I. Introduction

1. Capacitors

A capacitor is a passive electronic component that stores energy in the form of an electrostatic field. The unit of capacitance is the farad (coulomb/volt). Practical capacitor values usually lie in the picofarad (1 pF = 10^{-12} F) to microfarad (1 μF = 10^{-6} F) range. Recall that a current is a flow of charges. When current flows into one plate of a capacitor, the charges don't pass through (although to maintain local charge balance, an equal number of the same polarity charges leave the other plate of the device) but instead accumulate on that plate, increasing the voltage across the capacitor. The voltage V across the capacitor (capacitance C) is directly proportional to the charge Q stored on the plates:

\[ Q = CV \]

Since Q is the integration of current over time, we can write:

\[ v(t) = \frac{Q(t)}{C} = \int i(t) dt \]

Differentiating this equation, we obtain the I-V characteristic equation for a capacitor:

\[ i(t) = C \frac{dV(t)}{dt} \]  \hspace{1cm} (Eq. 1)

2. RC Circuits

An RC (resistor + capacitor) circuit will have an exponential voltage response of the form \( v(t) = A + B \ e^{-t/RC} \) where constant A is the final voltage and constant B is the difference between the initial and the final voltages. (\( e^x \) is \( e \) to the \( x \) power, where \( e \approx 2.718 \), the base of the natural logarithm.) The product \( RC \) is called the time constant (whose units are seconds) and is usually represented by the Greek letter \( \tau \). When the time has reached a value equal to the time constant, \( \tau \), then the voltage is \( B \ e^{-\tau/RC} = B \ e^{-1} = 0.368 \) * B volts away from the final value A, or about 5/8 of the way from the initial value (A + B) to the final value (A).

The characteristic “exponential decay” associated with an RC circuit is important to understand, because complicated circuits can oftentimes be modeled simply as resistor and a capacitor. This is especially true in integrated circuits (ICs).

A simple RC circuit is drawn in Figure 1 with currents and voltages defined as shown. Equation 2 is obtained from Kirchhoff’s Voltage Law, which states that the algebraic sum of voltage drops around a closed loop is zero. Equation 1 (above) is the defining I-V characteristic equation for a capacitor, and Equation 3 is the defining I-V characteristic equation for a resistor (Ohm’s Law).
Combining Equations 1 and 3 into 2, we obtain the following first-order linear differential equation:

\[ V_{in} = V_C + V_R \]  

(Eq. 2)

\[ V_R = IR = RC \frac{dV_C}{dt} \]  

(Eq. 3)

If \( V_{IN} \) is a step function at time \( t=0 \), then \( V_C \) and \( V_R \) are of the form:

\[ V_C = A + B e^{-t/RC} \]  

(Eq. 5)

\[ V_R = A' + B' e^{-t/RC} \]

If a voltage difference exists across the resistor (i.e. \( V_R < 0 \) or \( V_R > 0 \)), then current will flow (Eq. 3). This current flows through the capacitor and causes \( V_C \) to change (Eq. 1). \( V_C \) will increase (if \( I > 0 \)) or decrease (if \( I < 0 \)) exponentially with time, until it reaches the value of \( V_{IN} \) at which time the current goes to zero (since \( V_R = 0 \)). For the square-wave function \( V_{IN} \) as shown in Figure 2(a), the responses \( V_C \) and \( V_R \) are shown in Figure 2(b) and Figure 2(c), respectively.

Figure 1

\[ V_{in} = V_C + V_R \]  

Figure 2 (a)  

\[ V_R = IR = RC \frac{dV_C}{dt} \]  

Figure 2 (b)  

\[ V_C = A + B e^{-t/RC} \]  

Figure 2 (c)
Note that if the frequency of the square wave \( V_{IN} \) is too high (i.e. if \( f \gg 1/RC \)), then \( V_C \) and \( V_R \) will not have enough time to reach their asymptotic values. If the frequency is too low (i.e. if \( f \ll 1/RC \)), the decay time will be very short relative to the period of the waveform and thus the exponential decay will be difficult to observe. As a rough guideline, the period of the square wave should be chosen such that it is approximately equal to \( 10RC \), in order for the responses shown in Figure 2b-c to be readily observed on an oscilloscope.

3. Inductors

An inductor is a passive electronic device that stores energy in a magnetic field. The unit of inductance is the henry (volt-second/ampere). Practical values of inductance range from one microhenry (1 \( \mu \)H = 1 x 10\(^{-6} \) H) to one henry (1 H).

