A student is trying to figure out whether or not it’s going to rain (R). When it’s cloudy (C) it tends to rain (R), but if it’s windy (W) the clouds get blown away and it’s less likely to rain (R). The student can observe the clouds (C) but can’t directly observe the wind (the apartment window is jammed). However, if it’s windy (W) the tree outside tends to sway (S) a lot which can be observed, but sometimes the tree sways (S) even when it’s not windy (squirrels perhaps?).

\[
\begin{array}{c|c} 
C & P(C) \\
\hline 
c & 0.3 \\
\end{array} \quad \begin{array}{c|c} 
W & P(W) \\
\hline 
w & 0.7 \\
\end{array} \quad \begin{array}{c|c|c} 
C & W & P(R = r | C, W) \\
\hline 
c & w & 0.40 \\
c & \neg w & 0.80 \\
\neg c & w & 0.10 \\
\neg c & \neg w & 0.20 \\
\end{array} \quad \begin{array}{c|c} 
W & P(S = s | W) \\
\hline 
w & 0.9 \\
\neg w & 0.3 \\
\end{array}
\]

The student wants to decide whether or not wear a cool leather jacket (J). If he wears the jacket and it doesn’t rain his utility is 20, but if he wears the jacket and it does rain his utility is -25. If he doesn’t wear the jacket then his utility is 0 whether or not it rains.

a) Draw a decision diagram which incorporates the action J with the above Bayes Net. Make sure to draw the utility node as well. Also, write down the utility table.

b) Use variable elimination to compute \(P(R | c, s)\)

\[
P(R | c, s) \propto P(R, c, s) = \sum_W P(R, W, c, s) = \sum_W P(c)P(W)P(R | c, W)P(s | W)
\]
Eliminating $W$ we introduce a factor:

$$m_1(R) = \sum_{W} P(W)P(R|c,W)P(s|W) = P(w)P(R|c,w)P(s|w) + P(\neg w)P(R|c,\neg w)P(s|\neg w)$$

| $m_1(R)$ | $P(R|c,s)$ |
|---------|------------|
| $r$     | .324       |
| $\neg r$ | .396       |

**c)** Given no information other than the fact that it's cloudy and the tree is swaying, what is the student's maximum expected utility.

$$MEU(c, s) = \max_J EU(\text{do}(J), c, s)$$

$$= \max_J \sum_R U(\text{do}(J), R)P(R|c, s)$$

$$= \max_J U(\text{do}(J), r) \times .45 + U(\text{do}(J), \neg r) \times .55$$

$$= \max (-25 \times .45 + 20 \times .55, 0 \times .45 + 0 \times .55)$$

$$= \max(-12.5, 0) = 0$$

**d)** What is the maximum expected utility if the student knows that it's windy, cloudy, and the tree is swaying.

$$MEU(w, c, s) = \max_J EU(\text{do}(J), w, c, s)$$

Note that $P(R|w, c, s) = P(R|c, w)$:

$$= \max_J \sum_R U(\text{do}(J), R)P(R|c, w)$$

$$= \max_J U(\text{do}(J), r) \times .4 + U(\text{do}(J), \neg r) \times .6$$

$$= \max (-25 \times .4 + 20 \times .6, 0 \times .4 + 0 \times .6)$$

$$= \max(12, 0) = 2$$

**e)** What is the maximum expected utility if the student knows that it's not windy, cloudy, and the tree is swaying.

$$MEU(\neg w, c, s) = \max_J EU(\text{do}(J), \neg w, c, s)$$

$$= \max_J \sum_R U(\text{do}(J), R)P(R|c, \neg w)$$

$$= \max_J U(\text{do}(J), r) \times .8 + U(\text{do}(J), \neg r) \times .2$$

$$= \max (-25 \times .8 + 20 \times .2, 0 \times .8 + 0 \times .2)$$

$$= \max(-16, 0) = 0$$

**f)** Use variable elimination to compute $P(W|c, s)$
\[ P(W|c, s) \propto P(W, c, s) \]
\[ = \sum_R P(W, R, c, s) = \sum_R P(W)P(s|W)P(R|c, W)P(c) \]

Eliminating \( R \), we get the factor:
\[ m_1(W) = \sum_R P(R|c, W) \]

which is 1 for all \( W \). Thus,
\[ P(W; c, s) = P(c)P(W)P(s|W) \]
\[ P(W|c, s) \propto P(W)P(s|W) \]

| \( W \) | \( P(W|c, s) \) |
|-------|----------------|
| \( w \) | .875 |
| \( \neg w \) | .125 |

\( g \) Now suppose that the student could break his window to determine whether or not it's windy, but then he would have to pay to repair it which would have a utility of \(-x\). What is the minimum value of \( x \) such that the student would rather not break the window on a cloudy day and when the tree is swaying.

\[ VPI(W|c, s) = \sum_W MEU(W, c, s)P(W|c, s) - MEU(c, s) \]
\[ = MEU(w, c, s)P(w|c, s) + MEU(\neg w, c, s)P(\neg w|c, s) - MEU(c, s) \]
\[ = 2 \times .875 + 0 \times .125 - 0 \]
\[ = 1.75 \]

Thus, \( x \) must be at least 1.75.