N247 HW2 Solution

1 Problem 1

The filter is high pass. Using the function bode in matlab, we see that the passband ripple is 2dB, the stopband attenuation is 50dB, and the cutoff frequency 1MHz.(See 1,2)

$w_p[rad/s]$	Q
23.4M	-
6.3M	21.9
8.83M	4

Tab. 1: Pole Placements

$w_z[rad/s]$	Q
0	-
4.5M	∞
3.6M	∞

Tab. 2: Zero locations

we use the heuristics of lecture 6 and order the biquads in the signal flow starting with the lowest Q and increasing. The second order section is placed first. From Tables 1 and 2, we find that the highest Q poles correspond to the expression $s^2 + 5.831e5s + 4.066e13$ (Q=21.9). Therefore we partition the filter design as shown in figure 6.

Once the capacitors are fixed in the Tow-Thomas biquad we have 6 equations (a0,b0,a1,b1,a2,b2), and 8 unknowns ($R_1 ... R_8$). The equations read:

$$a_0 = \frac{R_8}{R_3 R_2 R_7 C_1 C_2} \tag{1}$$

$$a_1 = \frac{1}{R_1 C_1} \tag{2}$$

$$b_2 = \frac{R_8}{R_6} = 1 \tag{3}$$

$$b_1 = a_1 (b_2 - \frac{R_1 R_8}{R_4 R_7}) = 0 \tag{4}$$

$$b_0 = \frac{R_8}{R_3 R_5 R_7 C_1 C_2} \tag{5}$$

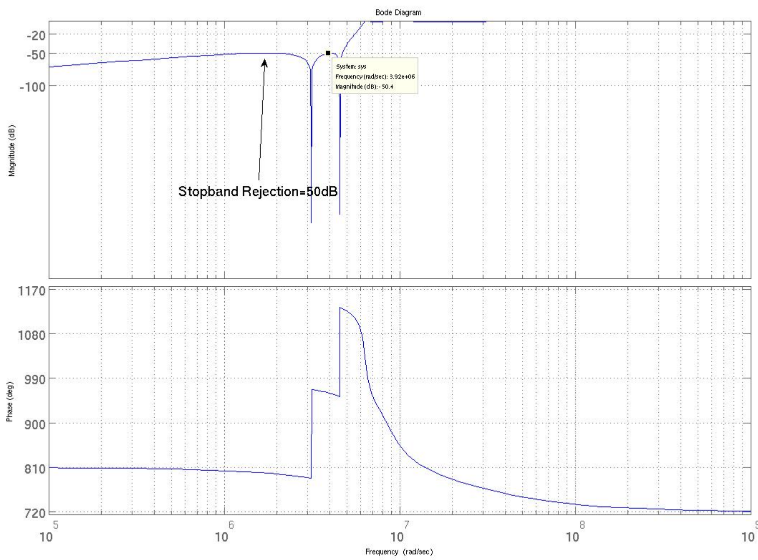


Fig. 1: Filter Frequency response

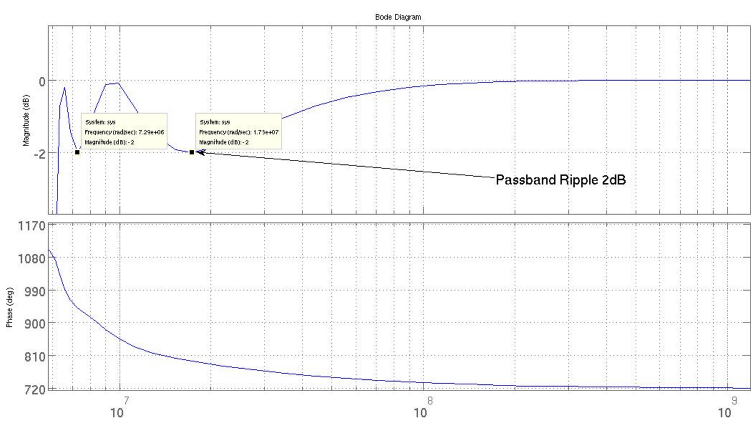


Fig. 2: Passband detail

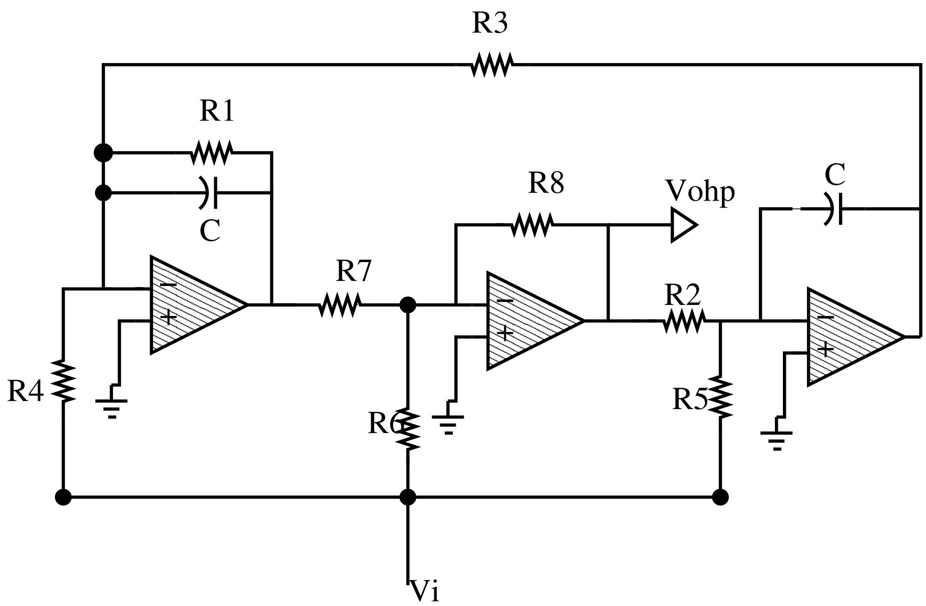


