First comes thought; then organization of that thought, into ideas and plans; then transformation of those plans into reality. The beginning, as you will observe, is in your imagination.

Napoleon Hill

### Implementation of Relational Operations

**CS186, Fall 2005**  
*R&G - Chapter 14*

### Relational Operations

- **We will consider how to implement:**
  - *Selection (σ)* Selects a subset of rows from a relation.
  - *Projection (π)* Deletes unwanted columns from a relation.
  - *Join (⋈)* Allows us to combine two relations.
  - *Set-difference (−)* Tuples in reln. 1, but not in reln. 2.
  - *Union (∪)* Tuples in reln. 1 and in reln. 2.
  - *Aggregation* (SUM, MIN, etc.) and GROUP BY

*Since each op returns a relation, ops can be composed. After we cover the operations, we will discuss how to optimize queries formed by composing them.*

### Simple Selections

- **Of the form** $σ_{p_{q in op value}}(R)$
- **Question: how best to perform?** Depends on:
  - what indexes/access paths are available
  - what is the expected size of the result (in terms of number of tuples and/or number of pages)

*Size of result (cardinality) approximated as* $\text{size of } R \times \text{reduction factor}$

- "reduction factor" is usually called *selectivity*.

### Schema for Examples

Sailors (sid: integer, sname: string, rating: integer, age: real)  
Reserves (sid: integer, bid: integer, day: dates, rname: string)

- **Similar to old schema; rname added for variations.**
- **Reserves:**
  - Each tuple is 40 bytes long, 100 tuples per page, 1000 pages.
- **Sailors:**
  - Each tuple is 50 bytes long, 80 tuples per page, 500 pages.

### Simple Selections (cont)

- **With no index, unsorted:**
  - Must essentially scan the whole relation
  - cost is $M$ (#pages in R). For "reserves" $= 1000$ I/Os.
- **With no index, sorted:**
  - cost of binary search + number of pages containing results.
  - For reserves $= 10$ I/Os + \( \lceil \text{selectivity} \times \text{#pages} \rceil \)
- **With an index on selection attribute:**
  - Use index to find qualifying data entries,
  - then retrieve corresponding data records.
  - Cost?
Using an Index for Selections

- Cost depends on qualifying tuples, and clustering.
  - With a clustered index, cost is little more than 100 I/Os;
  - If unclustered, could be up to 10000 I/Os!
  - Unless you get fancy...

Selections using Index (cont)

- Important refinement for unclustered indexes:
  1. Find qualifying data entries.
  2. Sort the rid’s of the data records to be retrieved.
  3. Fetch rids in order. This ensures that each data page is looked at just once (though # of such pages likely to be higher than with clustering).

General Selection Conditions

- (day<8/9/94 AND name='Paul') OR bid=5 OR sid=1

- Such selection conditions are first converted to conjunctive normal form (CNF):
  - (day<8/9/94 OR bid=5 OR sid=3) AND (name='Paul' OR bid=5 OR sid=3)

- We only discuss the case with no ORs (a conjunction of terms of the form attr op value).
- A B-tree index matches (a conjunction of) terms that involve only attributes in a prefix of the search key.
  - Index on <a, b, c> matches a=5 and b=3, but not b=3.
  - (For Hash index, must have all attr in search key)

Two Approaches to General Selections

- First approach: Find the most selective access path, retrieve tuples using it, and apply any remaining terms that don’t match the index:
  - Most selective access path: An index or file scan that we estimate will require the fewest page I/Os.
  - Terms that match this index reduce the number of tuples retrieved: other terms are used to discard some retrieved tuples, but do not affect number of tuples/pages fetched.

Most Selective Index - Example

- Consider day<8/9/94 AND bid=5 AND sid=3.
  - A B+ tree index on day can be used;
    - Then, bid=5 and sid=3 must be checked for each retrieved tuple.
  - Similarly, a hash index on <bid, sid> could be used;
    - Then, day<8/9/94 must be checked.
- How about a B+tree on <name, day>?
- How about a B+tree on <day, name>?
- How about a Hash index on <day, name>?

