Hash functions

- To do this, must have way to convert key to bucket number: a hash function.
- Example:
  - \( N = 200 \) data items.
  - keys are longs, evenly spread over the range \( 0..2^{64} - 1 \).
  - Want to keep maximum search to \( L = 2 \) items.
  - Use hash function \( h(K) = K \% M \), where \( M = N / L = 100 \) is the number of buckets: \( 0 \leq h(K) < M \).
  - So 100232, 433, and 10002332482 go into different buckets, but 10, 400210, and 210 all go into the same bucket.

External chaining

- Array of \( M \) buckets.
- Each bucket is a list of data items.
  - Not all buckets have same length, but average is \( N / M = L \), the load factor.
  - To work well, hash function must avoid collisions: keys that "hash" to equal values.
Open Addressing

- Idea: Put one data item in each bucket.
- When there is a collision, and bucket is full, just use another.
- Various ways to do this:
  - Linear probes: If there is a collision at \( h(K) \), try \( h(K)+m, h(K)+2m \), etc. (wrap around at end).
  - Quadratic probes: \( h(K) + m, h(K) + m^2, \ldots \)
  - Double hashing: \( h(K) + h'(K), h(K) + 2h'(K), \ldots \)
- Example: \( h(K) = K \% M \), with \( M = 10 \), linear probes with \( m = 1 \).
  - Add 1, 2, 11, 3, 102, 9, 18, 108, 309 to empty table.

Things can get slow, even when table is far from full.
- Lots of literature on this technique, but
- Personally, I just settle for external chaining.

Filling the Table

- To get (likely to be) constant-time lookup, need to keep \#buckets within constant factor of \#items.
- So resize table when load factor gets higher than some limit.
- In general, must re-hash all table items.
- Still, this operation constant time per item,
- So by doubling table size each time, get constant amortized time for insertion and lookup
- (Assuming, that is, that our hash function is good).

Hash Functions: Strings

- For String, "s0s1 \ldots s_{n-1}\" want function that takes all characters and their positions into account.
- What's wrong with \( s_0 + s_1 + \ldots + s_{n-1}\)?
- For strings, Java uses \( h(s) = s_0 \cdot 31^{n-1} + s_1 \cdot 31^{n-2} + \ldots + s_{n-1} \)
  computed modulo \( 2^{32} \) as in Java int arithmetic.
- To convert to a table index in \( 0 \ldots N - 1 \), compute \( h(s) \% N \) (but don't use table size that is multiple of 31!)
- Not as hard to compute as you might think; don't even need multiplication!
  ```java
  int r; r = 0;
  for (int i = 0; i < s.length (); i += 1)
    r = (r << 5) - r + s.charAt (i);
  ```

Hash Functions: Other Data Structures I

- Lists (ArrayList, LinkedList, etc.) are analogous to strings: e.g., Java uses
  ```java
  hashCode = 1; Iterator i = list.iterator();
  while (i.hasNext()) {
    Object obj = i.next();
    hashCode =
    31*hashCode
    + (obj==null ? 0 : obj.hashCode());
  }
  ```
- Can limit time spent computing hash function by not looking at entire list. For example: look only at first few items (if dealing with a List or SortedSet).
- Causes more collisions, but does not cause equal things to go to different buckets.
Hash Functions: Other Data Structures II

- Recursively defined data structures ⇒ recursively defined hash functions.
- For example, on a binary tree, one can use something like
  \[
  \text{hash}(T) = \begin{cases} 
  0 & \text{if } T = \text{null} \\
  \text{someHashFunction}(T.\text{label}) + 255 \times \text{hash}(T.\text{left}) + 255 \times 255 \times \text{hash}(T.\text{right}) & \text{otherwise}
  \end{cases}
  \]
- Can use address of object ("hash on identity") if distinct (\(!=\)) objects are never considered equal.
- But careful! Won’t work for Strings, because \(\text{.equal}\) Strings could be in different buckets:
  \[
  \text{String H = "Hello", S1 = H + ", world!", S2 = "Hello, world!";}
  \]
- Here \(S1.\text{equals}(S2)\), but \(S1 \neq S2\).

What Java Provides

- In class \(\text{Object}\), is function \(\text{hashCode()}\).
- By default, returns address of \(\text{this}\), or something similar.
- Can override it for your particular type.
- For reasons given on last slide, is overridden for type \(\text{String}\), as well as many types in the Java library, like all kinds of \(\text{List}\).
- The types \(\text{Hashtable}, \text{HashSet},\) and \(\text{HashMap}\) use \(\text{hashCode}\) to give you fast look-up of objects.

Characteristics

- Assuming good hash function, add, lookup, deletion take \(\Theta(1)\) time, amortized.
- Good for cases where one looks up equal keys.
- Usually bad for range queries: "Give me every name between Martin and Napoli." [Why?]
- But sometimes OK, if hash function is monotonic (i.e., when key \(k_1 > k_2\), then \(h(k_1) \geq h(k_2)\)). For example,
  - Items are time-stamped records; key is the time.
  - Hashing function is to have one bucket for every hour.
- Hashing is probably not a good idea for small sets that you rapidly create and discard [why?]

Comparing Search Structures

Here, \(N\) is \#items, \(k\) is \#answers to query.

<table>
<thead>
<tr>
<th>Function</th>
<th>Unordered List</th>
<th>Sorted Array</th>
<th>Bushy Search Tree</th>
<th>&quot;Good&quot; Hash Table</th>
<th>Heap</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{find})</td>
<td>(\Theta(N))</td>
<td>(\Theta(lg N))</td>
<td>(\Theta(lg N))</td>
<td>(\Theta(1))</td>
<td>(\Theta(N))</td>
</tr>
<tr>
<td>(\text{add})</td>
<td>(\Theta(1))</td>
<td>(\Theta(N))</td>
<td>(\Theta(lg N))</td>
<td>(\Theta(1))</td>
<td>(\Theta(lg N))</td>
</tr>
<tr>
<td>range query</td>
<td>(\Theta(N))</td>
<td>(\Theta(k + lg N))</td>
<td>(\Theta(k + lg N))</td>
<td>(\Theta(N))</td>
<td>(\Theta(N))</td>
</tr>
<tr>
<td>(\text{find largest})</td>
<td>(\Theta(N))</td>
<td>(\Theta(1))</td>
<td>(\Theta(lg N))</td>
<td>(\Theta(N))</td>
<td>(\Theta(1))</td>
</tr>
<tr>
<td>(\text{remove largest})</td>
<td>(\Theta(N))</td>
<td>(\Theta(1))</td>
<td>(\Theta(lg N))</td>
<td>(\Theta(N))</td>
<td>(\Theta(lg N))</td>
</tr>
</tbody>
</table>