Fig. 3: Tow-Thomas Biquad reference schematic

From noise considerations, we also choose $R_7 = R_8$ and $R_4 = R_3$. Using these equations, one obtains:

$$a_0 = \frac{1}{R_3 R_2 C_1 C_2} \rightarrow R_3 R_2 = \frac{1}{a_0 C_1 C_2} \tag{6}$$

$$a_1 = \frac{1}{R_1 C_1} \rightarrow R_1 (= R_2 = R_3) = \frac{1}{a_1 C_1} \tag{7}$$

$$\frac{a_0}{b_0} = \frac{R_5}{R_2} \tag{8}$$

And finally for the first and the second biquad When placed at the end of the

CELL	$R_1 = R_4 = R_3$	$R_6 = R_7 = R_8 = R_2$	R_5
1	$17.15K\Omega$	$143.4K\Omega$	$280K\Omega$
2	$2.35\text{e}3K\Omega$	$544K\Omega$	$2040K\Omega$

Tab. 3: Component values before scaling (C=100pF)

chain .The first order section can implemented in passive way by using a series C-R circuit. Notice that this is general not a very scalable implementation as it can be used in this case only because the filter drives an open-circuit load. For the same reason, placing the first order section before the first biquad also results in a transfer function error, due to the finite input impedance of the Tow-Thomas cell. In general, the cell in 4 includes a buffer and therefore can drive resistive loads(provided the buffer output impedance is low enough)-This is a general-