Intersection of Rids

- Second approach: if we have 2 or more matching indexes (w/Alternatives (2) or (3) for data entries):
  - Get sets of rids of data records using each matching index.
  - Then intersect these sets of rids.
  - Retrieve the records and apply any remaining terms.
  - Consider day<8/9/94 AND bid=5 AND sid=3. With a B+ tree index on day and an index on sid, we can retrieve rids of records satisfying day<8/9/94 using the first, rids of recs satisfying sid=3 using the second, intersect, retrieve records and check bid=5.
  - Note: commercial systems use various tricks to do this:
    - bit maps, bloom filters, index joins
Projection (DupElim)
- Issue is removing duplicates.
- Basic approach is to use sorting
  - 1. Scan R, extract only the needed attrs (why do this 1st?)
  - 2. Sort the resulting set
  - 3. Remove adjacent duplicates
- Cost: Reserves with size ratio 0.25 = 250 pages. With 20 buffer pages can sort in 2 passes, so
  \[1000 + 250 + 2 \times 250 + 250 = 2500 \text{ I/Os}\]
- Can improve by modifying external sort algorithm:
  - Modify Pass 0 of external sort to eliminate unwanted fields.
  - Modify merging passes to eliminate duplicates.
- Cost: for above case: read 1000 pages, write out 250 in runs of 40 pages, merge runs = 1000 + 250 + 250 = 1500.

Projection Based on Hashing
- Partitioning phase: Read R using one input buffer. For each tuple, discard unwanted fields, apply hash function \( h \) to choose one of \( B-1 \) output buffers.
  - Result is \( B-1 \) partitions (of tuples with no unwanted fields).
  - 2 tuples from different partitions guaranteed to be distinct.
- Duplicate elimination phase: For each partition, read it and build an in-memory hash table, using hash fn \( h2 \) (\( <> h1 \)) on all fields, while discarding duplicates.
  - If partition does not fit in memory, can apply hash-based projection algorithm recursively to this partition.
- Cost: For partitioning, read R, write out each tuple, but with fewer fields. This is read in next phase.

DupElim & Indexes
- Sort-based approach is the standard; better handling of skew and result is sorted.
- If an index on the relation contains all wanted attributes in its search key, can do index-only scan.
  - Apply projection techniques to data entries (much smaller!)
- If an ordered (i.e., tree) index contains all wanted attributes as prefix of search key, can do even better:
  - Retrieve data entries in order (index-only scan), discard unwanted fields, compare adjacent tuples to check for duplicates.
- Same tricks apply to GROUP BY/Aggregation

Joins
- Joins are very common
- Joins are very expensive (worst case: crossproduct)
- Many approaches to reduce join cost

Equality Joins With One Join Column
\[
\begin{align*}
\text{SELECT} & \quad \text{FROM} \\
\text{Reserves R1, Sailors S1} \quad \text{WHERE} \\
\text{R1.sid = S1.sid}
\end{align*}
\]
- In algebra: \( R \bowtie S. \) Common! Must be carefully optimized. \( R \times S \) is large; so, \( R \times S \) followed by a selection is inefficient.
- Note: join is associative and commutative.
- Assume:
  - \( M \) pages in \( R, p_r \) tuples per page
  - \( N \) pages in \( S, p_s \) tuples per page
  - In our examples, \( R \) is Reserves and \( S \) is Sailors.
- We will consider more complex join conditions later.
- Cost metric: \( \# \) of I/Os. We will ignore output costs.

Simple Nested Loops Join
\[
\begin{align*}
\text{foreach tuple} \ r \ \text{in} \ R \ \text{do} \\
\text{foreach tuple} \ s \ \text{in} \ S \ \text{do} \\
\text{if} \ r.\text{sid} = s.\text{sid} \ \text{then add} \ <r, s> \ \text{to result}
\end{align*}
\]
- For each tuple in the outer relation \( R \), we scan the entire \( inner \) relation \( S \).
- How much does this Cost?
  - \( (p_r \times M) \times N + M = 100 \times 1000 \times 500 + 1000 \text{ I/Os}. \)
  - At 10ms/IO, Total: ???
- What if smaller relation (\( S \)) was outer?
- What assumptions are being made here?
- Q: What is cost if one relation can fit entirely in memory?
Page-Oriented Nested Loops Join

foreach page \(b_r\) in \(R\) do
foreach page \(b_s\) in \(S\) do
foreach tuple \(r\) in \(b_r\) do
foreach tuple \(s\) in \(b_s\) do
if \(r_i = s_j\) then add \(\langle r, s \rangle\) to result

- For each page of \(R\), get each page of \(S\), and write out matching pairs of tuples \(\langle r, s \rangle\), where \(r\) is in \(R\)-page and \(s\) is in \(S\)-page.

