HMMs: Tracking a Jabberwock

You have been put in charge of a Jabberwock for your friend Lewis. The Jabberwock is kept in a large tugley wood which is conveniently divided into a $10 \times 10$ grid. It wanders freely around the 100 possible cells. At each time step $t = 1, 2, 3, \ldots$, the Jabberwock is in some cell $X_t \in \{1, \ldots, 10\}^2$, and it moves to cell $X_{t+1}$ randomly as follows: with probability 0.5, it chooses one of the (up to 4) valid neighboring cells uniformly at random; with probability 0.5, it uses its magical powers to teleport to a random cell uniformly at random among the 100 possibilities (it might teleport to the same cell). It always starts in $X_1 = (1, 1)$.

(a) Compute the probability that the Jabberwock will be in $X_2 = (2, 1)$ at time step 2.

$$P(X_2 = (2, 1)) =$$

(b) What about $Pr(X_2 = (4, 4))$?

$$P(X_2 = (4, 4)) =$$

(c) At each time step $t$, you dont see $X_t$ but see $E_t$, which is the row that the Jabberwock is in: that is, if $X_t = (r, c)$, then $E_t = r$. You still know that $X_1 = (1, 1)$. Suppose we see that $E_1 = 1$, $E_2 = 2$ and $E_3 = 10$. Fill in the following table with the distribution over $X_t$ after each time step, taking into consideration the evidence. Your answer should be concise. *Hint*: you should not need to do any heavy calculations.

| $t$ | $P(X_t|e_{1:t-1})$ | $P(X_t|e_{1:t})$ |
|-----|------------------|------------------|
| 1   |                   |                  |
| 2   |                   |                  |
(d) Assuming you now observe $E_3 = e_3$, how would you calculate $P(X_3|e_{1:3})$? (do not use explicit values for the probabilities in your answer)

(e) Now consider the images below, which correspond to probability distributions of the Jabberwock’s location. Match them with the most appropriate probabilities from this list

\[ P(X_1), \ P(X_1|e_1), \ P(X_2|e_1), \ P(X_2|e_1, e_2), \ P(X_3|e_1, e_2), \ P(X_3|e_1, e_2, e_3) \]
2 Used Car Purchase

[Adapted from problem 16.11 in Russell & Norvig]

A used car buyer can decide to carry out various investigations with various costs (e.g., kick the tires, take the car to a qualified mechanic) and then, depending on the outcome of the investigations, decide which car to buy. We will assume that the buyer is deciding whether to buy car \( c \) and that there is time to carry out at most one investigation which costs $50 and which can help to figure out the quality of the car. A car can be in good shape (of good quality \( Q = +q \)) or in bad shape (of bad quality \( Q = -q \)), and the investigation might help to indicate what shape the car is in. There are only two outcomes for the investigation: pass (\( I = \text{pass} \)) or fail (\( I = \text{fail} \)). Car \( c \) costs $1,500, and its market value is $2,000 if it is in good shape; if not, $700 in repairs will be needed to make it in good shape. The buyers estimate is that \( c \) has 70\% chance of being in good shape.

(a) Draw the decision network that represents this problem. (The only action is whether to buy or not the car).

(b) Calculate the expected net gain from buying car \( c \), given no investigation.
Investigations can be described by the probability that the car will pass or fail the investigation given that the car is in good or bad shape. We have the following information:

\[ P(I = \text{pass}|Q = +q) = 0.9 \]
\[ P(I = \text{pass}|Q = -q) = 0.2 \]

From this we can determine the following about whether the car will pass (or fail) its investigation, and the probability that it is in good (or bad) shape given each possible investigation outcome:

\[
P(I = \text{pass}) = \sum_q P(I = \text{pass}, Q = q) = 0.69
\]
\[
P(I = \text{fail}) = 1 - P(I = \text{pass}) = 0.31
\]
\[
P(Q = +q|I = \text{pass}) = \frac{P(I = \text{pass}|Q = +q)P(Q = +q)}{P(I = \text{pass})} \approx 0.91
\]
\[
P(Q = +q|I = \text{fail}) = \frac{P(I = \text{fail}|Q = +q)P(Q = +q)}{P(I = \text{fail})} \approx 0.22
\]

(c) Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

(d) Calculate the value of (perfect) information of the investigation. Should the buyer pay for an investigation?