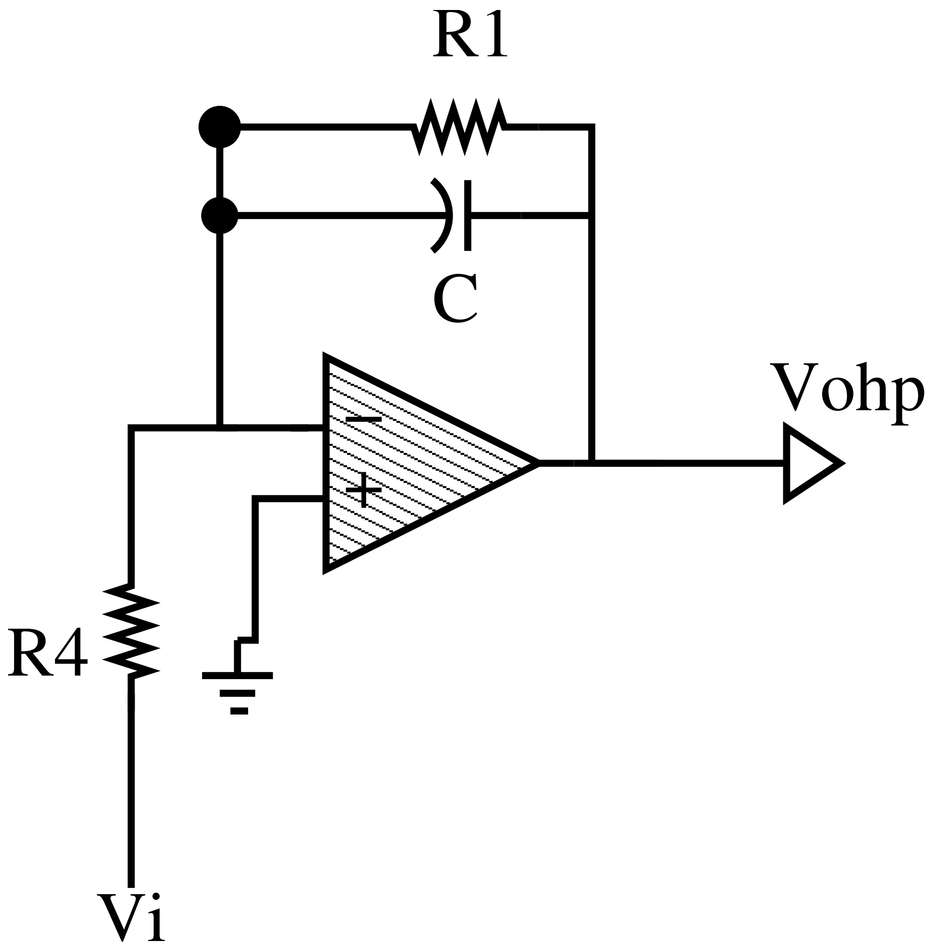


Fig. 4: Active first order cell schematic

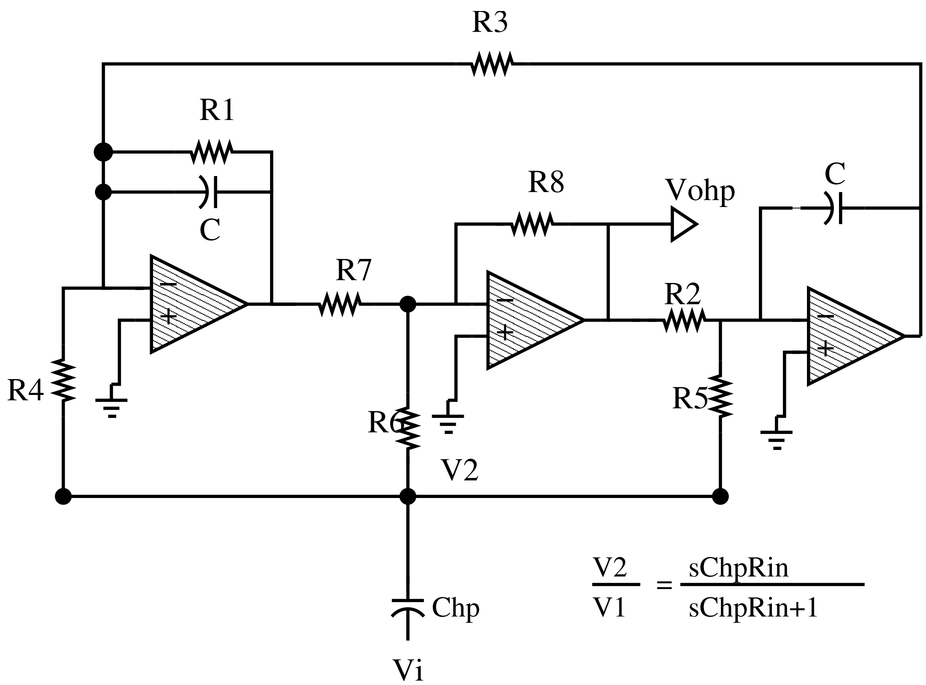
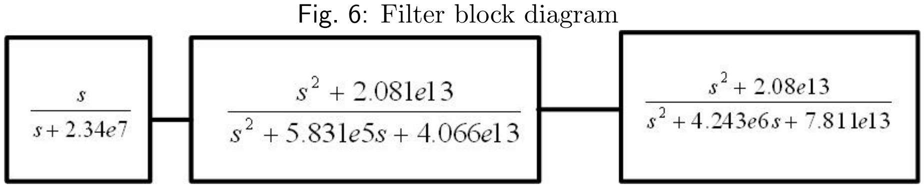