Inductors are usually made by highly coiled wires, in which changing current generate a magnetic field. By Lenz’s Law, the changing magnetic flux produces a back-EMF (electromotive force), or a potential in the opposite direction of current flow and magnetic flux. While capacitors act to oppose changes in voltage, inductors oppose changes in current. The voltage \( v(t) \) across an inductor (with inductance \( L \)) is equal to the inductance multiplied by the change in current, \( di(t)/dt \) through the inductor:

\[
v(t) = L \frac{di(t)}{dt}
\]

(Eq. 6)

4. Second-Order Circuits

Second order circuits have both inductor and capacitor components, which produce one or more resonant frequencies, \( \omega_0 \). In general, a differential equation for the circuit can be written in the form:

\[
\frac{d^2v(t)}{dt^2} + 2\alpha \frac{dv(t)}{dt} + \omega_0^2 v(t) = f(t)
\]

(Eq. 7)

Where \( \alpha \) and \( \omega_0 \) depend on values for \( R, L, \) and \( C \) present in the circuit, and the forcing function \( f(t) \) depends on these values as well as voltage sources present. This differential equation follows the same form as that for a damped harmonic oscillator (i.e. a mass oscillating due to a spring (resonance), but experiencing a damping force (friction), and possibly a driving force (on the mass to counteract the spring)). In the case of our second order circuit, the damping is produced by a resistor burning energy, resonance occurs between the inductive and capacitive components, and driving force would be a voltage source to force the circuit against the damping by the resistor.

For a series RLC circuit, the damping factor \( \alpha = \frac{R}{2L} \) and resonant frequency \( \omega_0 = \frac{1}{\sqrt{LC}} \)
The solution to the differential equation (Eq. 7) depends on these parameters $\alpha$ and $\omega_0$, in that if:

- $\alpha > \omega_0$: The circuit is **overdamped**, and the solution is:
  \[ v(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} \]

- $\alpha = \omega_0$: The circuit is **critically damped**, and the solution is:
  \[ v(t) = K_1 e^{s_1 t} + K_2 t e^{s_2 t} \]

- $\alpha < \omega_0$: The circuit is **underdamped**, the solution is:
  \[ v(t) = e^{-\alpha t} \left( K_1 \cos(\omega_{n} t) + K_2 \sin(\omega_{n} t) \right) \]

The roots $s_1$ and $s_2$ of the over- and critically damped solutions are given by:

\[ s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \]

The natural frequency $\omega_{n}$ of the underdamped case is given by:

\[ \omega_{n} = \sqrt{\omega_0^2 - \alpha^2} \]

In this lab, we will observe the step response of a series RLC circuit. The step response is how the circuit behaves in response to a forcing function that is a step function (f(t) = 0 for $t < 0$, and f(t) = 1 if $t \geq 0$) which we apply periodically to our circuit with a square wave. The solution is shown in Figure 2.6, where the values are normalized (amplitude of the forcing function divided out).
II. Hands On

1. Determining the RC Circuit Configuration

In this part of the experiment, you will make ohmmeter measurements to see if you can
discover a method to determine if a resistor and capacitor are connected in series or in
parallel.

Get a 5 kΩ resistor and 220 μF capacitor from your TA.

Recall that an ohmmeter has a built-in current source that sends a small current into
the circuit under test. The ohmmeter reads the voltage across the circuit under test and
determines the resistance of the circuit using Ohm’s Law.

Build the circuit shown in Figure 3. Note that the ohmmeter’s current source keeps on
charging up the capacitor. (For small values of capacitance, the capacitor will be fully
charged almost instantly.)

**Question 1:** Are you able to measure the value of the resistor? If not, explain the
reason why you cannot make the measurement.

Build the circuit shown in Figure 4. Note that the capacitor stops charging when the
current through the resistor is equal to the current from the ohmmeter.

**Question 2:** Explain how you got your ohmmeter reading for the circuit in Figure 4.
Why does it take some time before the ohmmeter’s reading stabilizes?

![Figure 2.6: Normalized Step Response of 2\textsuperscript{nd} order circuit](image)
**Question 3:** Given a black box with either a series or parallel RC circuit, can you determine the RC configuration using an ohmmeter? If so, how?

Identifying Physical Values in a Series RC Circuit Black Box and a Parallel RC Circuit Black Box

The TA will give you two “black boxes” (if available). One contains a series RC circuit and the other contains a parallel RC circuit. Determine the basic resistor-capacitor configuration in each black box using an ohmmeter. If you are instructed to build the “black box” yourself, note that you are not allowed access to the node shared by $R_B$ and $C_B$, so you can’t measure $R_B$ directly. Please use the breadboard.

**Question 4:** Determine whether each “black box” is a series or parallel RC circuit.

2. Series RC Circuit Black Box

Construct the circuit below for the black box that contains the unknown resistor $R_B$ and capacitor $C_B$ in series. There is an $R_S = 50$ ohms source resistor inside the function generator (might be negligible) and $R_x$ is the external resistor that is suggested by the TA. (Use $R_x = 500$ ohms) Measure and record $R_X$.

Set the amplitude of the square wave to about $5 V_{pp}$ (not critical), with 0 VDC offset. Adjust the frequency of the square wave and the scope viewing scale until you see the exponential characteristic decay. Make sure that you are using the waveform from the OUTPUT terminal of the function generator, not the SYNC terminal.

