} + 1\text{mV}\ \ \text{f}_s/2 \text{ square wave}$$
$$v(t) = y(t) \times V_{ref} = \text{fundamental}$$
$$+\ 0.05\% \text{ of spectrum at f}_s/2$$

AM Modulation in DAC



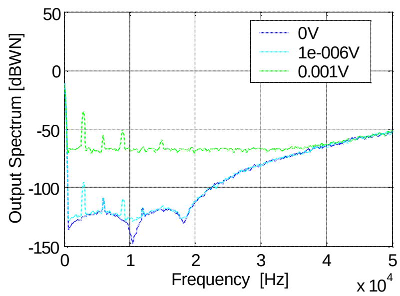
V_{ref} Interference

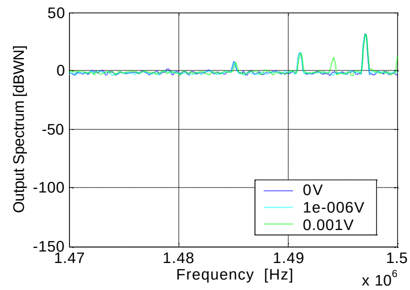


V_{ref} Interference

- Simulation are for specified amounts of $f_s/2$ interference in the DAC reference
- As predicted interference demodulates the high-frequency tones
- Since the high frequency tones are strong, a small amount (1μV) of interference suffices to create huge base-band tones
- Stronger interference (1mV) rises the noise floor also
- Amplitude of demodulated tones is proportional to interference

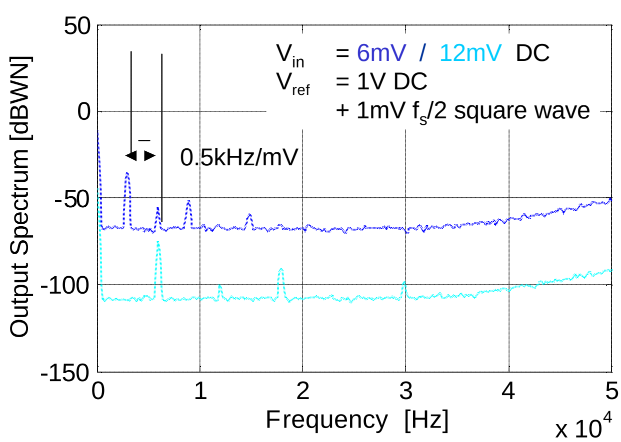
V_{ref} Interference





Symmetry of the spectra at $f_s/2$ and DC confirm that this is AM modulation

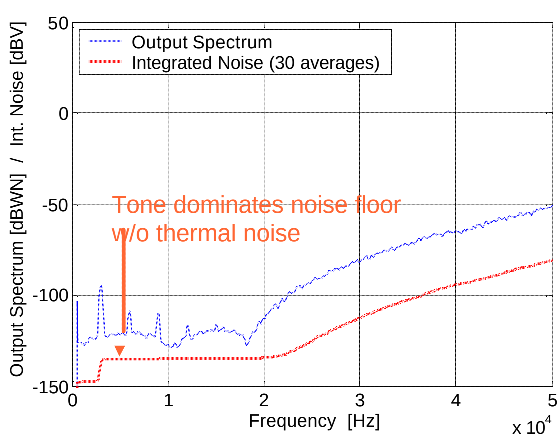
V_{ref} Tone Velocity



V_{ref} Tone Velocity

- The velocity of AM demodulated tones is half that of the native tone
- Such differences help debugging of real silicon
- How clean does the reference have to be?

V_{ref} Interference



V_{ref} Interference

- 120dB of clock-to-V_{ref} isolation is not sufficient for digital audio applications
- Achieving this level of performance requires careful engineering
- Getting an accurate requirement is the first (and an essential) step
- See

  E. Swanson, N. Sooch, and D. Knapp, "Method for Reducing Effects of Electrical Noise in Analog-to-Digital Converter," U.S. Patent 4746899, 1988

  for more ideas

Summary

- Our stage 2 model can drive almost all capacitor sizing decisions
  - Gain scaling
  - kT/C noise
  - Dither
- Dither removes effectively in-band tones
  - Actual tonality determined by demodulation of limit cycles near $f_s/2$
- Next we will add relevant component imperfections, e.g.
  - Real capacitors aren't perfect
  - Real opamps aren't ideal
- We'll model nonlinearities in the ΣΔ system next time …