1. Consider the state $|\psi\rangle = 1/\sqrt{2}|0\rangle + e^{i\theta}/\sqrt{2}|1\rangle$. To estimate the phase $\theta$, we measure $|\psi\rangle$ in the sign basis. Analyze the probability that the outcome of the measurement is $|+\rangle$.

2. Prove that the Bell state $|\psi^-\rangle$ is rotationally invariant: i.e. $|\psi^-\rangle = 1/\sqrt{2}(|v^+\rangle - |v^-\rangle)$. Here $|v\rangle = \alpha|0\rangle + \beta|1\rangle$ and $|v^\perp\rangle = \bar{\beta}|0\rangle - \bar{\alpha}|1\rangle$.

3. Recall that in Bell’s experiment, Alice is given as input a random bit $r_A$ and Bob a random bit $r_B$. Without communicating with each other, Alice and Bob wish to output bits $a$ and $b$ respectively such that $r_A \cdot r_B = a + b \mod 2$.

   (a) Suppose Alice and Bob are classical and deterministic (they cannot make random choices). Show that they cannot succeed with probability greater than $3/4$.

   (b) Now suppose that Alice and Bob share a random string $R$ drawn from some probability distribution. They consult $R$ in addition to their respective inputs to determine their output bits $a$ and $b$. Show that their expected probability of success is still no greater than $3/4$.

   (Hint: Can you show that if the expected success probability were greater than $3/4$, then there must be some fixed shared string $R$ such that the success probability is greater than $3/4$. Why does this lead to a contradiction?)

4. (a) Consider a CNOT gate whose control qubit (first input) is $1/\sqrt{2}|0\rangle + 1/\sqrt{2}|1\rangle$ and target qubit (second input) is $1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle$. What are the states of the two output qubits? Repeat when the control qubit is $|+\rangle = 1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle$. Can you describe the action of the CNOT gate on the first qubit when the target qubit is $|+\rangle = 1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle$. What about when the target qubit is $|+\rangle$.

   (b) This brings us to an important concept. We can consider what the action of a quantum gate looks like when we write the qubits in a different basis. Show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.