Note that the voltage across $R_x$ follows the shape of $V_{IN} - V_C$ (try the math and look at the nice pictures).
(d) From voltage $V_X$ across $R_X$, you could use Ohm’s Law to obtain the current $I_X$, but both $I_X$ and $V_X$ will have the same time constant. You should trigger on A1 edge and set Main Time Ref to Left, and always use the knobs to zoom in upon your cursor measurements. Your TA should review use of the CURSOR function: Set cursor V1 to ground, set cursor V2 to the peak and record $\Delta V(A1)$. Next, move V2 to 0.368 of that value. Move t1 to the start, move t2 to intersect V2. $\Delta t$ is the time constant.

**Figure 7:** How to measure the time constant $\tau_1$ on the oscilloscope

**Question 5:** What is the time constant $\tau_1$? What is the value of $R_X$?
Now increase the value of \( R_X \) by about a factor of ten. Measure the new \( R_{X2} \).

**Question 6:** What is the time constant \( \tau_2 \)? What is the value of \( R_{X2} \)?

**Question 7:** Solve for the resistance \( R_B \) and capacitance \( C_B \) using the formula for the time constant in both trials. \( \tau = C_B (R_S + R_B + R_X) \). Ask your TA for the resistance and capacitance values. Are they in good agreement with the values you have obtained experimentally? Explain if there are any significant differences.

3. **Parallel RC Circuit Black Box**

Build the Parallel “Black Box” using unknown \( R_B \) and \( C_B \) from your TA. Measure the resistance of the circuit inside the black box using the ohmmeter. The rest of the measurements go toward finding the capacitance \( C_B \).

**Question 8:** What is the measured value of the resistor inside the black box?

Select a resistor \( R_X \) (suggested value 5Kohms) of a value comparable to the resistor \( R_B \) and construct the circuit below. Measure the time constant of the circuit with \( R_X \).

![Figure 9: Setup for finding R and C of an unknown parallel RC circuit](image)

**Question 9:** Measure the time constant of the circuit with \( R_X \), then solve for the value of the capacitor \( C_B \). Ask your TA for the values of the resistor and the capacitor inside the black box. Are they in good agreement with the values you have obtained experimentally? Explain if there are any significant differences.
4. Series RLC Circuit

Construct the circuit from pre-lab (shown below) using $L = 10 \text{ mH}$, $C = 1 \text{nF}$. For the values of $R = 200 \Omega$, $6.3 \text{k}\Omega$, and $6.9 \text{k}\Omega$, measure your resistors, and fill in the chart on the lab report with calculated values. Apply a $V_{in}$ square wave at 6.0 Vpp on the function generator (remember that the actual output will be 12 Vpp). For the overdamped and critically damped cases, use a frequency of 3 kHz. For the underdamped case, we need a slower frequency and longer period to observe the damped oscillation, so use a frequency of 1 kHz.

![Series RLC Circuit](image)

**Figure 10:** Series RLC Circuit, with DC source

Use the oscilloscope to view the voltage across the capacitor. Make sure you zoom in on the voltage and time scales to observe the damping of the signal. You may need to reposition the plot (adjust the time and/or voltage offsets) to get a more complete view of the waveform. It will help to raise the trigger level to a level that will provide a stable waveform. Also, you can press the Stop button on the oscilloscope to freeze a frame for better viewing.

**Question 10:** Fill in below with determined values, and plot the observed waveforms.

<table>
<thead>
<tr>
<th>Values</th>
<th>Waveform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overdamped $(R, \alpha)$</td>
<td></td>
</tr>
<tr>
<td>Critically damped $(R, \alpha)$</td>
<td></td>
</tr>
<tr>
<td>Underdamped $(R, \alpha, \omega_n)$</td>
<td></td>
</tr>
</tbody>
</table>

By zooming in on the waveform, and using the cursors (like we did for the RC circuit), we can find the oscillation frequency for the underdamped circuit. By finding the $\Delta t$ for one period, we can obtain the frequency of the signal (Hz) by $f = 1/\Delta t$. We may also find the angular frequency by the relation $\omega = 2\pi f$. 
**Question 11:** For the underdamped case, what is the measured observed oscillation frequency in Hz and rad/s? (Show your calculations) How does this compare to the resonant frequency, $\omega_0$? How does it compare to the natural frequency, $\omega_n$?

Is the damping observed in the underdamped case actually occurring at the rate our solution predicts? Use the cursors to find the voltages of 3 consecutive peaks in the waveform. Let the first value be your maximum voltage, say at time $t = 0$. These peaks occur periodically after the maximum voltage, and we know the period of the signal. Thus, we can predict when and how high the subsequent peaks should be, by using our expression for the underdamped solution: they should decay at a rate proportional to $e^{-\alpha t}$.

**Question 12:** According to the 3 observed peaks, do the consecutive values decay as predicted? List your observed values and predicted values based on the first, reference value. Explain why or why not the predictions are observed.