CHECKMATE!

A Brief Introduction to Game Theory

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The World

Kasparov

Game Theory:
Economic or Combinatorial?

- Economic
  ○ von Neumann and Morgenstern’s 1944 Theory of Games and Economic Behavior
  ○ Matrix games
  ○ Prisoner’s dilemma
  ○ Incomplete info, simultaneous moves
  ○ Goal: Maximize payoff

- Combinatorial
  ○ Sprague and Grundy’s 1939 Mathematics and Games
  ○ Board (table) games
  ○ Nim, Domineering
  ○ Complete info, alternating moves
  ○ Goal: Last move

Combinatorial Game Theory

History

- Early Play
  ○ Egyptian wall painting of Senet (c. 3000 BC)

- Theory
  ○ C. L. Bouton’s analysis of Nim [1902]
  ○ Sprague and Grundy [1939] Impartial games and Nim
  ○ Knuth Surreal Numbers [1974]
  ○ Conway On Numbers and Games [1976]
  ○ Prof. Elwyn Berlekamp (UCB), Conway, & Guy Winning Ways [1982]

What is a combinatorial game?

- Two players (Left & Right) move alternately
- No chance, such as dice or shuffled cards
- Both players have perfect information
  ○ No hidden information, as in Stratego & Magic
- The game is finite – it must eventually end
- There are no draws or ties
- Normal Play: Last to move wins!

What games are out, what are in?

- Out
  ○ All card games
  ○ All dice games
- In
  ○ Nim, Domineering, Dots-and-Boxes, Go, etc.
  ○ 1, 2, …, 10, Kayles, Toads & Frogs, Snake, Tactix, Poison
  ○ In, but not normal play
  ○ Chess, Checkers, Othello, Tic-Tac-Toe, etc.

“Computational” Game Theory (for non-normal play games)

- Large games
  ○ Can theorize strategies, build AI systems to play
  ○ Can study endgames, smaller version of original
    • Examples: Quick Chess, 9x9 Go, 6x6 Checkers, etc.
- Small-to-medium games
  ○ Can have computer solve and teach us strategy
  ○ GAMESMAN does exactly this
    • It can solve BOTH normal and non-normal play games
Computational Game Theory

- Simplify games / value
  - Store turn in position
  - Each position is (for player whose turn it is)
    - Winning (3 losing child)
    - Losing (All children winning)
    - Tying (1 losing child, but 1 tying child)
    - Drawing (can't force a winner be forced to lose)

Exciting Game Theory Research at Berkeley

- Combinatorial Game Theory Workshop
  - MSRI July 24-28th, 2000: Son of Games of No Chance
  - 1994 Workshop book: Games of No Chance
- Prof. Elwyn Berlekamp
  - Dots & Boxes, Go endgames
  - Economist’s View of Combinatorial Games
- Dr. Dan Garcia
  - Undergraduate Game Theory Research Group
    - http://www.cs.berkeley.edu/~ddgarcia/research/gametheory/current/