Fig. 5: Tow Thomas Biquad with embedded first order cell. $R_{in} = R_4 \| R_5 \| R_6$.

Fig. 6: Filter block diagram



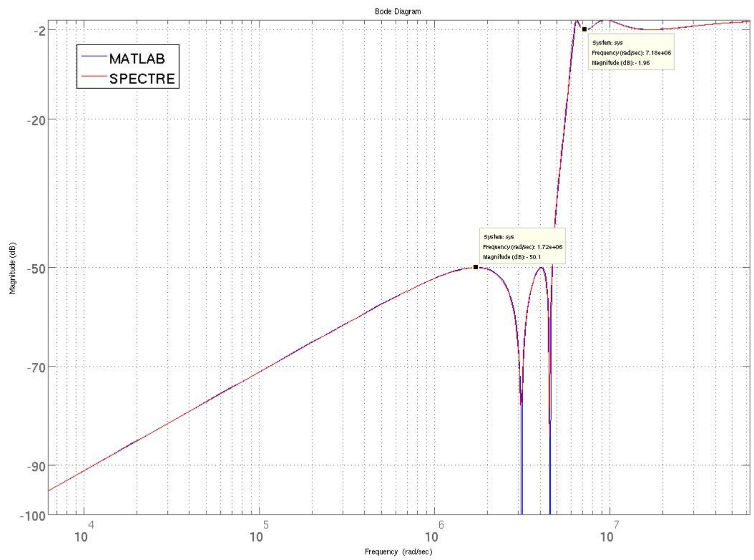


Fig. 7: MATLAB and SPECTRE responses

purpose solution that comes at a noise penalty. Alternatively, one could place a passive first order cell before the first biquad, by relaxing the constraint that the capacitor values be all 100pF. This way, the input impedance of the Tow-Thomas biquad is used as the resistive part in the $C-R$ circuit. For the first Biquad in this implementation, $R_{in} = R_4 \| R_5 \| R_6 = 1.1885K\Omega$ and in $C_{hp} = 36pF$(See 5. For the active first order cell, instead $C = 100pF, R = \frac{1}{w_p C} = 421\Omega$. The response of the implemented filter is shown in figure 7, compared with the MATLAB prediction.

When using the passive first order cell, the simulated integrated noise from 10KHz to 50MHz for a filter built using operational amplifiers with $A_v = 80dB, f_u = 1GHz$ is $V_n^2 = 19nV^2$. Since $V_n^2 \alpha \frac{1}{C}$, to meet the noise target $C^1 = 16.9pF$, and $R^1 = R_0 * 100/16.9 \approx 5.9R_0$. When replacing the opamps with components with $f_u = 20MHz$, the stopband attenuation is almost unaffected, while the passband ripple increases to 9dB. Noise is also totally dominated by the opamps. In order to meet noise and error , I found from simulation

$$C_{oa} = 48pF \tag{9}$$

$$R_n \leq 250\Omega \tag{10}$$

$$f_u \geq 128MHz \tag{11}$$

2 Problem 2

(C_{oa} indicates the filter capacitance to be used with op-amps).

$$f_S \geq 12MHz \tag{12}$$

$$y(k) = \cos(2\pi.5/5K) + \cos(2\pi 3/5K) + \cos(2\pi 6/5K) \tag{13}$$

$$= \cos(2\pi.5/5K) + \cos(2\pi 3/5K) + \cos(2\pi 1/5K) \tag{14}$$