Problem 16.11

(a)

Here the result node denotes the result of the test, which could be “pass”, “fail”, or “unknown” (in the case that the test wasn’t run).

(b) \[ E(gain) = -1500 + P(q+) \times ($2000) + P(q-) \times ($2000 - 700) \]
\[ = -1500 + 0.70 \times ($2000) + 0.30 \times ($1300) \]
\[ = 290 \]

(c) \[ P(\text{pass}(c_1, t_1)) = P(\text{pass}(c_1, t_1)|q^+(c_1))P(q^+(c_1)) + P(\text{pass}(c_1, t_1)|q^-(c_1))P(q^-(c_1)) \]
\[ = 0.8 \times 0.70 + 0.35 \times 0.30 \]
\[ = 0.665 \]
\[ P(fail(c_1, t_1)) = 1 - P(pass(c_1, t_1)) = 0.335 \]

\[ P(q^+(c_1)|pass(c_1, t_1)) = \frac{P(pass(c_1, t_1)|q^+(c_1))P(q^+(c_1))}{P(pass(c_1, t_1))} = \frac{0.8 \times 0.70}{0.665} = 0.8421 \]

\[ P(q^-(c_1)|pass(c_1, t_1)) = 1 - P(q^+(c_1)|pass(c_1, t_1)) = 0.1579 \]

\[ P(q^+(c_1)|fail(c_1, t_1)) = \frac{P(fail(c_1, t_1)|q^+(c_1))P(q^+(c_1))}{P(fail(c_1, t_1))} = (1 - 0.8) \times \frac{0.70}{0.335} = 0.4179 \]

\[ P(q^-(c_1)|fail(c_1, t_1)) = 1 - P(q^+(c_1)|fail(c_1, t_1)) = 0.5821 \]

(d) Assuming we run the test then since the test costs $50,

\[ E(gain|fail, not buy) = E(gain|pass, not buy) = -50 \]

\[ E(gain|pass, buy) = -1500 + P(q^+|pass) \times (2000) + P(q^-|pass) \times (2000 - 700) - 50 \]
\[ = -1500 + 0.8421 \times 2000 + 0.1579 \times 1300 - 50 \]
\[ = 339.47 \]

\[ E(gain|fail, buy) = -1500 + P(q^+|fail) \times (2000) + P(q^-|fail) \times (2000 - 700) - 50 \]
\[ = -1500 + 0.4179 \times 2000 + 0.5821 \times 1300 - 50 \]
\[ = 42.53 \]

The optimal decision in both cases is to buy the car.
Problem 17.1

This problem is related to the task of projecting a Hidden Markov Model as follows. We want to maintain a probability distribution over the 12 states of the 4 by 3 grid world. Let vector \( p \) represent the probability of being in a particular state at a particular time. \( p \) is initialized to have a 1 in the position corresponding to the \((1,1)\) (start) cell in the grid. Now consider the 12 by 12 transition matrices \( T_{up} \) and \( T_{right} \).

\[
T_{up}(i, j) = \text{probability of landing in cell } i \text{ when moving Up from cell } j.
\]

\[
T_{right}(i, j) = \text{probability of landing in cell } i \text{ when moving Right from cell } j.
\]

The probability of being in each square after the \([Up,Up,Right,Right,Right]\) sequence is \(T_{right}^3 T_{up}^2 p =
\[
\begin{bmatrix}
0.0252 & 0.0622 & 0.1799 & 0.3279 \\
0.1805 & 0 & 0.0444 & 0.0137 \\
0.0246 & 0.0282 & 0.0263 & 0.0869
\end{bmatrix}
\]

Value Iteration

\[
\begin{bmatrix}
0.7948 & 0.8436 & 0.8912 & 0.9382 & 0.8912 \\
0.8050 & 0.8625 & 0.9149 & 1.0000 & 0.9149 \\
0.7550 & 0 & 0.6577 & -1.0000 & 0.6577 \\
0.6987 & 0.6487 & 0.6053 & 0.3947 & 0.5840
\end{bmatrix}
\]