N247 HW1 Solution

1 Question 1

$$Q = \frac{\omega_0}{BW} = \frac{1MHz}{250KHz} = 4$$

2 Question 3

The equations simplify to

$$\omega_0 = \frac{\sqrt{2}}{RC} \tag{1}$$

$$Q = \frac{\sqrt{2}}{4 - K} \tag{2}$$

$$G = \frac{K}{4 - K} \tag{3}$$

For C=1pF, we find

$$R = \frac{\sqrt{2}}{\omega_0 C} \approx 220 K\Omega \tag{4}$$

$$K = 4 - \frac{\sqrt{2}}{Q} \approx 3.65 \tag{5}$$

$$G = \frac{K}{4 - K} = 10.42 \tag{6}$$

The sensitivity of the Quality Factor to the value of K can be calculated as

$$S_K^Q = \frac{\partial Q}{\partial K} \frac{K}{Q} = K \cdot \frac{\partial \log(Q)}{\partial K} = \frac{K}{4 - K} = G = 10.42 \tag{7}$$

Spectre simulations results: Command used the bandiwdth and center frequency are reported in SK.ocn(see website). Comment: the variation in Q is larger than predicted by sensitivity analysis. The reason is nonlinear dependance of Q on K. The last column of the table above reports calculated values of Q with changing K, considering the nonlinear relation. The agreement with simulations is almost perfect.

| Κ | $\omega_0 [\text{Hz}]$ | BW | Q | G[dB] | Q(calc) |
|----------------|-------------------------|--------------------|------|-------|---------|
| 3.65 | 1M | 255K | 4 | 20.67 | 4 |
| $3.65^{*}1.05$ | 1M | $117.2 \mathrm{K}$ | 8.44 | 27 | 8.44 |
| 3.65*0.95 | 1M | 376K | 2.65 | 16.27 | 2.5 |

| Tab. 1: 1 | Simulation | Results |
|-----------|------------|---------|
|-----------|------------|---------|

3 Question 4

Simulating with 4 50fF stray capacitors I obtained: $\omega_0=912KHz, Q=2.82, PG=17dB$

4 Question 5

Since the sensitivity of Q to K for this example is approximately 10, in order to control Q within 10% we need to control K within 1%. Since for an amplifier with finite gain A_v , $K \approx K_{id}(1 - K_{id}/Av)$, $K_{id}/A_v \leq .01$ or $A_v \geq 364$. Since $A_v^{Min} = 2/3A_{v0}$, $A_{v0} \geq 560$.