- What is the cost of this approach?
- \(M \times N + M = 1000 \times 500 + 1000\)
  - If smaller relation \((S)\) is outer, cost = \(500 \times 1000 + 500\)

Examples of Index Nested Loops

- \(B^+\)-tree index (Alt. 2) on \(sid\) of Sailors (as inner):
  - Scan Reserves: 1000 page I/Os, 100*1000 tuples.
  - For each Reserves tuple: 2 I/Os to get data entry in index, plus 1 I/O to get the exactly one matching Sailors tuple.
  - Total:

- \(B^+\)-Tree index (Alt. 2) on \(sid\) of Reserves (as inner):
  - Scan Sailors: 500 page I/Os, 80*500 tuples.
  - For each Sailors tuple: 2 I/Os to find index page with data entries, plus cost of retrieving matching Reserves tuples. Assuming uniform distribution, 2.5 reservations per sailor (100,000 / 40,000). Cost of retrieving them is 1 or 2.5 I/Os depending on whether the index is clustered.
  - Totals:

“Block” Nested Loops Join

- Page-oriented NL doesn’t exploit extra buffers.
- Alternative approach: Use one page as an input buffer for scanning the inner \(S\), one page as the output buffer, and use all remaining pages to hold “block” (think “chunk”) of outer \(R\).
- For each matching tuple \(r\) in \(R\)-chunk, \(s\) in \(S\)-page, add \(\langle r, s \rangle\) to result. Then read next \(R\)-chunk, scan \(S\), etc.

Examples of Block Nested Loops

- Cost: \(\text{Scan of outer} + \#\text{outer chunks} \times \text{scan of inner}\)
  - \#outer chunks = \(\left\lceil \frac{\#\text{of pages of outer/chunksize}}{2} \right\rceil\)
- With Reserves \((R)\) as outer, and 100 pages of \(R\):
  - Cost of scanning \(R\) is 1000 I/Os; a total of 10 chunks.
  - Per chunk of \(R\), we scan Sailors (\(S\)); 10*500 I/Os.
  - If space for just 90 pages of \(R\), we would scan \(S\) 12 times.
- With 100-page chunk of Sailors as outer:
  - Cost of scanning \(S\) is 500 I/Os; a total of 5 chunks.
  - Per chunk of \(S\), we scan Reserves; 5*100 I/Os.
- If you consider seeks, it may be best to divide buffers evenly between \(R\) and \(S\):
  - Disk arm “jogs” between read of \(S\) and write of output
  - If output is not going to disk, this is not an issue

Sort-Merge Join \((R \bowtie S)\)

- Sort \(R\) and \(S\) on the join column, then scan them to do a “merge” (on join col.), and output result tuples.
- Useful if:
  - One or both inputs already sorted on join attribute(s)
  - Output should be sorted on join attribute(s)
- General scheme:
  - Do (Advance scan of \(R\) until current \(R\)-tuple \(>\) current \(S\)-tuple; Advance scan of \(S\) until current \(S\)-tuple \(>\) current \(R\)-tuple;)
  - Until current \(R\)-tuple = current \(S\)-tuple.
  - At this point, all \(R\) tuples with same value in \(R_i\) (current \(R\) group) and all \(S\) tuples with same value in \(S_j\) (current \(S\) group) matched.
  - Output \(\langle r, s \rangle\) for all pairs of such tuples.
    - Like a mini nested loops
    - Then resume scanning \(R\) and \(S\).
- \(R\) is scanned once; each \(S\) group is scanned once per matching \(R\) tuple. (Multiple scans of an \(S\) group will probably find needed pages in buffer.)
Example of Sort-Merge Join

<table>
<thead>
<tr>
<th>sid</th>
<th>bid</th>
<th>day</th>
<th>name</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>8</td>
<td>12/4/96</td>
<td>dustin</td>
</tr>
<tr>
<td>28</td>
<td>103</td>
<td>11/3/96</td>
<td>yuppy</td>
</tr>
<tr>
<td>31</td>
<td>101</td>
<td>10/10/96</td>
<td>lubber</td>
</tr>
<tr>
<td>44</td>
<td>101</td>
<td>10/11/96</td>
<td>lubber</td>
</tr>
<tr>
<td>58</td>
<td>103</td>
<td>11/12/96</td>
<td>dustin</td>
</tr>
</tbody>
</table>

- Cost: \( M \log M + N \log N + (M+N) \)
  - The cost of scanning, \( M+N \), could be \( M^N \) (very unlikely)
  - With 35, 100 or 300 buffer pages, both Reserves and Sailors can be sorted in 2 passes; total join cost: 7500.

