Illumination Models

So far considered mainly local illumination
- Light directly from light sources to surface

Global Illumination: multiple bounces
- Already ray tracing: reflections/refractions

Global Illumination

Diffuse interreflection, color bleeding [Cornell Box]

Global Illumination

Caustics: Focusing through specular surface

Overview of lecture

- **Theory** for all methods (ray trace, radiosity)
  - We derive **Rendering Equation** [Kajiya 86]
    - Major theoretical development in field
    - Unifying framework for all global illumination
  - Discuss existing approaches as special cases

Outline

- **Reflection Equation (review)**
- **Global Illumination**
- **Rendering Equation**
  - As a general Integral Equation and Operator
  - Approximations (Ray Tracing, Radiosity)
  - Surface Parameterization (Standard Form)
Reflection Equation

\[ L_r(x, \omega_r) = L_i(x, \omega_i) + \sum L_i(x, \omega_i) f(x, \omega_i, \omega_r)(\omega_i \cdot \mathbf{n}) \]

Reflected Light (Output Image)  Emission  Incident Light (from light source)  BRDF  Cosine of Incident angle

Reflection Equation

\[ L_r(x, \omega_r) = L_e(x, \omega_r) + \int \omega_i \neq x \int L_r(x', -\omega_i) f(x', \omega_i, \omega_r) \cos \theta \, d\omega_i \]

Reflected Light (Output Image)  Emission  Reflected Light (from surface)  BRDF  Cosine of Incident angle

Rendering Equation

\[ L_r(x, \omega_r) = L_r(x, \omega_r) + \int \omega_i \neq x \int L_r(x', -\omega_i) f(x', \omega_i, \omega_r) \cos \theta \, d\omega_i \]

Reflected Light (Output Image)  Emission  Reflected Light (from surface)  BRDF  Cosine of Incident angle

Rendering Equation (Kajiya 86)

\[ L_r(x, \omega_r) = L_r(x, \omega_r) + \int \omega_i \neq x \int L_r(x', -\omega_i) f(x', \omega_i, \omega_r) \cos \theta \, d\omega_i \]

Reflected Light (Output Image)  Emission  Reflected Light (from surface)  BRDF  Cosine of Incident angle

Figure 6: A sample image. All objects are rendered gray. Fiber on the object below to creating a glow effect and inter color bleeding from the image.
Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)

The material in this part of the lecture is fairly advanced and not covered in any of the texts. The slides should be fairly complete. This section is fairly short, and I hope some of you will get some insight into solutions for general global illumination.

Rendering Equation as Integral Equation

\[ L_r(x, \omega_r) = L_e(x, \omega_r) + \int L_r(x', \omega') K(x, x') \cos \theta \, d\omega' \]

Reflected Light (Output Image)  
Emission  
Reflected Light  
BRDF  
Cosine of Incident angle

UNKNOWN  
KNOWN  
UNKNOWN  
KNOWN  
KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

\[ l(u) = e(u) + \int l(v) K(u, v) \, dv \]

Kernel of equation

Solution Techniques

All global illumination methods try to solve (approximations of) the rendering equation

- Too hard for analytic solution: numerical methods
- General theory of solving integral equations

Radiosity (next lecture; usually diffuse surfaces)

- General class numerical finite element methods (divide surfaces in scene into a finite set elements or patches)
- Set up linear system (matrix) of simultaneous equations
- Solve iteratively

Ray Tracing and extensions

- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

\[ L = E + KL \]

\[ (I - KL) = E \]

\[ L = (I - K)^{-1} E \]

Binomial Theorem

\[ L = (I + K + K^2 + K^3 + ...) E \]

\[ L = E + KE + K^2 E + K^3 E + ... \]

Ray Tracing

\[ L = E + KE + K^2 E + K^3 E + ... \]

Emission directly From light sources

Direct Illumination on surfaces

Global Illumination (One bounce indirect) [Mirrors, Refraction]

(Time bounce indirect) [Caustics etc]
Ray Tracing

\[ L = E + KE + K^2E + K^3E + \ldots \]

- Emission directly from light sources
- Direct illumination on surfaces
- OpenGL Shading

Outline
- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
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Rendering Equation

\[ \omega_i \cdot \omega_r(x, \omega_i) = \omega_e(x, \omega_r) + \Omega \int L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i \]

- Reflected Light (Output Image)
- Emission Reflected Light
- BRDF Cosine of Incident angle
- Surfaces (interreflection)

Change of Variables

\[ L(x, \omega_r) = L(x, \omega_i) + \int L(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[ \omega_i \rightarrow \omega_r \quad d\omega_i = \frac{dA'}{|x - x'|^2} \cos \theta_i \]

Change of Variables

\[ L(x, \omega_r) = L(x, \omega_i) + \int L(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[ L(x, \omega_r) = L(x, \omega_i) + \int L(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i \]

Domain integral awkward. Introduce binary visibility function

\[ V(x, \omega_i, \omega_r) = \frac{dA'}{|x - x'|^2} \cos \theta_i \]

Rendering Equation: Standard Form

\[ L(x, \omega_r) = L(x, \omega_i) + \int L(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[ L(x, \omega_r) = L(x, \omega_i) + \int L(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i \]

Same as equation 2.52 Cohen Wallace. It swaps primed and unprimed, omits angular args of BRDF, - sign.

\[ G(x, x') = G(x', x) = \frac{\cos \theta_i \cos \theta_r}{|x - x'|^2} \]
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