(BNL cost: 2500 to 15000 I/Os)

Refinement of Sort-Merge Join

- We can combine the merging phases in the sorting of R and S with the merging required for the join.
  - Allocate 1 page per run of each relation, and 'merge' while checking the join condition
  - With \( B > \sqrt{L} \), where \( L \) is the size of the larger relation, using the sorting refinement that produces runs of length \( 2B \) in Pass 0, \( \# \) runs of each relation is \( < B/2 \).
  - Cost: read+write each relation in Pass 0 + read each relation in (only) merging pass (+ writing of result tuples).
  - In practice, cost goes down from 7500 to 4500 I/Os.
  - In practice, cost of sort-merge join, like the cost of external sorting, is linear (very few passes)

Hash-Join

- Partition both relations using hash fn: \( R \) tuples in partition \( i \) will only match \( S \) tuples in partition \( i \).

  - Read in a partition of \( R \), hash it using \( h_2(\ldots) \). Scan matching partition of \( S \), probe hash table for matches.

<table>
<thead>
<tr>
<th>Original Relation</th>
<th>INPUT</th>
<th>OUTPUT</th>
<th>Partitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disk</td>
<td>B main memory buffers</td>
<td>Disk</td>
<td></td>
</tr>
</tbody>
</table>

Observations on Hash-Join

- \#partitions \( k < B \), and \( B-1 > \) size of largest partition to be held in memory. Assuming uniformly sized partitions, and maximizing \( k \), we get:
  
  \[ k = B-1, \text{ and } M/(B-1) < B-2, \text{ i.e., } B \text{ must be } > \sqrt[M]{2} \]

- If we build an in-memory hash table to speed up the matching of tuples, a little more memory is needed.

- If the hash function does not partition uniformly, one or more \( R \) partitions may not fit in memory. Can apply hash-join technique recursively to do the join of this \( R \)-partition with corresponding \( S \)-partition.

Cost of Hash-Join

- In partitioning phase, read+write both rels; \( 2(M+N) \).
  - In matching phase, read both rels; \( M+N \) I/Os.
- In our running example, this is a total of 4500 I/Os.

  - Sort-Merge Join vs. Hash Join:
    - Given a minimum amount of memory (what is this, for each?) both have a cost of \( 3(M+N) \) I/Os. Hash Join superior on this count if relation sizes differ greatly. Also, Hash Join shown to be highly parallelizable.
    - Sort-Merge less sensitive to data skew; result is sorted.

General Join Conditions

- Equalities over several attributes (e.g., \( R.sid=S.sid \) AND \( R.rname=S.name \)):
  - For Index NL, build index on \( <sid,sname> \) (if \( S \) is inner); or use existing indexes on \( sid \) or \( sname \).
  - For Sort-Merge and Hash Join, sort/partition on combination of the two join columns.

- Inequality conditions (e.g., \( R.rname < S.sname \)):
  - For Index NL, need (clustered!) B+ tree index.
    - Range probes on inner; \# matches likely to be much higher than for equality joins.
    - Hash Join, Sort Merge Join not applicable!
  - Block NL quite likely to be the best join method here.
### Set Operations
- Intersection and cross-product special cases of join.
- Union (Distinct) and Except similar; we’ll do union.
- Sorting based approach to union:
  - Sort both relations (on combination of all attributes).
  - Scan sorted relations and merge them.
  - Alternative: Merge runs from Pass 0 for both relations.
- Hash based approach to union:
  - Partition $R$ and $S$ using hash function $h$.
  - For each $S$-partition, build in-memory hash table (using $h_2$), scan corresponding $R$-partition and add tuples to table while discarding duplicates.

### Impact of Buffering
- If several operations are executing concurrently, estimating the number of available buffer pages is guesswork.
- Repeated access patterns interact with buffer replacement policy.
  - e.g., Inner relation is scanned repeatedly in Simple Nested Loop Join. With enough buffer pages to hold inner, replacement policy does not matter. Otherwise, pinning a few pages is best, LRU is worst (sequential flooding).
  - Does replacement policy matter for Block Nested Loops?
  - What about Index Nested Loops? Sort-Merge Join?
  + REMEMBER THIS!

### Summary
- A virtue of relational DBMSs: queries are composed of a few basic operators; the implementation of these operators can be carefully tuned (and it is important to do this!).
- Many alternative implementation techniques for each operator; no universally superior technique for most operators.
- Must consider available alternatives for each operation in a query and choose best one based on system statistics, etc. This is part of the broader task of optimizing a query composed of several ops.